Longitudinal and transverse parton momentum distributions for hadrons within relativistic constituent quark models

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Relativistic quark models for pion form factor
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Relativistic quark models for nucleon form factors
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Pion Generalized Parton Distributions
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A wealth of information on the partonic structure of hadrons is encoded in the Generalized Parton Distributions. Extensive theoretical and experimental research programs are being pursued to gain information on GPD’s.

Our strategy:

- to obtain the quark-hadron vertex functions from an investigation of hadron EM form factors in the spacelike region
- to use the obtained hadron vertex functions to evaluate GPD’s

★ Unpolarized GPD’s for the pion: three models

- analytic covariant model bare photon vertex
- Mandelstam inspired light-front model dressed photon vertex
- light-front Hamiltonian dynamics model bare photon vertex

★ Longitudinal and transverse quark momentum distributions in the nucleon

Fock decomposition of the pion state

\[ |\pi\rangle = |q\bar{q}\rangle + |q\bar{q} q\bar{q}\rangle + |q\bar{q} g\rangle \ldots. \]

valence nonvalence
Isoscalar and isovector pion GPD’s in the light-cone gauge are

\[
H^0_\pi(x, \xi, t) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}_q(-\frac{z}{2}) \gamma^+ \psi_q(\frac{z}{2}) | p \rangle \bigg|_{\tilde{z}=0}
\]

\[
H^1_\pi(x, \xi, t) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}_q(-\frac{z}{2}) \gamma^+ \tau_3 \psi_q(\frac{z}{2}) | p \rangle \bigg|_{\tilde{z}=0}
\]

\[\tilde{z} \equiv \{z^+ = z^0 + z^3, z_\perp \}, \quad \psi_q(z) = \text{quark field isodoublet}\]

\[t = \Delta^2 \quad \Delta = p' - p\]

\[\xi = -\frac{\Delta^+}{2P^+} \quad 2P = p + p'\]

\[x = \frac{k^+}{P^+} \quad (1 \geq x \geq -1)\]

LF time-ordered analysis of the pion GPD

active-quark valence region [DGLAP]

\[1 \geq x \geq |\xi|\]

diagonal in the Fock space

nonvalence region [ERBL]

\[|\xi| > x > -|\xi|\]

non diagonal in the Fock space
We use a pion Bethe-Salpeter amplitude (BSA) suggested by an effective Lagrangian [Frederico, Miller, PRD 45 (1992) 4207]

\[ \Psi(k - P, p) = -\frac{m}{f_\pi} S(k - \Delta/2) \gamma^5 \Lambda(k - P, p) S(k - P) \]

\( m = 220 \text{ MeV} \) quark mass \( f_\pi = 92.4 \text{ MeV} \) decay constant

The quark propagator includes the instantaneous term, \( \gamma^+ / 2k^+ \)

\[ S(k) = \frac{k + m}{k^2 - m^2 + i\epsilon} = \frac{k_{on} + m}{k^+(k^- - k_{on}^- + \frac{i\epsilon}{k^+})} + \frac{\gamma^+}{2k^+} \]

Two covariant symmetric forms for \( \Lambda(k - P, p) \) are used:

i) a sum form

\[ \Lambda_1 = \frac{C_1}{[(k - \Delta/2)^2 - m_R^2 + i\epsilon]} + \frac{C_1}{[(P - k)^2 - m_R^2 + i\epsilon]} \]

ii) a product form

\[ \Lambda_2 = \frac{C_2}{[(k - \Delta/2)^2 - m_R^2 + i\epsilon] \ [(P - k)^2 - m_R^2 + i\epsilon]} \]

The parameter \( m_R \) is used to fit \( f_\pi \)

The \( u \)-quark GPD is given in Impulse Approximation by

\[ H^u(x, \xi, t) = -i N_c \mathcal{R} \times \int \frac{d^4k}{2(2\pi)^4} \delta(P^+ x - k^+) \ V^+ \Lambda(k - P, p') \ \Lambda(k - P, p) \]

\( N_c = 3 \) is the number of colors \( \mathcal{R} = 2m^2/f^2_\pi \)

The \( \delta \) function imposes the active quark support \( -|\xi| \leq x \leq 1 \)
\[ V^+ = Tr \{ S (k - P) \gamma^5 S (k + \Delta/2) \gamma^+ S (k - \Delta/2) \gamma^5 \} \]

We adopt a Breit frame with \( \Delta^+ = -\Delta^- \geq 0 \)

\[ \xi^2 = \frac{-\Delta^2 - |\Delta_\perp|^2}{-\Delta^2 + 4m^2_\pi} \]

The whole kinematical range can be explored.

