Novel Charmonium and Bottomonium Spectroscopies due to Deeply Bound Hadronic Molecules from Single Pion Exchange

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Abstract. Pion exchange in S-wave between hadrons that are themselves in a relative S-wave are shown to shift energies by hundreds of MeV. In the case of charmed mesons $D, D^*, D_0, D_1$, a spectroscopy of quasi-molecular states may arise consistent with enigmatic charmonium states observed above 4 GeV in $e^+e^-$ annihilation. A possible explanation of $Y(4260) \rightarrow \psi\pi\pi$ and $Y(4360) \rightarrow \psi'\pi\pi$ is found. Searches in $D\bar{D}\pi$ and $B\bar{B}\pi$ channels are recommended to test this hypothesis. An exotic $1^{-+}$ in $D\bar{D}\pi\pi$ (non $D^*\bar{D}^*$) is predicted.

Keywords: Anomalous States, Molecular Models, Meson Exchange

INTRODUCTION

The last decade has seen the appearance of many anomalous states in the charm mass region, the first observed state being the $X(3872)$[1] (a thorough review of the observed states is beyond the scope of this proceedings). Many of these states do not fit easily within the quark model spectrum[2], and as such are possibly non-$q\bar{q}$. Naturally the possibility of non-$q\bar{q}$ states and the resulting theoretical implications have sparked intense interest in modeling these states including molecular models, tetraquark models, and hybrid models. This work focuses on a molecular model.

It was pointed out long ago that one pion exchange (OPE) might bind pairs of flavored mesons into molecular states called “deusons”[3, 4]. These initial studies suggested, much as in the case of the deuteron, that the binding energies would be of order 1 MeV. With the discovery of the $X(3872)$ which is nearly degenerate with the mass of $D$ and $D^*$, it has been popular to model anomalous states as bound states of nearby 2 meson thresholds.

However, it has recently been pointed out that OPE might bind certain pairs of mesons very deeply[5]. In particular the pair of mesons must be of opposite parity for deep binding to be possible. Liu et. al.[6] were the first to apply this idea to an isovector deuson, but the channel they considered has a repulsive core and their technique suggested no binding. If one simply considered the isoscalar channel, deep binding resulted[5].

In this proceedings we preliminarily report on our efforts[7] to understand the implications of S-wave pion exchange for the $1^{-+}$ channel, focusing specifically on the hidden charm combination of $D_1$ and $D^*$. First, we describe the chiral model, its parameterization and neglected effects for this system. Then we present the results of solving the Schrödinger equation for this system and in a variety of channels. Finally we give our conclusions.

MODEL

Chiral Lagrangians provide a systematic way of describing the coupling between pairs of the flavored charm mesons and the pion field. We accept the chiral potential derived in ref.[6] for a $D_1D^*$ (from here on $D_1\bar{D}^*$ will be taken to imply inclusion of the charge conjugate state molecular system:

$$V_S(q) = \frac{\hbar^2}{2f_\pi^2} \frac{(m_{D_1} - m_{D^*})^2}{|q|^2 + \mu^2 + i\epsilon} (\tau_i \cdot \tau_j) \mathcal{F}(q)^2. \quad (1)$$

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where \( h/(\sqrt{2}f_\pi) \) is the \( D_1D^*\pi \) coupling constant (up to a phase), \( f_\pi = 132 \text{ MeV} \), \( q \) is the exchanged three-momentum, \( \mu^2 \equiv -(m_{D_1} - m_{D^*})^2 + m_{\pi}^2 (\mu^2 < 0 \text{ for the } D_1D^* \text{ system}) \), and \((\tau_i, \tau_j)\) is the usual contraction of Pauli matrices resulting from the exchange of an isovector by two isospin-half particles. \( \mathcal{F} \) is the model dependent form-factor which regulates the potential and would be unity in the chiral model.

We make several additional assumptions which restrict the dynamics of the system to the potential given by Eq. (1). We assume that the inelastic vertices \((D_1 \to D^*\pi, \text{etc.)} \) are dominant and give only an S-wave contribution. Also, due to \( \mu^2 < 0 \) the intermediate pion can go on-shell. This should give an imaginary potential and a width with the bound state system. For now, we ignore this additional complication.

