Longitudinal Rescaling, Confinement and Soft Scattering

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This talk is about ideas on confinement (starting about five years ago) and high-energy soft scattering (in the last few years).

1. The starting point was Mandelstam’s approach to confinement in the axial gauge.

2. 1+1-dimensional integrability and exact form factors are useful (and the techniques parallel some in condensed-matter physics).

3. Connection with decades-old ideas of Verlinde and Verlinde and by McLerran and Venugopalan (Color-Glass-Condensate) in which the gauge-field action is longitudinally rescaled.
Some papers...

1., 2. discuss (3+1)-dimensional collisions. The others concern confinement in 2+1 dimensions.

1. **Longitudinal Rescaling and High-Energy Effective Actions**, with Jing Xiao,

2. **Near-Integrability and Confinement for High-Energy Hadron-Hadron Collisions**,

3. **Composite Strings in (2+1)-Dimensional Anisotropic Weakly-Coupled Yang-Mills Theory**,  

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4. Glueball Masses in (2+1)-Dimensional Anisotropic Weakly-Coupled Yang-Mills Theory,

5. String Tensions and Representations in Anisotropic 2+1-Dimensional Weakly-Coupled Yang-Mills Theory,

6. Integrable Models and Confinement in (2+1)-Dimensional Weakly-Coupled Yang-Mills Theory,

7. Lattice QCD_{2+1},

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Coupling Constant Renormalization

If the cut-off $\Lambda$ is very large (say of order of the Planck mass) and the coupling is $g$ we can integrate out some degrees of freedom to find an effective theory at some smaller cut-off $\tilde{\Lambda}$. In this effective theory, the coupling is $\tilde{g}$.

In perturbation theory: \[ \frac{1}{\tilde{g}^2} = \frac{1}{g^2} - \frac{11C_N}{48\pi^2} \ln \frac{\Lambda}{\tilde{\Lambda}}. \]

For large $\tilde{\Lambda}$, perturbation theory is good, and the coupling $\tilde{g}$, is small. For small $\tilde{\Lambda}$, perturbation theory can no longer be trusted. It suggests, however, that the coupling $\tilde{g}$ GROWS.

Effective strong-coupling theories have been used to try to understand hadron phenomenology. These include Hamiltonian lattice theories in the late 70’s and AdS/QCD approaches in the last ten years. These are models. They are not real QCD.

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Longitudinally rescaled QCD

Verlinde + Verlinde (’93), McLerran and Venugopalan (’94)

\[ x^L = (x^0, x^3), \quad x^\perp = (x^1, x^2) \]

\[ x^L \rightarrow \lambda x^L, \quad x^\perp \rightarrow x^\perp \]

The center-of-mass energy squared changes as \( s \rightarrow \lambda^{-2}s \). Since we wish to consider high energies, we take \( \lambda \ll 1 \).

Rescaled action:

\[
S = \frac{1}{2g_0^2} \int d^4x \, \text{Tr} \left( \sum_{j=1}^{2} F_{0j}^2 - \sum_{j=1}^{2} F_{j3}^2 + \lambda^{-2} F_{03}^2 - \lambda^2 F_{12}^2 \right),
\]

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]. \]
Hamiltonian:

\[ H = \int d^3x \left[ \frac{g_0^2}{2} \mathcal{E}_\perp^2 + \frac{1}{2g_0^2} \mathcal{B}_\perp^2 + \lambda^2 \left( \frac{g_0^2}{2} \mathcal{E}_3^2 + \frac{1}{2g_0^2} \mathcal{B}_3^2 \right) \right] \]

\[ = H_0 + \lambda^2 H_1. \]

\[ (\partial_\perp \cdot \mathcal{E}_\perp + \partial_3 \mathcal{E}_3 - \rho) \Psi_{\text{Physical}} = 0, \]

Gauge-invariant UV cut-off and \( \lambda \ll 1 \implies \text{mass gap and confinement.} \)

Furthermore \( \lambda = 0 \implies \text{integrability.} \)
We put this theory on a lattice. Then:

\( H_0 \) is a set of 1+1-dimensional SU(\( N \)) principal-chiral sigma models,

\[
\mathcal{L} = \frac{1}{2g_0^2} \int d^2 x \operatorname{Tr} \partial^\mu U^\dagger \partial_\mu U, \quad \mu = 0, 3, 
\]

hence the \( \lambda \to 0 \) limit of the gauge theory, is completely integrable.

We can treat \( H_1 \) as an interaction Hamiltonian, provided

\[
\lambda^2 \ll g_0^{-3} \exp -\frac{4\pi}{g_0^2 N}.
\]
1+1-dimensional PC Sigma Model

Mass Spectrum: \( m_r = m_1 \frac{\sin(\pi r/N)}{\sin(\pi/N)} \), \( r = 1, \ldots, N - 1 \).