At \( \xi = -1 \Rightarrow x = \frac{2k^+_q}{\Delta^+} \). Only the \( q\bar{q} \) pair production contributes.

Then at high \( |t| \) a maximum of GPD’s around \( x \sim 1 \) is expected.

Indeed large \( |t| \) values mean large \( \Delta^+ = \Delta_z \approx 2k_{zq} \).

As a consequence \( x \sim \frac{k^+_q - k^+_{\bar{q}}}{2k_{zq}} \sim \frac{E_q}{k_{zq}} = \frac{\sqrt{m^2 + k^2}}{k_{zq}} \rightarrow 1 \)

**Mandelstam-inspired pion light-front model**

We take a Breit frame where \( \Delta_\perp = 0 \) and assume \( m_\pi = 0 \).

Then \( \xi = -1 \). The quark mass \( m = 200 \text{ MeV} \) is adopted.

The previous expression for \( H^u(x, \xi, t) \) is used, but \( \gamma^+ \) is replaced by a microscopic Vector Meson (VM) dominance vertex

\[ \Gamma^+(k, \Delta) = \sqrt{2} \sum_{n, \lambda} \left[ \epsilon_\lambda \cdot \hat{V}_n(k, P_n) \right] \Lambda_n(k, P_n) \frac{[\epsilon^+_\lambda]^* f_{Vn}}{(t - M^2_n)} \]

\( f_{Vn} \) decay constant of the n-th VM (calculated in the model)

\( P^\mu_n \equiv \{ M^2_n/\Delta^+, \Delta^+, 0_\perp \} \) VM four-momentum with \( P^2_n = M^2_n \)

\( \epsilon_\lambda(P_n) \) VM polarization \( \hat{V}_n(k, P_n) \) proper VM Dirac structure

Up to 20 vector mesons are considered.
In the $k^-$ integration, only the propagators poles are considered, i.e. the BSA analytic structure is disregarded: i) in the initial and final pion, and ii) in the quark-photon vertex.

For the tridimensional reduction of the VM BSA in the valence sector, $0 < k^+ < P_n^+$, we take the eigenfunction, $\psi_n(k^+, k_\perp; P_n)$, of a relativistic square mass operator [Frederico, Pauli, Zhou, PRD 66 (2002) 116011], normalized as in [PRD 73 (2006) 074013].

$$\frac{P_n^+ \Lambda_n(k, P_n)|_{k^- = k_{on}^-}}{[M_n^2 - M_0^2(k^+, k_\perp; P_n)]} = \psi_n(k^+, k_\perp; P_n),$$

$$M_0^2(k^+, k_\perp; P_n) = P_n^+ \left[ k_{on}^- + (P_n - k)^- \right] \quad \text{free } q\bar{q} \text{ mass}$$

For the pion in the valence sector the corresponding mass operator eigenfunction is used. For the nonvalence pion vertex (■), we assume a constant [Choi & Ji (PLB 513 (2001) 330)].

A parameter, $w = -1$, which modulates the relative weights of the instantaneous contributions is used to fit the form factor.

**Light-front Hamiltonian dynamics model**

Drell-Yan $\Delta^+ = 0$ reference frame $\Rightarrow x_q = x$, since $\xi = 0$.

**Pion light-front wave function**

$$\Psi_\pi = \psi_\pi(x, \kappa_\perp) \sum_{\mu_q, \bar{\mu}\bar{q}} \left( \frac{1}{2} \mu_q \frac{1}{2} \bar{\mu} \bar{q} |00\right) D_{\mu_q \bar{\lambda}_q}^{1/2} \left[ R(\kappa) \right] D_{\bar{\mu} \bar{\bar{q}} \bar{\bar{\lambda}}_{\bar{q}}}^{1/2} \left[ R(-\kappa) \right]$$

$$\kappa \equiv \{ \kappa_\perp, \kappa_z \} \quad \kappa_z = M_0(x, \kappa_\perp) \left( x - \frac{1}{2} \right) \quad \text{intrinsic variables}$$
\[ M_0^2(x, \kappa) = \frac{m^2 + |\kappa|^2}{x(1-x)} \quad M_0(x, \kappa) = \text{pion free mass} \]

The Melosh rotations \( R(\kappa) \) convert the instant-form spins into LF spins and ensure the rotational invariance.