Because our model is focused on the inelastic vertices, the couplings are simply connected to the real decay process \( D_1 \to D^*\pi \). The chiral formula for that decay width is[8]:

\[
\Gamma(D_1^0 \to D^{*+}\pi^-) = \frac{h^2}{8\pi f_\pi} \frac{|q|m_{D^*}^2}{m_{D_1}^2} (m_{D_1}^2 - m_{D^*}^2) \times \frac{1}{3} \left( 2 + \frac{(m_{D_1} + m_{D^*})^2}{4m_{D_1}^2m_{D^*}^2} \right). \tag{2}
\]

The total width is \( 384_{-110}^{+130}\text{ MeV}[9] \) which we assume is saturated by the \( D_1 \to D^*\pi \). A charged decay width, Eq. (2), is related to the total decay width by a simple factor of \( \frac{3}{2} \)[10]. Plugging in the numbers, we would naively extract \( h = 0.80_{-0.17}^{+0.20} \).

Generally processes such as \( D_1 \to D^*\pi \) include form-factors which suppress the exclusive single pion decay relative to other modes as the \( |q| \) increases. Therefore, to estimate this effect, we incorporate a quark model form-factor,

\[
\mathcal{F}(q) = \left( 1 - \frac{2}{9} \frac{q^2}{\beta^2} \right) \exp \left\{ - \frac{q^2}{12\beta^2} \right\}, \tag{3}
\]

and everywhere replace \( h \) with \( h\mathcal{F} \). \( \beta \) is the width of the SHO wavefunction and we take \( \beta = 0.4 \text{ GeV} \). Effectively this renormalizes \( h \) by a factor of \( 1/\mathcal{F}(|q|) \), giving \( h = 1.0_{-0.3}^{+0.7} \).

Having a value for the parameter \( h \), we can now focus on taking the potential, Eq. (1), and transforming it into coordinate space. Under a Fourier transform, we would expect the potential to become complex due to the possibility of real pion emission. We ignore that possibility for now and take only the real part of the Fourier transformed potential:

\[
V_S(r) = \frac{h^2(m_{D_1} - m_{D^*})^2}{8\pi f_\pi} \frac{\cos(|\mu|r)}{r} (\tau_i, \tau_j). \tag{4}
\]

The mesons are spatially extended objects. As such, the potential will be regulated by the convolution of their wave functions, making it finite near the origin. Following refs.[4, 11] we introduce a dipole form-factor with the static approximation,

\[
\mathcal{F} = \left( \frac{\Lambda^2 - m_{\pi}^2}{\Lambda^2 - q^2} \right) \approx \left( \frac{\Lambda^2}{\Lambda^2 + \mu^2 - m_{\pi}^2 + q^2} \right). \tag{5}
\]

The parameter \( \Lambda \) is entirely phenomenological and often takes values between .8 and 2 GeV in the literature. When incorporated into the potential and Fourier transformed, the coordinate space potential becomes,

\[
V_S(r) = \frac{h^2(m_{D_1} - m_{D^*})^2}{8\pi f_\pi^2} \left[ \frac{\cos(|\mu|r)}{r} - e^{-Xr} - \frac{(\Lambda^2 - m_{\pi}^2)}{2X} e^{-Xr} \right] (\tau_i, \tau_j). \tag{6}
\]

where \( X^2 \equiv \Lambda^2 + \mu^2 - m_{\pi}^2 = \Lambda^2 - (m_{D_1} - m_{D^*})^2 \).

The parameter \( \Lambda \) can have a profound impact on the potential which we plot as a function of \( r \) in Fig. 1. The solid black line is the point particle potential. We decrease \( \Lambda \) from 1.5 GeV to .5 GeV and wipe out the attractive core of the potential and replace it with a repulsive core. This means that we have total control over whether the potential is attractive near the origin and broad control over how attractive it is. Worse, in the isovector case we could take a potential with a repulsive core and generate strong attraction. These extremes only appear to occur for \( \Lambda \lesssim .8 \text{ GeV} \) and we will exclude these values from consideration.