“Elementary” soliton-like particles (\( r = 1 \)) are color dipoles (like a quark-antiquark pair). Other (\( r > 1 \)) particles are bound states. The elementary antiparticle has \( r = N - 1 \).

\[ (r=1) \text{ by } (r=1) \text{ S-matrix:} \]

\[ \mathcal{S}_{11}(\theta) = \frac{\sin(\theta/2-\pi i/N)}{\sin(\theta/2+\pi i/N)} \mathcal{S}_{\text{CGN}}(\theta) \otimes \mathcal{S}_{\text{CGN}}(\theta), \]

where

\[ \mathcal{S}_{\text{CGN}}(\theta) = \frac{\Gamma(i\theta/2\pi+1)\Gamma(-i\theta/2\pi-1/N)}{\Gamma(i\theta/2\pi+1-1/N)\Gamma(-i\theta/2\pi)}(1 - \frac{2\pi i}{N\theta}P). \]

Other S-matrix elements are found by fusion and crossing (\( \theta \rightarrow \pi i - \theta \)).

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Gauss Law + Mass Gap $\implies$ Confinement

One coupling $g'' = g_0/\lambda$, is strong, so we haven’t proved confinement in (3+1)-dimensional QCD ($\mathcal{B}_3$ fluctuates wildly). In 2+1 dimensions, there is no $g''$, and confinement takes place at weak coupling.

**Diffraction:**

If $\lambda = 0$, particles move only longitudinally, not transversely, and scattering is only in the forward direction. As $\lambda$ is increased, the diffraction peak broadens.

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The blue bullets are massive soliton-like excitations of transverse electric flux. The red lines are longitudinal electric flux. In meson and baryon states, flux terminates on a quarks.
$d = 2 + 1$ PHASE DIAGRAM

$g_0' = \lambda g_0$

CRITICAL POINT

SOLVABLE

CROSSOVER

STRONG COUPLING

Integrable Line

$g_0$

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Summary of Results: $d=2+1$ and (mostly) $N = 2$

Longitudinal string tension:

$$\sigma_L = \frac{3(g'_0)^4}{8K},$$

$$K = \frac{(g'_0)^2 a^2}{4} + \frac{1}{3m^2\pi^2} \left(\frac{g'_0}{g_0}\right)^4 \exp \left[-2 \int_0^\infty \frac{d\xi}{\xi} e^{-\xi \tanh^2 \frac{\xi}{2}}\right].$$

Transverse string tension:

$$\sigma_{\perp} = \frac{ma}{a} - \frac{2\sqrt{3}}{\pi} \frac{g'_0}{g_0^3 a^2}.$$ 

An exact form factor of the sigma model (Karowski, Weisz 1977) was used to find these string tension corrections.

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Summary of Results: \( d=2+1 \) and (mostly) \( N = 2 \)
(CONTINUED)

Low-lying mass spectrum:

\[
M_n = 2m_1 + \left[ \epsilon_n^{1/3} - \frac{3(3-2\ln 2)\sigma_\perp}{4\pi m} \epsilon_n^{-1/3} \right]^2, \\
\epsilon_n = \frac{3\pi\sigma_\perp(n+\frac{1}{2})}{4m^{1/2}} + \left\{ \frac{3\pi\sigma_\perp}{4m^{1/2}(n+\frac{1}{2})} \right\}^2
+ \frac{1}{8} \left[ \frac{3(3-2\ln 2)\sigma_\perp}{2\pi m} \right]^3 \right\}^{1/2}.
\]

\( k \)-string tensions (for \( N > 2 \)):

- **Casimir law** for longitudinal string tensions.
- **Sine law** for transverse string tensions.
The rescaling $x^L \rightarrow \lambda x^L$, is classical. How can we do a quantum rescaling?

This can be done by an anisotropic renormalization technique, in which longitudinal directions ($x^0, x^3$) are integrated out “more” than transverse directions ($x^1, x^2$).
Rescaling of field theory on a lattice with $\lambda = 1/2$. First, a Kadanoff transformation increases the longitudinal lattice spacing. The spacing is then restored to its original value by a longitudinal rescaling.

$$\mathcal{L}_{\text{eff}} = \frac{1}{4g_{\text{eff}}^2} \text{Tr} \left( F_{01}^2 + F_{02}^2 - F_{13}^2 - F_{23}^2 + \lambda^{-2+\frac{17C_N}{48\pi^2}g_0^2} F_{03}^2 - \lambda^{2+\frac{7C_N}{48\pi^2}g_0^2} F_{12}^2 \right) + \cdots .$$

$$\frac{1}{g_{\text{eff}}^2} = \frac{1}{g_0^2} - \frac{11C_N}{48\pi^2} \ln \frac{\Lambda}{\tilde{\Lambda}} - \frac{C_N \ln \tilde{b}}{64\pi^2} = \frac{1}{\tilde{g}_0^2} \tilde{b} - \frac{C_N}{64\pi^2} \tilde{g}_0^2 + \cdots .$$
Some Problems Under Investigation

In $2+1$ dimensions, we want to

BEAT THE CROSSOVER!

For the $Z_2$ (Ising) gauge theory, this has been done by Konik and Adamov. I am trying to solve this problem by redoing the renormalization, keeping finite parts. This would give a solution of the isotropic theory at weak dimensionless coupling.

Exact SU($N$) sigma-model form factors are necessary, if the results are to be extended to $N = 3$ or even $N = \infty$.

In $3+1$ dimensions, we want the forward elastic scattering amplitude. We need the distribution of soliton-like particles in the transverse plane and in rapidity space.