Molecule-charmonium mixing for the $X(3872)$ in QCD Sum Rules

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Abstract.

We use QCD sum rules to test the nature of the meson $X(3872)$, assumed to be a mixture between charmonium and exotic molecular $[c\bar{q}][q\bar{q}]$ states with $J^P_C = 1^{++}$. We find that only a small range for the values of the mixing angle, $5^\circ \leq \theta \leq 13^\circ$, can provide simultaneously good agreement with the experimental value of the mass and the decay width, within the errors. In this range we get $m_X = (3.77 \pm 0.18)$ GeV and $\Gamma(X \rightarrow J/\psi \pi^+ \pi^-) = (9.3 \pm 0.9)$ MeV.

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INTRODUCTION

The state $X(3872)$ has been first observed by the Belle collaboration in the decay $B^+ \rightarrow X(3872)K^+ \rightarrow J/\psi \pi^+ \pi^- K^+$ [1]. This observation was later confirmed by CDF, D0 and BaBar [2]. The current world average mass is $m_X = (3871.4 \pm 0.6)$ MeV, and the total decay width is $\Gamma < 2.3$ MeV at 90% confidence level. Further studies from Belle, Babar and CDF strongly favor the quantum numbers $J^P_C = 1^{++}$ or $2^{++}$ [3].

It is established that the mass of the state does not fit into constituent quark model predictions [4]. Another important observation is that the decay rates of processes $X(3872) \rightarrow J/\psi \pi^+ \pi^- \pi^0$ and $X(3872) \rightarrow J/\psi \pi^+ \pi^-$ are comparable [3]:

$$\frac{\Gamma(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\Gamma(X \rightarrow J/\psi \pi^+ \pi^-)} = 1.0 \pm 0.4 \pm 0.3.$$  \hspace{1cm} (1)

This ratio indicates a strong isospin and G parity violation, which is incompatible with a $c\bar{c}$ structure for $X(3872)$. These are strong evidences that the $X(3872)$ is not a conventional $q\bar{q}$ state, and attempts to treat this state as a multiquark state are being pursued.

One proposal to describe the quark structure of the $X(3872)$ is a molecular $(D^{*0}\bar{D}^0 - \bar{D}^{*0}D^0)$ bound state with small binding energy [5, 6]. There are also evidences that seem to indicate the existence of a $c\bar{c}$ component in the $X(3872)$ structure. In ref. [7] it was shown that, because of the very loose binding of the molecule, the production rates of a pure $X(3872)$ molecule should be at least one order of magnitude smaller than what is seen experimentally. Also, the recent observation, reported by BaBar [8], of the decay $X(3872) \rightarrow \psi(2S)\gamma$ at a rate: \( \mathcal{B}(X \rightarrow \psi(2S)\gamma)/\mathcal{B}(X \rightarrow \psi\gamma) = 3.4 \pm 1.4 \), is much bigger than the molecular prediction $\Gamma(X \rightarrow \psi(2S)\gamma)/\Gamma(X \rightarrow \psi\gamma) \sim 4 \times 10^{-3}$ [9].

In this work we will follow ref. [10] and consider a mixed charmonium-molecular current to study the $X(3872)$ in the QCD Sum Rule framework. As it will be seen, in order to be consistent with $X$ decay data, we must consider a second mixing between: \( (c\tau) + (D^{*0}\bar{D}^0 - \bar{D}^{*0}D^0) \) and \( (\pi\pi) + (D^+\bar{D}^- - \bar{D}^+D^-) \). With all these ingredients we perform a calculation of the mass of the $X(3872)$ and its decay width into $J/\psi(2\pi)$ and $J/\psi(3\pi)$

THE MASS OF THE $X(3872)$ STATE IN QCD SUM RULES

The mass of a state can be calculated in QCD sum rules [11] from the two-point correlation function:

$$\Pi_{\mu\nu}(q) = i \int d^4x \, e^{iq\cdot x} \langle 0 | T[J^\dagger_\mu(x)J_\nu(0)] | 0 \rangle$$ \hspace{1cm} (2)