The potentials in the \( C = \pm \) and isovector/isoscalar channels are related by a simple constant. The potentials of isovector and isoscalar channels are related by a \( \tau_i, \tau_j \) factor while the potential in channels with opposite charge conjugation are related by a relative phase and the double charm system is degenerate with the \( C = - \) system (for a discussion of the charge conjugation phase see ref.[12]). For reference we plot the potential in all for isospin/charge conjugation channels in Fig. 2.
FIGURE 1. The potential, Eq. (6), plotted against $r$ in the isoscalar $1^{-}$ channel for $h = 0.8$ and a variety of $\Lambda$s. The solid line is the point particle result – in effect $\Lambda \to \infty$; the dashed line is the result for $\Lambda = 1.5$ GeV; the dash-dot line is for $\Lambda = 1.0$ GeV; the dash-dot-dot line is for $\Lambda = 0.75$ GeV; and the dotted line is for $\Lambda = 0.5$ GeV.

FIGURE 2. The potential, Eq. (6), plotted against $r$ in the various $1^{-}$ channels for $h = 1$ and $\Lambda = 1$ GeV. The dotted lines are the isovector potentials while the solid lines are the isoscalar potentials. The left panel shows the $C = -$ potentials and the right panel gives the $C = +$ potentials.

RESULTS

We present the results for the binding energy as a function of $h$ and $\Lambda$ for the 1S and 2S isoscalar $1^{-} D_1 \bar{D}^*$ states in the left and right panels of Fig. 3. The horizontal axes are $1/\Lambda$ so that the the origin corresponds to the point-like case. As one can see, the ground state binding energy falls off rapidly with decreasing $\Lambda$. Eventually the ground state binding energy finds a stable point and remains at approximately that energy for the rest of the considered values of $\Lambda$. This behavior is sharply contrasted by the binding energy of the 2S state. The 2S binding energy is initially insensitive to a decrease in $\Lambda$ before falling steeply and finally becoming insensitive again.

This behavior can be understood from the behavior of the potential in Fig. 1. As $\Lambda$ is decreased from $\infty$, the potential is increasingly regulated. This manifests as overwriting the initial attraction from the potential and eventually replacing it by an entirely repulsive core interaction for $\Lambda \lesssim 800$ MeV. Thus we would expect a steep fall off in ground state binding energy as $\Lambda$ is decreased. In contrast, the 2S state is bound primarily by the second attractive well, which is unaffected by decreasing $\Lambda$ as long as $\Lambda \gtrsim 800$ MeV. Thus, we would expect the 2S binding energy to be relatively stable against decreasing $\Lambda$. At some point, which is $h$ dependent, there will no longer be enough attraction in the first attractive well to bind the system, and so the ground state will begin to require presence in the second attractive well in order to bind, displacing the 2S state and decreasing its binding energy. When the first attractive well is completely overwritten, both the 1S and 2S states should have a relatively stable binding energy as the attractive wells (second and third) which bind them are stable against decreased $\Lambda$. Indeed the binding energies of the 1S state decrease slightly as $\Lambda \to 500$ MeV corresponding with the alteration of the second attractive well in Fig. 1.

We present our results for the $D_1 \bar{D}^*$ isospin and charge conjugation channels with $\Lambda = 1$ GeV in Table 1. First, we
Y should be in the vicinity of the Y system respectively. If we presume these two states are dominated by a molecular of all degenerate. Significant binding is found in all channels, usually for all values of $h$ the bottomonium states are consistently more deeply bound. This is what would be expected when binding a system.

Interestingly for $h = 1.3$ and $\Lambda = 1$ GeV, the isoscalar $1^{-+}$ system has three bound states, the first two of which are consistent with the $Y(4260)$[13] and $Y(4360)$[14] as the ground state and first radially excited state of a $D_1\bar{D}^*$ system respectively. If we presume these two states are dominated by a molecular $D_1\bar{D}^*$ and bound by the mechanism described here we can make some further predictions. Perhaps the most interesting is a deeply bound, exotic $1^{-+}$ which should be in the vicinity of the $Y(4260)$. We would also expect an exotic $0^{-+}$ isoscalar degenerate with the $Y(4260)$ and $Y(4360)$ and an isovector that’s less bound in both the charmonium-like and manifestly charmed channels. The $Y(4260)$ and $Y(4360)$ should have manifestly charmed partners. There’s a large spectrum of crypto-exotic states as well in this model. Also, the $1^{-+}$ isovector is compatible, as a charge conjugate partner, with the $Z(4430)^+[15]$.

