Finite width induced modification to the electromagnetic form factors of spin-1 particles.

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Abstract. The inclusion of the unstable features of a spin-1 particle, without breaking the electromagnetic gauge invariance, can be properly accomplished by including higher order contributions as done in the so-called fermion loop scheme (for the $W$ gauge boson), and the boson loop scheme (for vector mesons). This induces a non trivial modification to the electromagnetic vertex of the particle. Considering the modified electromagnetic vertex, we obtain general expressions for the corresponding multipoles as a function of the mass of the particles in the loop. For the $W$ gauge boson no substantial deviations from the stable tree level case is observed. For the $\rho$ meson the mass of the particles in the loop makes a significant effect.

Keywords: Form factors, unstable particles, gauge invariance

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1. INTRODUCTION

The description of unstable states in an electromagnetic gauge invariant way requires the modification of the propagator and the electromagnetic vertex, the unstable feature is parameterized by the finite decay width, $\Gamma$. Approaches like the so called fermion loop scheme [1, 2, 3] and the boson loop scheme [4] (suitable for the $W$ and $\rho$ bosons respectively) have been developed to address this problem. These schemes are based in two main observations: In quantum field theory the width is naturally included in the imaginary part of the self-energy of the particles and, the Ward identity is respected at all orders in perturbation theory. These facts are exploited in these schemes by including the resummation of the fermion/boson loops in the propagator and the corrections in the electromagnetic vertex. Then, imaginary part of the fermion/boson loops introduces the tree level width in the gauge boson propagator and, the gauge invariance is not violated since the fermion/boson loops obey the ward identity order by order.

At tree level the propagator for a vector boson of mass $M_V$ can be set as

$$D_{\mu\nu}^{0}(q) = -\frac{i T^{\mu\nu}(q)}{q^2 - M_V^2} + \frac{i L^{\mu\nu}(q)}{M_V^2},$$

where $T^{\mu\nu}(q) \equiv g^{\mu\nu} - q^\mu q^\nu / q^2$ and $L^{\mu\nu}(q) \equiv q^\mu q^\nu / q^2$, are the transversal and longitudinal projectors, respectively.

The vertex for the process $V(q_1) \rightarrow V(q_2) \gamma(k)$ is defined from the electromagnetic current

$$<V(q_2)|J^\mu(0)|V(q_1)> = A^\nu_\lambda(q_2)A^\lambda_\mu(q_1)\Gamma^{\mu\nu\lambda}.$$

At tree level, the vertex can be set as that given by the standard model for the $W$ boson:

$$\Gamma^{\mu\nu\lambda} = g^{\mu\nu}(q_1 + q_2)^\lambda - g^{\mu\lambda}(q_1 + k)^\nu - g^{\nu\lambda}(q_2 - k)^\mu;$$

These expressions satisfy the Ward identity

$$k_\mu \Gamma_{0}^{\mu\nu\lambda} = \left[iD_{0}^{\nu\lambda}(q_1)\right]^{-1} - \left[iD_{0}^{\nu\lambda}(q_2)\right]^{-1};$$

Upon the inclusion of the finite width of the boson, by considering the loop contributions, the propagator is modified in a generic form as:

$$D^{\mu\nu}(q) = -\frac{i T^{\mu\nu}(q)}{q^2 - M_V^2 + i\text{Im}T(q^2)} + \frac{i L^{\mu\nu}(q)}{M_V^2 - i\text{Im}L(q^2)},$$
where $\text{Im} \Pi (q^2)$ and $\text{Im} \Pi (q^2)$, are the transverse and longitudinal part of the absorptive contribution of the self-energy induced by the particles in the loop. Similarly, the vertex becomes
\[
t e \Gamma^{\mu \nu \lambda} = t e (\Gamma_0^{\mu \nu \lambda} + \Gamma_1^{\mu \nu \lambda}),
\]
where $\Gamma_1^{\mu \nu \lambda}$ contains the loop corrections. The Ward identity relates the loop contributions by requiring to satisfy
\[
k_\mu \Gamma_1^{\mu \nu \lambda} = t \text{Im} \Pi^{\nu \lambda} (q_1) - t \text{Im} \Pi^{\nu \lambda} (q_2).
\]

For a boson like the $W$, the scheme consider that such loops are produced by fermions, while for vector mesons, like the $\rho$, the natural particles in the loop are bosons.

Under these schemes, the electromagnetic vertex $VV\gamma$ is modified respect to the tree level form in a non-trivial way. This implies that the electromagnetic structure itself suffers modifications. In the present work we analyze the most general results for the electromagnetic vertices obtained in the loop schemes, which include the mass of the particles in the loops, and extract the expressions for the modified form factors. We compare our results for the magnetic dipole moment (MDM) and electric quadrupole moments of these particles with others computed in the literature for contributions of different nature. The gauge invariance along with the modification in the propagator, inherent to the schemes, keep the electric charge free of radiative correction contributions.