The current $J^\dagger_\mu$ in Eq. (2) creates the states with the quantum numbers of the hadron. In this work we will study the $X(3872)$ state considering a mixed charmonium-molecular current. For the charmonium part we use the conventional
Then, by inserting intermediate states for the meson \( X \), the correlation function is written in terms of quark and gluon fields and a Wilson’s operator product expansion (OPE) may be described at both quark and hadron levels. At the hadronic level (the phenomenological side) the correlation current, \( J^{(2a)} \), and from these two currents we build the following mixed charmonium-molecular current for the \( X(3872) \):

\[
J^{(2)}_{\mu}(x) = \bar{c}_{\mu}(x)\gamma_{\mu}c_{a}(x).
\]

The \( D^0 \ D^{\ast 0} \) molecule is interpolated by [12]:

\[
j^{(4a)}_{\mu}(x) = \frac{1}{\sqrt{2}}\left[ \bar{u}_{\mu}(x)\gamma_{\mu}c_{a}(x)\bar{c}_{b}(x)\gamma_{\mu}u_{b}(x) - \bar{u}_{\mu}(x)\gamma_{\mu}c_{a}(x)\bar{c}_{b}(x)\gamma_{\mu}u_{b}(x) \right].
\]

As in ref. [10] we define the normalized two-quark current as

\[
j^{(2a)}_{\mu} = \frac{1}{6\sqrt{2}}(\bar{u}u)j^{(2)}_{\mu},
\]

and from these two currents we build the following mixed charmonium-molecular current for the \( X(3872) \):

\[
J^{(2)}_{\mu}(x) = \sin(\theta)j^{(4a)}_{\mu}(x) + \cos(\theta)j^{(2a)}_{\mu}(x).
\]

The sum rule approach is based on the principle of duality. It consists in the assumption that the correlation function may be described at both quark and hadron levels. At the hadronic level (the phenomenological side) the correlation function is calculated introducing hadron characteristics such as masses and coupling constants. At the quark level, the correlation function is written in terms of quark and gluon fields and a Wilson’s operator product expansion (OPE) is used to deal with the complex structure of the QCD vacuum.

The phenomenological side is treated by first parameterizing the coupling of the axial vector meson \( 1^{++} \), \( X \), to the current, \( J^{(2)}_{\mu} \), in Eq. (6) in terms of the meson-current coupling parameter \( \lambda^{a} \):

\[
\langle 0|J^{(2)}_{\mu}|X \rangle = \bar{c}_{\mu}e_{\mu}.
\]

Then, by inserting intermediate states for the meson \( X \), we can write the phenomenological side of Eq. (2) as

\[
\Pi^{\text{phen}}_{\mu\nu}(q) = \frac{\langle \lambda \rangle^{2}}{m^{2}_{X} - q^{2} - i\epsilon} - g_{\mu\nu} + \frac{2g_{\mu\nu}}{m^{2}_{X}} + \cdots,
\]

where the Lorentz structure projects out the \( 1^{++} \) state. The dots denote higher mass axial-vector resonances. This resonances will be dealt with through the introduction of a continuum threshold parameter \( s_{0} \).

In the OPE side we work up to dimension 8 at the leading order in \( \alpha_{s} \). The light quark propagators are calculated in coordinate-space and then Fourier transformed to the momentum space. The charm quark part is calculated directly into the momentum space, with finite \( m_{c} \), and combined with the light part. The correlator in Eq. (2) can be written as:

\[
\Pi^{(i)}_{\mu\nu}(q) = \int d^{4}x e^{q\mu x}(0|\langle j^{(i)}_{\mu}(x)j^{(j)}_{\nu}(0)|0\rangle).
\]

After making a Borel transform of both sides of the sum rule, and transferring the continuum contribution to the OPE side, the sum rule for the axial vector meson up to dimension-eight condensates can be written as:

\[
\langle \lambda \rangle^{2}e^{-m^{2}_{X}/M^{2}} = \frac{\langle \bar{u}u \rangle}{6\sqrt{2}}\cos^{2}(\theta)\Pi^{(2,2)}_{\mu\nu}(M^{2}) + \frac{\langle \bar{u}u \rangle}{6\sqrt{2}}\sin^{2}(\theta)\Pi^{(4,4)}_{\mu\nu}(M^{2}).
\]