For the momentum dependence of the pion w.f. we adopt an exponential form

\[ \psi_{\pi}(x, \kappa) = \left[ \frac{2(2\pi)^3}{\pi^{3/4}/\beta^{3/2}} \left( \frac{M_0(x, \kappa)}{4x(1-x)} \right) \right]^{1/2} \exp \left( -\kappa^2 / (2\beta^2) \right). \]

The constituent quark mass \( m = 250 \text{ MeV} \) is adopted and the parameter \( \beta \) is adjusted to fit \( f_\pi \).

The pion GPD \( H^u(x, \xi = 0, t) \) in the range \( 0 \leq x \leq 1 \) is given by

\[ H^u(x, \xi = 0, t) = \sum_{\{\lambda_i\}} \int \frac{d\kappa}{2(2\pi)^3} \left[ \Psi^*_\pi(x, \kappa; \{\lambda_i\}) \Psi_\pi(x, \kappa; \{\lambda_i\}) \right] \]

\( \lambda_i \) are the spin projections. Initial and final transverse components of active quark momenta in the intrinsic frame are related by

\[ \kappa'_\perp = \kappa_\perp + (1-x) \Delta_\perp \]

Then at high \( |t| \), i.e. at high \( |\Delta_\perp| \), \( H^u(x, \xi = 0, t) \) is expected not to be vanishing only for \( x \sim 1 \).
\[ \int_{-1}^{1} dx \ H^{I=1}(x, \xi, t) = F_{\pi}(t) \]

Thin dashed line: covariant model, with the sum-form for BSA
Dotted line: covariant model, with the product-form BSA
Blue line and Red line: monopole and faster than monopole fit to
Dot-dashed line: LF Mandelstam-inspired model with a pion wf
eigenstate of LF mass operator.
Thick dashed line: LFHD model pion wave function.

\[ F_{\text{mon}}(t) = 1/(1 + |t|/m_{\rho}^2) \quad m_{\rho} = 770 \text{ MeV} \]

Models that show an asymptotic decay slower than \( F_{\text{mon}}(t) \), as the
covariant sum-form model, yield a divergent density at short range.
Pion longitudinal momentum distribution

\[ u(x) = H^u(x, 0, 0) = 2 H^{I=1}(x, 0, 0) \]

\[ u(x) = \int d\mathbf{k}_\perp f_1(x, |\mathbf{k}_\perp|^2), \quad (x \geq 0). \]

At \( \xi = 0 \) the variable \( x \) coincides with the longitudinal fraction \( x_q \)

Thin dashed line: covariant model with the sum-form BSA
Dotted line: covariant model with the product-form BSA
Thick dashed line: LFHD model with a Gaussian wave function

Sum-form BSA is unable to yield vanishing values at the end points.

The covariant product-form model with a \( |k_\perp|^4 \) decay of the BSA, compatible with a BSA kernel dominated by the one-gluon-exchange (OGE), gives a consistent description of the tail of the form factor and of the end-point fall-off of the parton distribution.

Both at \( -t \to \infty \) and at \( x \to 0 \) or at \( x \to 1 \) the high momentum part of the pion state is probed.
Transverse-momentum dependent function, $f_1(x, |k_\perp|^2)$

Covariant symmetric model

Sum-form BSA

Product-form BSA

$$f_1(x, k_\perp)/G(k_\perp) \text{ (GeV/c)}^{-2}$$

$$G(|k_\perp|) = \frac{1}{1 + |k_\perp|^2/m^2_\rho}^4$$

$$k_{\perp} = |k_\perp|$$

The normalization is given by

$$\int_0^1 dx \int d|k_\perp| f_1(x, |k_\perp|^2) = 1$$

The product-form model has a faster $|k_\perp|$ falloff than the sum-form model.
Generalized Parton Distributions \( |\xi| = 1, \ m_\pi = 0 \)

**Isoscalar**

covariant symmetric model  
(product-form BSA)

\[
H_{\pi}^{I=0}(x,|\xi|=1,t)/F_{\text{mon}}(t)
\]