## 2. FINITE WIDTH EFFECTS

The CP conserving electromagnetic vertex can be decomposed in the following general Lorentz structure
\[
\Gamma^{\mu \nu \lambda} = \alpha (k^2) \gamma^{\nu \lambda} (q_1 + q_2)^\mu + \beta (k^2) (g^{\mu \nu} k^\lambda - g^{\mu \lambda} k^\nu) - \gamma (k^2) (q_1 + q_2)^\mu k^\nu k^\lambda.
\]

where $\alpha (k^2)$ is the electric charge form factor, $\beta (k^2) \equiv 1 + \kappa + \lambda$ is the magnetic dipole moment form factor (in e/2$M_V$ units) and the electric quadrupole form factor is $\kappa - \gamma (k^2) M_V^2 \equiv \kappa - \lambda$ (in e$/2M_V^2$ units) (the $\kappa$ and $\lambda$ parameters are commonly used in the literature) [5, 6]. The static electromagnetic properties of a particle are defined for the case when the particle is on-shell and in the limit of $k \to 0$. At tree level, for example, the standard model predicts for the $W$ to have $\alpha (0) = 1$, $\beta (0) = 2$ and $\gamma (0) = 0$ (corresponding to $\kappa = 1$ and $\lambda = 0$). Deviations from these values are generically called anomalous and are produced by the inclusion of higher order contributions [7]. In the present case such contributions are exclusively those required to maintain the electromagnetic gauge invariance.

### 2.1. W boson form factors

Let us identify the modification to the $W$ boson form factors introduced by the correction to the electromagnetic vertex. For that purpose we consider the explicit expression of the vertex obtained in ref. [3], where the mass $m$ of the emitting particles in the loop and its weak partner $m'$ have been considered. The Lorentz structure, transversality and on-shell condition of the boson along with the proper limit for $k \to 0$ leads to the following expressions for the form factors, defined in equation (8):

- Electric charge
  \[
  \alpha (0) = 1 + \tilde{Q}_e \sum_i \frac{\Gamma_i}{M_W} \left( M_W^4 (M_W^4 + \Delta^4) + 2M_W^4 (\Sigma^4 - M_W^4) - 2\Delta^8 \right),
  \]
  where $\Sigma^2 \equiv m^2 + m'^2$, $\Delta^2 \equiv m^2 - m'^2$ and $Q_e$ is the electric charge of the radiating particle in the loop. We have defined $\Gamma_i/M_W \equiv g_i^2/4\pi$ as the partial decay width for the modes corresponding to the particles in the loop, $g_i$ denotes the coupling. This expression is exact in the case of massless particles. A sum over all the allowed flavors and color degeneracies is explicitly included. Since the schemes consider the particles in the loop to be on-shell, the flavors include all the leptons and the $u, d, s$ and $c$ quarks.

- Magnetic dipole moment
\[
\beta(0) = 2 + iQ \sum_i \frac{\Gamma_i}{M_W} \left\{ \Delta^8 - \Delta^4 (4m^2 + m'^2) M_W^2 + m^2 (3m^2 + m'^2) M_W^4 + 2M^2M_W^6 - 2M_W^8 \right\} \frac{M_W^6}{M_W^6 \sqrt{-2\Sigma^2 M_W^2 + M_W^4 + \Delta^4}}
\]

(10)

• Electric quadrupole moment

\[
\gamma(0) = -i \sum_i Q_i \frac{\Gamma_i}{M_W} \frac{M_W^4 (\Sigma^2 + \Delta^2)(M_W^2 - \Sigma^2 - 2\Delta^2) + 2M_W^4 \Delta^4(2\Sigma^2 + \Delta^2) - 2\Delta^8}{M_W^6 \sqrt{-2\Sigma^2 M_W^2 + M_W^4 + \Delta^4}}
\]

(11)

2.2. \(\rho\) Meson

Proceeding along the same lines of the \(W\) gauge boson, we obtain the modifications to the \(\rho\) meson form factors, taking the complete expressions for the modified vertex computed in [4]:

• Electric charge

\[
\alpha_\rho(0) = 1 + i \frac{\Gamma_\rho}{M_\rho} \frac{-2\Delta^4 + \Sigma^2 M_\rho^2 + M_\rho^4}{M_\rho^6} \sqrt{\Delta^4 + M_\rho^4 - 2\Sigma^2 M_\rho^2};
\]

(12)

where, as in the \(W\) case, \(\Gamma_\rho/M_\rho = g^2/48\pi\), \(g\) is the effective \(\rho\pi\pi\) coupling constant, and \(\Delta^2 \equiv m_\pi^2 - m_\rho^2\) and \(\Sigma^2 \equiv m_\pi^2 + m_{\rho'}^2\).