By taking the derivative of Eq. (11) with respect to \( 1/M^{2} \) and dividing the result by Eq. (11) we can obtain the mass of \( m_{X} \) without worrying about the value of the meson-current coupling \( \lambda^{a} \). The expression thus obtained is analyzed numerically using the following values for quark masses and QCD condensates [13, 14]: \( m_{c}(m_{c}) = (1.23 \pm 0.05) \) GeV, \( \langle \bar{u}u \rangle = (-0.23 \pm 0.03) \) GeV, \( \langle \bar{u}G_{\mu\nu}u \rangle = m^{2}_{0}\langle \bar{u}u \rangle, m^{2}_{0} = 0.8 \) GeV², \( \langle G^{2} \rangle = 0.88 \) GeV⁴. We have used \( s_{0}^{1/2} = 4.4 \) GeV and \( \theta = 9^\circ \) to determine the Borel Window. By comparing the contributions of the terms of the OPE grouped by condensate dimensions divided by the RHS of Eq. (11) we can determine that the OPE is converging for values of \( M^{2} \geq 2.6 \) GeV². The upper limit to the value of \( M^{2} \) comes by imposing that the QCD pole
The contribution should be bigger than the continuum contribution. The condition obtained is $M^2 \leq 3.2 \text{ GeV}^2$, but in this case, the dependence on the choice of $\theta$ is very strong. Taking into account the variation of $\theta$ we have determined that, for $5^\circ \leq \theta \leq 13^\circ$, the QCDSR are valid in the following region:

$$2.6 \text{ GeV}^2 \leq M^2 \leq 3.0 \text{ GeV}^2$$  \hspace{1cm} (12)$$

In Fig. 1, we show the $X$ meson mass in this region. We see that the results are reasonably stable as a function of $M^2$. From Fig. 1 we obtain $m_X = (3.80 \pm 0.08) \text{ GeV}$ where the error includes the variation of both $s_0$ and $M^2$. If we also take into account the variation of $\theta$ in the region $5^\circ \leq \theta \leq 13^\circ$ we get:

$$m_X = (3.77 \pm 0.18) \text{ GeV},$$  \hspace{1cm} (13)$$

which is in a good agreement with the experimental value. The value obtained for the mass grows with the value of the mixing angle $\theta$, but for $\theta \geq 30^\circ$ it reaches a stable value being completely determined by the molecular part of the current.

From Eq. (11) we can also obtain $\lambda^u$ by fixing $m_X$ equal to the experimental value ($m_X = 3.87 \text{ GeV}$). Using the same region in $\theta$, $s_0$ and $M^2$ that we have used in the mass analysis we obtain:

$$\lambda^u = (3.6 \pm 0.9) \times 10^{-3} \text{ GeV}^5.$$  \hspace{1cm} (14)$$

**DECAY OF THE X(3872) AND THE THREE POINT CORRELATOR**

In this section we discuss the calculation of the decay of the state $X(3872)$ in QCD sum rules. The width for the decay $X \to J/\psi V \to J/\psi F$ where $F = \pi^+ \pi^- (\pi^+ \pi^- \pi^0)$ for $V = \rho (\omega)$ is given by [15, 16]:

$$\frac{d\Gamma}{ds}(X \to J/\psi f) = \frac{1}{8\pi m_X^2} |\mathcal{M}|^2 B_{V \to F} \frac{\Gamma_V m_V}{\pi} \frac{p(s)}{(s - m_V^2)^2 + (m_V \Gamma_V)^2},$$  \hspace{1cm} (15)$$

where $p(s) = \sqrt{\lambda(m_X^2, m_\psi^2, s)}/2m_X$, with $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. The invariant amplitude squared is given by:

$$|\mathcal{M}|^2 = 8^2 m_X \frac{1}{2} \left(4m_X^2 - m_\psi^2 + s\right) + \frac{(m_X^2 - m_\psi^2)^2}{2s} + \frac{(m_X^2 - s)^2}{2m_\psi^2} \left(m_X^2 - m_\psi^2 + s\right).$$  \hspace{1cm} (16)
From the Eq. (15), using $B_{\rho-\pi\pi\pi} = 0.89$, $\Gamma_{\pi} = 8.49$ GeV, $m_{\rho} = 782.6$ MeV, $B_{\rho-\pi\pi} = 1$, $\Gamma_{\rho} = 149.4$ GeV and $m_{\rho} = 775.5$ MeV, the ratio in Eq. (1) can be obtained:

$$\frac{\Gamma(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\Gamma(X \rightarrow J/\psi \pi^+ \pi^-)} = 0.118 \left(\frac{g_{X\psi\omega}}{g_{X\psi\rho}}\right)^2. \quad (17)$$