**Isovector**

Mandelstam-inspired model

\[
H_{\pi}^{I=1}(x,|\xi|=1,t)/F_{\text{mon}}(t)
\]

\(|\xi| = 1 \Rightarrow \) nonvalence region
Generalized Parton Distributions $\xi = 0$

Isoscalar  

\textbf{covariant symmetric model} (product-form BSA)

\[
H_{\pi}^{I=0}(x,\xi=0,t)/F_{\text{mon}}(t)
\]

\[
H_{\pi}^{I=1}(x,\xi=0,t)/F_{\text{mon}}(t)
\]

Isovector  

Light-Front Hamiltonian model

\[
H_{\pi}^{I=0}(x,\xi=0,t)/F_{\text{mon}}(t)
\]

\[
H_{\pi}^{I=1}(x,\xi=0,t)/F_{\text{mon}}(t)
\]

$\xi = 0 \Rightarrow$ valence region

As already noticed, as $-t \to \infty$, the maximum of GPD’s moves from $x = 0.5$ towards $x = 1$. 
At $|\xi| = x$ one explores the transition from valence to nonvalence region. This kinematical regime should be relevant to study single spin asymmetry [Diehl, Phys.Rep. 388 (2003) 41].

**covariant symmetric model** (product form BSA)

Again, as $-t \rightarrow \infty$, the maximum of GPD moves from $x = 0.5$ towards $x = 1$.

The covariant product-form model is able to reproduce the GPD’s evaluated at $|\xi| = 1$ with the Mandelstam-inspired model and at $\xi = 0$ with the LFHD model. One could argue that it contains the main ingredients for the description of the constituents inside the pion and could be applied to study experimental data.
The Dirac structure of the quark-nucleon vertex is suggested by an effective Lagrangian (de Araujo et al., PLB B478 (2001) 86).

The symmetrized Bethe-Salpeter amplitude for the nucleon is approximated as follows

\[ \Phi_N^\sigma(k_1, k_2, k_3, P_N) = \imath \left[ S(k_1) \tau_y \gamma^5 S_C(k_2)C \otimes S(k_3) + S(k_3) \tau_y \gamma^5 S_C(k_1)C \otimes S(k_2) + S(k_3) \tau_y \gamma^5 S_C(k_2)C \otimes S(k_1) \right] \]

\[ \times \Lambda(k_1, k_2, k_3) \chi_{\tau N} U_N(P_N, \sigma) \]

\[ \Lambda(k_1, k_2, k_3) \] describes the symmetric momentum dependence of the vertex function upon the quark momentum variables, \( k_i \).

\[ U_N(P_N, \sigma) \] is the nucleon spinor and \( \chi_{\tau N} \) the isospin eigenstate.

The matrix elements of the macroscopic current are approximated microscopically by the Mandelstam formula

\[ \langle \sigma', P'_N | j^\mu | P_N, \sigma \rangle = \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} \sum \left\{ \Phi_N^\sigma(k_1, k_2, k_3, P'_N) \right. \]

\[ \times S^{-1}(k_1) S^{-1}(k_2) T^\mu(k_3, q) \Phi_N^\sigma(k_1, k_2, k_3, P_N) \right\} 3 N_c \]

where \( T^\mu(k_3, q) \) is the quark-photon vertex.

Breit reference frame: \( q_\perp = 0 \quad q^+ = |q^2|^{1/2} \)

Quark mass: \( m_u = m_d = 200 \, MeV \).
We project out the Mandelstam Formula by the integration on \( k_1^- \) and \( k_2^- \), taking into account only the poles of the propagators. Then the vertex functions have only a three-momentum dependence.

Triangle contr.  
Pair contribution

\[
\begin{align*}
\gamma^* & \quad = \quad k_3 + q \\
\gamma^* & \quad + \quad k_3 \\
& \quad \quad (\text{val.)} \quad 0 < k_i^+ < P_N^+ \\
& \quad \quad 0 > k_3^+ > -q^+ \\
& \quad \quad \times \quad \Rightarrow \quad k \text{ on the mass shell: } k_{on}^- = (m^2 + k^2_{\perp})/k^+
\end{align*}
\]

Quark-Photon Vertex : \( \mathcal{I}^\mu = \mathcal{I}_{IS}^\mu + \tau_z \mathcal{I}_{IV}^\mu \)