<table>
<thead>
<tr>
<th>State</th>
<th>Isospin</th>
<th>$h = 0.8$</th>
<th>$h = 0.9$</th>
<th>$h = 1.0$</th>
<th>$h = 1.1$</th>
<th>$h = 1.2$</th>
<th>$h = 1.3$</th>
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</thead>
<tbody>
<tr>
<td>1S $(0, 1, 2)^-$</td>
<td>0</td>
<td>12</td>
<td>20</td>
<td>29</td>
<td>60</td>
<td>110</td>
<td>160</td>
</tr>
<tr>
<td>2S</td>
<td>1.6</td>
<td>3.6</td>
<td>23</td>
<td>39</td>
<td>51</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>3S</td>
<td>–</td>
<td>0.7</td>
<td>6.7</td>
<td>11</td>
<td>15</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>1S $(0, 1, 2)^{-+}$</td>
<td>1</td>
<td>4.2</td>
<td>8.8</td>
<td>15</td>
<td>21</td>
<td>29</td>
<td>38</td>
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<tr>
<td>2S</td>
<td>–</td>
<td>–</td>
<td>23</td>
<td>39</td>
<td>51</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>3S</td>
<td>–</td>
<td>0.7</td>
<td>6.7</td>
<td>11</td>
<td>15</td>
<td>21</td>
<td></td>
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<tr>
<td>1S $(0, 1, 2)^{-+}$</td>
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<td>67</td>
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<td>120</td>
<td>150</td>
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<td>8.1</td>
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<td>19</td>
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<td>6.1</td>
<td>10</td>
<td>14</td>
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<td>1.6</td>
<td>3.4</td>
<td>5.9</td>
<td>8.9</td>
</tr>
<tr>
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<td>–</td>
<td>–</td>
<td>0.1</td>
<td>0.4</td>
<td>0.9</td>
<td></td>
</tr>
</tbody>
</table>

For completeness we consider the application of our idea to the bottomonium analogs. There’s no data on the width of $B_1 \to B^* \pi$ and so we assumed it was identical to the charmed case as a first approximation. As can be seen from Table 2 the bottomonium states are consistently more deeply bound. This is what would be expected when binding a system.
with a heavier mass under otherwise identical conditions. This would suggest that the bottom system would have a
rich spectrum and may provide an explanation for some resonances, such as the $\Upsilon(10.86) \rightarrow \Upsilon\pi\pi$ enhancement[16].

CONCLUSIONS

In general we find that deeply bound molecules in the $D_1\bar{D}^*$ system should occur as a result of $\pi$ exchange in S-wave,
leading to a potentially rich spectroscopy. Whether such states are narrow enough to show up above background is a
question that experiment may resolve. We note however that the emerging data on the $1^{-+}$ states known as $Y(4260)$
and $Y(4360)$ are consistent with being examples of these molecular states. If the identification is confirmed then several
manifestly exotic states are expected along with a number of crypto-exotic states as shown in Table 1.

As long as one picks and chooses which datum one will fit, it is possible to fit it in a molecular model. A reason is
that binding energies are very sensitive to parameters that are not well determined elsewhere. Thus a model designed
to fit a single state has limited appeal. The more relevant test is whether a group of states share a common heritage,
and their production or decay properties reveal the underlying molecular structure. In the particular case here, one can
fit the masses and decay widths in tetraquark, hybrid and molecular models. As such the existence of these states does
discern among them.

The saturation of the $D_1$ width by the $D^*\pi$ channel implies that $D\bar{D} + 3\pi$ is the dominant decay mode of the
molecular state. Thus, if our dynamics is significant to the $Y(4260)$ and $Y(4360)$ significant strength must be found
in the $D\bar{D} + 3\pi$ channels. If that decay channel does not have a significant branching fraction, this model is wrong.
Therefore, if our hypothesis is correct, we expect significant strength in the $e^+e^- \rightarrow D\bar{D} + 3\pi$ channels in the 4 to 5
GeV region. Such evidence may already exist among the data sets for $e^+e^-$ annihilation involving ISR at BaBar and Belle.

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