The couplings, $g_{X\psi\omega}$, can be evaluated through a QCDSR calculation for the vertex, $X(3872)J/\psi V$, that centers in the three-point function given by

$$\Pi_{\mu
u\alpha}(p, p', q) = \int d^4x d^4y \, e^{ip'x} e^{iqy} \langle 0|T\{j^\mu_{\alpha}(x)j^\nu_{\beta}(y)j^\alpha_{\lambda}(0)\}|0\rangle \quad (18)$$

where $p = p' + q$ and the interpolating fields are given by:

$$j^\mu_{\alpha} = \bar{c}_\alpha G_{\mu\alpha}, \quad (19)$$

$$J^\nu_{\alpha} = \frac{N_V}{2} \bar{u}_\alpha G_{\nu\alpha} + (-1)^{\nu} \bar{d}_\alpha G_{\nu\alpha}, \quad \text{with} \quad N_V = 1, I_\rho = 1, N_\omega = 1/3, I_\omega = 0. \quad (20)$$

If $X(3872)$ is a pure $D^0\bar{D}^{*0}$ molecule, $J^\nu_{\alpha}$ is given by Eq. (4). In this case the only difference in the OPE side of the sum rule is the factor $N_V$ and, therefore, regardless the approximations made in the OPE side and the number of terms considered in the sum rule one has

$$\Pi_{\mu
u\alpha}(p, p', q) = N_V \Pi_{\mu
u\alpha}^{\text{OPE}}(p, p', q). \quad (21)$$

To evaluate the phenomenological side of the sum rule we insert, in Eq. (18), intermediate states for $X, J/\psi$ and $V$. We get [16]:

$$\Pi_{\mu
u\alpha}^{\text{phen}}(p, p', q) = \frac{i\lambda_X m_{\psi} f_{\psi} m_{\psi} f_{V} g_{X\psi\psi}}{(p^2 - m_X^2)(p'^2 - m_X^2)(q^2 - m_V^2)} \left( -\epsilon^{\alpha\mu\nu\sigma}(p'_\sigma + q_\sigma) - \epsilon^{\alpha\mu\nu\sigma'}(p'_\sigma + q_\sigma) - \epsilon^{\alpha\nu\lambda\gamma'}(p'_\sigma + q_\sigma) \right) \Pi_{\mu
u\alpha}^{\text{OPE}}(p, p', q), \quad (22)$$

Therefore, for a given structure the sum rule is given by:

$$\frac{i\lambda_X m_{\psi} f_{\psi} m_{\psi} f_{V} g_{X\psi\psi}}{(p^2 - m_X^2)(p'^2 - m_X^2)(q^2 - m_V^2)} = N_V \Pi_{\mu
u\alpha}^{\text{OPE}}(p, p', q), \quad (23)$$

from where, considering $m_\rho \approx m_\omega$ and using $f_\rho = 157$ MeV and $f_\omega = 46$ MeV we obtain the ratio $g_{X\psi\omega}/g_{X\psi\rho} = 1.14$, and using this result in Eq. (17) we finally get that the rate of Eq. (1) is not reproduced by the pure molecule

$$\frac{\Gamma(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\Gamma(X \rightarrow J/\psi \pi^+ \pi^-)} \approx 0.15. \quad (24)$$

The same conclusion is achieved if we consider the $c\bar{c}$ admixture in the $D^0\bar{D}^{*0}$ molecule.

In the following, to be able to reproduce the data in Eq.(1), instead of the admixture of $pJ/\psi$ and $\omega J/\psi$ components to the $D^0\bar{D}^{*0}$ molecule, as done by Swanson [9], we will consider a small admixture of $D^+ D^{*-}$ and $D^- D^{*+}$ components. In this case, instead of Eq.(6) we have

$$j^X_{\mu}(x) = \cos \alpha j^\rho_{\mu}(x) + \sin \alpha j^\omega_{\mu}(x), \quad (25)$$

with $J^\rho_{\mu}(x)$ and $J^\omega_{\mu}(x)$ given by Eq.(6). By inserting $j^X_{\mu}(x)$ given by Eq. (25), in Eq. (18) and considering the quarks $u$ and $d$ to be degenerate, one has that the only contribution to the three-point function is given by the molecular term:

$$\Pi_{\mu
u\alpha}(p, p', q) = \sin(\theta) \frac{N_V}{2V^2} (\cos \alpha + (-1)^{\nu} \sin \alpha) \Pi_{\mu
u\alpha}^{\text{mol}}(p, p', q). \quad (26)$$