Each term contains a valence contribution and a contribution corresponding to the pair production. The last one can be decomposed in a bare term + a Vector Meson Dominance term, viz

\[
\mathcal{I}_i^\mu(k, q) = N_i \theta(P_N^+ - k^+) \theta(k^+) \gamma_i^\mu + \\
+ \theta(q^+ + k^+) \theta(-k^+) \left\{ Z_B \, N_i \gamma_i^\mu + Z_{VM}^i \Gamma^\mu[k, q, i] \right\}
\]

\( i = IS, IV \)

with \( N_{IS} = 1/6 \) and \( N_{IV} = 1/2 \). The constants \( Z_B \) (bare term) and \( Z_{VM}^i \) (VMD term) are unknown weights to be fitted.
Momentum Dependence of qqq-N vertexes

In the valence vertex, the spectator quarks are on the $k^-$-shell, and the momentum dependence, reduced to a 3-momentum dependence by the $k^-$ integrations, is approximated through a Nucleon Wave Function a la Brodsky (PQCD inspired), namely

$$
\Psi_N(\tilde{k}_1, \tilde{k}_2, P_N) = P_N^+ \frac{\Lambda(k_1, k_2, k_3)|_{(k_{1on}, k_{2on})}}{[M_N^2 - M_0^2(1, 2, 3)]} = 
$$

$$
= P_N^+ \mathcal{N} \frac{(9 m^2)^\alpha}{(\xi_1 \xi_2 \xi_3)^p [\beta^2 + M_0^2(1, 2, 3)]^\alpha}
$$

where $\tilde{k}_i \equiv (k_i^+, k_{i\perp})$, $M_0(1, 2, 3)$ is the free mass of the three-quark system,

$$
\xi_i = k_i^+ / P_N^+ \quad (i = 1, 2, 3)
$$

and $\mathcal{N}$ a normalization constant.

The powers $\alpha = 7/2$ and $p = 0.13$ are chosen to have an asymptotic decrease of the triangle contribution faster than the dipole.

Only the triangle diagram determines the magnetic moments, weakly dependent on $p$. Then $\beta = 0.645$ can be fixed by $\mu_p$ and $\mu_n$

Proton : $\mu_p = 2.87 \pm 0.02$ (Exp. 2.793)
Neutron : $\mu_n = -1.85 \pm 0.02$ (Exp. -1.913)
For the pair-production contribution, the nonvalence vertex is described through the available invariants, i.e. the free mass, $M_0(1, 2)$, of the $(1,2)$ quark pair and the free mass, $M_0(N, \bar{3})$, of the (nucleon - quark $\bar{3}$) system entering the nonvalence vertex. Then the nonvalence vertex is assumed to be

$$\Lambda_{NV}^{SL}(k_1, k_2, k_3) = [g_{12}]^2 [g_{N\bar{3}}]^{7/2 - 2} \left[ \frac{k_{12}^+}{P_N^{'+}} \right] \left[ \frac{P_N^+}{k_3^+} \right]^r \left[ \frac{P_N^+}{k_3^+} \right]^r$$

$$k_{12}^+ = k_1^+ + k_2^+ \quad g_{AB} = \frac{(m_A m_B)}{[\beta^2 + M_0^2(A, B)]}$$

The power 2 of $[g_{12}]^2$ is suggested from counting rules. The power $3/2$ of $[g_{N\bar{3}}]^{3/2}$ and the parameter $r = 0.17$ are chosen to have a dipole behavior for the nonvalence contribution.

**Adjusted parameters**

- the weights for the pair-production terms:
  $$Z_B = Z_{VM}^{IV} = 2.283 \quad \text{and} \quad Z_{VM}^{IS}/Z_{VM}^{IV} = 1.12$$
- the power $p = 0.13$ of $\xi_i$ in the valence amplitude
- the power $r = 0.17$ of the ratio $P_N^+/k_3^+$ in the spacelike nonvalence vertex.

$$\Rightarrow \quad \chi^2 = 1.7$$
Nucleon form factors

Solid line: full calculation $\equiv F_\triangle + F_{bare} + F_{VMD}$

Dotted line: $F_\triangle$ (triangle contribution only)

Data: www.jlab.org/cseely/nucleons.html and Refs. therein.

The pair-production contribution is essential for the result.