• Magnetic dipole moment

\[
\beta_\rho(0) = 2 + i \frac{\Gamma_\rho}{M_\rho} \frac{2M_\rho^4 + 2\Delta^4 - M_\rho^2 (\Sigma^2 + 3\Delta^2)}{2M_\rho^6} \sqrt{\Delta^4 + M_\rho^4 - 2\Sigma^2 M_\rho^2}.
\]

(13)

• Electric quadrupole moment

\[
\gamma_\rho(0) = -i \frac{\Gamma_\rho}{M_\rho} \frac{M_\rho^4 (\Sigma^2 + \Delta^2)(M_\rho^2 - \Sigma^2 - 2\Delta^2) + 2M_\rho^4 \Delta^4(2\Sigma^2 + \Delta^2) - 2\Delta^8}{M_\rho^6 \sqrt{\Delta^4 + M_\rho^4 - 2\Sigma^2 M_\rho^2}}.
\]

(14)

2.3. Chiral limit

The chiral limit correction to the vertex is known to be proportional to the tree level, in both the Fermion and boson loops corrections [4], therefore in this limit we can write the modification to the form factors in a generic form as follows:

\[
\Gamma^{\mu\nu\lambda} = (1 + i \frac{\Gamma}{M_V}) \Gamma^{\mu\nu\lambda}_0
\]

(15)

3. NUMERICAL RESULTS

The correction to the vertex seems to induce a modification to the electric charge. However, since the Ward identity is fulfilled, the modification to the vertex is followed by a modification to the propagator, which produces an exact
cancelation of the correction to the electric charge. Let us illustrate this point in more detail, for definiteness we consider the expression in the chiral limit: The modified propagator can be seen as a renormalization of the vector field by the inclusion of the finite width $\Gamma$. Therefore, gauge invariance requires that $Z\alpha'(k^2) = 1$, which is the case, and the electric charge does not receive any correction. An analysis for unstable spin-1/2 particles has also been performed in ref. [9], pointing out to complex renormalization factors as a requirement for properly defined physical quantities. Further considerations on the renormalizability of the wave function can be seen in ref. [10].

The proper values of the modifications to the form factors are then found by the expressions given in the previous section divided by $\Gamma$. As a by product, the mean square radius can be computed following [11] as, $<R^2> = (\kappa + \lambda)/M F_0$.

Using the general expressions for the finite width modified form factors, we have computed their corresponding numerical values. Since the particles in the loops are restricted to be on-shell, for the $W$ we have included all the leptons and the $u, d, s$ and $c$ quarks. In table 1, we present the corresponding results. It is worth to mention that, for the $W$ case, the general results are similar to those obtained in the chiral limit.

As a reference to the magnitude of the modification, we recall several results obtained for contributions of different nature: Ref. [12] finds a correction to the MDM induced by the Higgs of $\mu = 2 - 0.0151$, reference [13] finds a correction from quarks, leptons and Higgs loops in the SM of $\mu = 2 + 0.00258$.

For the $\rho$ meson, pions are the only on-shell particles allowed in the loops. In table 2, we present the numerical results for the multipoles. In this case the pion to $\rho$ mass ratio is not as small as the corresponding for the fermion to $W$ gauge boson, and therefore a significant effect from the mass of the particles in the loop is expected. For comparison, the $\rho$ meson MDM computed including dynamical quark contributions are: $2 + 0.14$ (in a covariant framework [15]), $2 - 0.08$ (in a light-front quark model [16]) and $2 - 0.01$ (using Schwinger-Dyson equations [17]).

Regarding the modification to the mean square radius, we obtain a deviation of $-0.0016 f m$ respect to the normal value (defined for $\kappa = 1$ and $\lambda = 0$), which can be compared, for example, with the one computed in reference [17], where they observe a correction of 0.06 f m, due to the inclusion of the pion contribution respect to a pure quark-antiquark state.

### 4. Conclusions

The inclusion of the unstable features of spin-1 particles, without breaking the electromagnetic gauge invariance, can be properly accomplished by including higher order contributions, as done in the so-called fermion loop scheme for the $W$ gauge boson, and the boson loop scheme for the vector mesons. In these approaches a non trivial modification to the electromagnetic vertex of the particle is induced. In this work we have considered the general expressions for
such vertices, which include the explicit mass dependence of the particles in the loop, to extract the corresponding modifications to the multipole structure. The modifications in both the propagator and electromagnetic vertex in combination with the Gauge invariance show that the properly defined form factors can be seen as accompanied by a complex renormalization of the vector fields. The electromagnetic properties of the $W$ are predicted by the symmetry structure of the standard model, while for the $\rho$ several predictions exist, based on effective models of the strong interaction binding the quarks and from lattice-QCD. Therefore, these properties can help to understand the symmetry structure and interactions of the fundamental particles. In order to draw definite conclusions, a complete study of the effects due to the instability is mandatory. Our numerical results for the $W$ gauge boson multipoles shows no substantial deviations from the stable tree level case. For the $\rho$ meson, the mass of the particles in the loop can make a significant effect. Our results suggest that the unstable nature of the vector mesons can be as relevant as other dynamical effects and should be considered in refinements when accounting for their properties. A more detailed analysis of the results will be published elsewhere [18].

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