The OPE side is then evaluated considering condensates up to dimension 5, and working in the structure $\epsilon^{\alpha\nu\lambda\gamma'}(p'_\sigma + q_\sigma)$. In the phenomenological side, considering the definition of $\lambda^u$ in Eq.(7) and the definition of the current in (25), we can define the meson-current coupling parameter as

$$\lambda_X = \cos \alpha \lambda^u + \sin \alpha \lambda^d = (\cos \alpha + \sin \alpha) \lambda^q, \quad (27)$$
where $\lambda^q$ was evaluated in Sec. III, and is given in Eq. (14).

By inserting Eq.(27) and Eqs. (26 in Eq.(22), and ) we get the following relation between the coupling constants:

$$g_{\psi\omega f_\alpha} = \frac{N_\alpha (\cos \alpha + \sin \alpha)}{N_\rho (\cos \alpha - \sin \alpha)}.$$  

(28)

Using the previous result in Eq. (17) and the numerical values for $f_\alpha$ and $f_\rho$ we have

$$\frac{\Gamma(X \to J/\psi \pi^+ \pi^- \pi^0)}{\Gamma(X \to J/\psi \pi^+ \pi^-)} \simeq 0.15 \left(\frac{\cos \alpha + \sin \alpha}{\cos \alpha - \sin \alpha}\right)^2.$$  

(29)

This relation determines the angle $\alpha \sim 20^0$, reproducing the experimental result in Eq.(1). Similar results were obtained in [15, 16].

We can now evaluate the decay rates. We choose to work with $X \to J/\psi \omega$ since the combination $\cos \alpha + \sin \alpha$ appears in both sides of the sum rule and the result for $g_{\psi\omega}$ is independent of $\alpha$. In order to evaluate the coupling constant we follow the procedure adopted in [16], where the coupling constant is defined as the value of a form factor at the meson pole, $g_{\psi\omega}(Q^2 = -m_\omega^2)$. Parameterizing the form factor as a monopole, we perform the numerical fitting of both sides of the sum rule for $s_0/2 = 4.4$ GeV, in the region $3.0$ GeV$^2 \leq M^2 \leq 3.5$ GeV$^2$, and varying $\theta$ in the range $5^0 \leq \theta \leq 13^0$. The result for the coupling constant is then given by:

$$g_{\psi\omega} = g_{\psi\omega}(-m_\omega^2) = 5.4 \pm 2.4.$$  

(30)

The decay width for the process $X \to J/\psi \pi^+ \pi^- \pi^0$ can now be obtained using Eqs. (15) and (30), and the result is:

$$\frac{\Gamma(X \to J/\psi \pi^+ \pi^- \pi^0)}{\Gamma(X \to J/\psi \pi^+ \pi^-)} = (9.3 \pm 6.9) \text{ MeV}.$$  

(31)

The result in Eq. (31) is in complete agreement with the experimental upper limit.

In conclusion, we have presented a QCDSR analysis of the two-point and three-point functions of the $X(3872)$ meson, by considering a mixed charmonium-molecular current. We find that the sum rules results in Eqs. (13) and (1) are compatible with experimental data. These results were obtained by considering the mixing angle in Eq. (6) in the range $5^0 \leq \theta \leq 13^0$.

We have also studied the mixing between the $D^0 \bar{D}^0$, $D^0 \bar{D}^0$ and $D^+ D^{-}, D^- D^+$ states by imposing the ratio in Eq. (1). In accordance with the findings in ref. [15] we found that the mixing angle in Eq. (25) is $\alpha \sim 20^0$.

With the knowledge of these two mixing angles we conclude that the $X(3872)$ is basically a $c\bar{c}$ state ($\sim 97\%$) with a small, but fundamental, admixture of molecular $D^0 \bar{D}^0$, $D^0 \bar{D}^0$ ($\sim 88\%$) and $D^+ D^-, D^- D^+$ ($\sim 12\%$) states.

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**REFERENCES**