The possible zero in $G_E^p \mu_p / G_M^p$ is strongly related to the pair-production contribution, i.e. to higher Fock components.

$$G_D = 1/[1 - q^2/(0.71 \ (GeV/c)^2)]^2$$
Transverse momentum distributions in the proton

\[ f_1^{u(d)}(x, k_\perp) = -\frac{N_c}{(2\pi)^6} \int_0^{1-x} d\xi_2 \frac{C_{u(d)}}{(1 - x - \xi_2)\xi_2} \frac{1}{x^2} \int d\mathbf{k}_{2\perp} \]
\[ \times \frac{1}{P_N^+} |\Psi_N(\tilde{k}_1, \tilde{k}_2, P_N)|^2 \left( \mathcal{H}_u(d) \right)_{(\tilde{k}_{1\perp}, \tilde{k}_{2\perp})} \]

\( \mathcal{H}_u \) and \( \mathcal{H}_d \) are proper traces of propagators and of the currents \( I^+_u \) and \( I^+_d \), respectively.

**u quark**

\[ f_1(x, k_{\perp})/G(k_{\perp}) \text{ (GeV/c)}^2 \]

**d quark**

\[ f_1(x, k_{\perp})/G(k_{\perp}) \text{ (GeV/c)}^2 \]

\[ G(k_{\perp}) = (1 + k_\perp^2/m_\rho^2)^{-5.5} \]

The decay of our \( f_1(x, k_\perp) \) vs \( k_\perp \) is faster than in diquark models of nucleon (Jacob et al., Nucl Phys. A626 (1997) 937), while it is slower than in factorization models for the transverse momentum distributions (Anselmino et al., PRD 74 (2006) 074015).
Longitudinal momentum distributions

For $P'_N = P'_N$ the unpolarized GPD $H_q(x, \xi, t)$ reduces to the longitudinal parton distribution function $q(x)$

$$H^q(x, 0, 0) = \int \frac{dz^-}{4\pi} e^{ixP^+_N z^-} \langle P_N|\overline{\psi}_q(0) \gamma^+ \psi_q(z)|P_N\rangle|_{\bar{z}=0}$$

$$= q(x) = \int d{k}_\perp f^q_1(x, k_\perp)$$

an average on the nucleon helicities is understood.

proton preliminary results

$u$ quark

$\begin{array}{c}
\text{Dashed lines : our longitudinal momentum distributions} \\
\text{Thick solid lines : our model after evolution to } Q^2 = 1.6 \text{ (GeV/c)}^2 \\
\text{Thin solid lines : CTEQ4 fit to data [Lai et al., PRD 51 (1995) 4753]} \\
\end{array}$

$\begin{array}{c}
d quark \\
\end{array}$
Conclusions & Perspectives

- Pion and nucleon relativistic quark models able to give a good description of the em form factors have been proposed
- The obtained quark-hadron vertexes have been used to evaluate the longitudinal and transverse parton momentum distributions
- The relevance of a $q$-$\pi$ vertex compatible with the OGE dominance to describe the tail of the form factor and to obtain a vanishing parton distribution at the end points has been shown
- In the pion case unpolarized isoscalar and isovector GPD’s have been obtained within three different models
- An interesting feature of pion GPD’s has been shown: as $-t$ increases the maximum of the GPD’s migrates from $x \sim 0.5$ towards $x \sim 1$
  The kinematical origin of this behaviour has been demonstrated for $\xi = 0$ and $|\xi| = 1$. Then at high $|t|$ a maximum around $x \sim 1$ should be found in the nucleon case as well.

Next steps:

- to calculate pion polarized GPD’s
- to develop more refined covariant models based on Nakanishi representation in order to calculate GPD’s to be used in the description of experimental data
Further possible developments

- different Dirac structures of the effective quark-nucleon Lagrangian could be considered

- different approximations for the nucleon wave function could be tested

- a model for the vector meson spectrum, able to account for a possible resonance at $M_n = 2.050 \, GeV$ should be investigated, possibly for a better description of the quark-photon vertex.
According to \( i \) the VMD term \( \Gamma^\mu [k, q, \bar{\nu}] \) includes isovector or isoscalar mesons.

\[
\Gamma = \sum_n \left\{ \Lambda_n(k, P_n) \right\}
\]

The decay constant, \( f_{Vn} \), is evaluated assuming that:

i) \( \Lambda_n(k, P_n) \) does not diverge in the \( k^- \) complex-plane for \( |k^-| \to \infty \),

and ii) the contributions of its singularities in the \( k^- \) integration are negligible.

\[
f_{Vn} = -\frac{N_c P_n^+}{4(2\pi)^3} \int_0^{P_n^+} \frac{dk^+ d\mathbf{k}_\perp}{k^+ (P_n^+ - k^+)} \frac{\Lambda_n(k, P_n)|_{[k^- = k_{on}^-]}}{M_n^2 - M_0^2(k^+, \mathbf{k}_\perp; P_n^+, \mathbf{P}_{n\perp})} \cdot Tr \left[ (\not{k} - \not{P}_n + \not{m})\gamma^+(\not{k} + \not{m}) \hat{V}_{nz}(k, k - P_n) \right].
\]
\[ \Gamma^\mu(k, q, i) = \sqrt{2} \sum_{n, \lambda} [\epsilon_\lambda \cdot \mathcal{V}(k, q)] \Lambda^i_n(k, P^i_n) \times \]
\[ \frac{[\epsilon_\lambda^\mu]^* f^i_{Vn}}{(q^2 - M^2_{i,n} + i M_{i,n} \tilde{\Gamma}^i_n(q^2))} \]  

(1)

- \( f^i_{Vn} \) is the decay constant of the n-th vector meson into a virtual photon (to be calculated in our model!), \( M_{i,n} \) the mass, \( \tilde{\Gamma}^i_n(q^2) = \Gamma^i_n q^2 / M^2_{i,n} \) (for \( q^2 > 0 \)) the corresponding total decay width and \( \epsilon_\lambda(P^i_n) \) the VM polarization
- \( [\epsilon_\lambda(P^i_n) \cdot \mathcal{V}(k, q)] \Lambda^i_n(k, P^i_n) \equiv \) VM vertex function.

To have a conserved current, we take

\[ \mathcal{V}^\mu(k, q) = V^\mu(k, q) - \frac{q^\mu}{q^2} q \cdot V(k, q) \quad (q \cdot \mathcal{V} = 0) \]

This definition generate no divergence at the photon point in our reference frame: \( q^\perp = 0 \quad q^+ = |q^2|^{1/2} \)

We assume

\[ V^\mu(k, q) = \gamma^\mu - \frac{k^\mu_{on} - (q - k)^\mu_{on}}{M_0(k^+, k^\perp; q^+, q^\perp) + 2m} \]

to generate the proper Melosh rotations for \(^3 S_1\) states. \( M_0 \) is the standard light-front free mass. [W. Jaus, PRD 41 (1990) 3394]

\( \Lambda^i_n(k, P^i_n) \) is the momentum-dependent part of the VM Bethe-Salpeter amplitude.
In the valence sector, $0 < k^+ < P_{n}^{i+}$, the on-shell amplitude of the VM has been related to the light-front VM wave function

$$
\frac{P_{n}^{i+} \Lambda_{n}^{i}(k, P_{n}^{i})}{[M_{n}^{2} - M_{0}^{2}(k^+, k_{\perp}; P_{n}^{i+}, P_{n\perp}^{i})]} = \psi_{n}^{i}(k^+, k_{\perp}; P_{n}^{i+}, P_{n\perp}^{i})
$$

is

- eigenfunction of a relativistic CQ square mass operator (Frederico, Pauli & Zhou, PRD 66 (2002) 116011), with confinement (harmonic oscillator potential) and $\pi - \rho$ splitting (Dirac-delta interaction in the pseudoscalar channel). A natural explanation of the "Iachello-Anisovitch law"

$$(M_{n}^{2} \sim M_{gr}^{2} + \omega (n - 1)); \ n \ \text{is the radial quantum number})$$

is obtained.

In the $\pi$ form factor calculation no isospin breaking was considered ($\rho \equiv \omega$).

- normalized to the probability of the lowest ($q\bar{q}$) Fock state (i.e. the valence component), roughly estimated in a simple model (de Melo et al., PRD 73 (2006) 074013) that reproduces the "Iachello-Anisovitch law", for the VM mass spectra.

Vector-meson valence probabilities $P_{q\bar{q};n}$ for the first resonances.

<table>
<thead>
<tr>
<th>n</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{q\bar{q};n}$</td>
<td>0.77</td>
<td>0.31</td>
<td>0.29</td>
<td>0.27</td>
<td>0.22</td>
<td>0.18</td>
<td>0.18</td>
</tr>
</tbody>
</table>
Fixed parameters

\[ m_u = m_d = 0.200 \text{ GeV} \]

Experimental vector-meson masses, \( M_{n}^{IV} \), and widths, \( \Gamma_{n}^{IV} \), for the first four isovector vector mesons.

<table>
<thead>
<tr>
<th>Meson</th>
<th>( M_{n}^{IV} ) (MeV)</th>
<th>( M_{n}^{\text{exp}} ) (MeV)</th>
<th>( \Gamma_{n}^{IV} ) (MeV)</th>
<th>( \Gamma_{n}^{\text{exp}} ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho(770) )</td>
<td>770</td>
<td>775.8 ± 0.5</td>
<td>146.4</td>
<td>146.4 ± 1.5</td>
</tr>
<tr>
<td>( \rho(1450) )</td>
<td>1497*</td>
<td>1459.0 ± 11.0</td>
<td>226*</td>
<td>147 ± 40</td>
</tr>
<tr>
<td>( \rho(1700) )</td>
<td>1720</td>
<td>1720.0 ± 20.0</td>
<td>220</td>
<td>250 ± 100</td>
</tr>
<tr>
<td>( \rho(2150) )</td>
<td>2149</td>
<td>2149.0 ± 17</td>
<td>230**</td>
<td>363 ± 50</td>
</tr>
</tbody>
</table>

Isoscalar masses, \( M_{n}^{IS} \), and widths, \( \Gamma_{n}^{IS} \), for the first 3 IS mesons.

<table>
<thead>
<tr>
<th>Meson</th>
<th>( M_{n}^{IS} ) (MeV)</th>
<th>( M_{n}^{\text{exp}} ) (MeV)</th>
<th>( \Gamma_{n}^{IS} ) (MeV)</th>
<th>( \Gamma_{n}^{\text{exp}} ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega(782) )</td>
<td>782</td>
<td>782.65 ± 0.12</td>
<td>8.44</td>
<td>8.49 ± 0.08</td>
</tr>
<tr>
<td>( \omega'(1420) )</td>
<td>1420</td>
<td>1425.0 ± 25.0</td>
<td>225</td>
<td>215 ± 35</td>
</tr>
<tr>
<td>( \omega''(1650) )</td>
<td>1720</td>
<td>1670.0 ± 30.0</td>
<td>250</td>
<td>315 ± 35</td>
</tr>
</tbody>
</table>


20 isoscalar and 20 isovector vector mesons are taken into account to reach convergence up to \( q^2 = 10 \text{ (GeV/c)}^2 \).
For \( m_\pi \to 0 \), only instantaneous contributions survive, since on-shell terms give vanishing contributions to the trace in \( j^\mu \).

Instantaneous contributions to the timelike em form factor of a massless pion. The instantaneous quark line (vertical line) is attached to the pion vertex in (a) and to VM vertex in (b). The shaded circle represents the dressed photon vertex.

We assume \( \Lambda^{ist} \sim C \Lambda^{full} \)

The constant \( C \) is thought to roughly describe the effects of the short-range interaction.

We use the relative weight, \( w_{VM} = C_{VM}/C_\pi \), as a free parameter.
Results for nucleon radii

\[ r_p = (0.903 \pm 0.004) \text{ fm} \quad r_p^{exp} = (0.895 \pm 0.018) \text{ fm} \]

\[- \left[ \frac{dG_E^n(Q^2)}{dQ^2} \right]^{th} = (0.501 \pm 0.002) (\text{GeV/c})^{-2} \]

\[- \left[ \frac{dG_E^n(Q^2)}{dQ^2} \right]^{exp} = (0.512 \pm 0.013) (\text{GeV/c})^{-2} \]
Nucleon timelike form factors
parameter free results

Missing strength at \( q^2 = 4.5 \text{ (GeV/c)}^2 \) and \( q^2 = 8 \text{ (GeV/c)}^2 \)

\[ G_{\text{eff}}^p(q^2)/G_D(q^2) = |G_M(q^2)|^2 + |G_E(q^2)|^2 \frac{2m_N^2}{q^2} \left( 1 + \frac{2m_N^2}{q^2} \right) \]