Lattice QCD, Photo Couplings and Radiative Transitions

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Abstract. We review recent progress in calculating radiative transition form factors using lattice QCD. We discuss results in the charmonium region involving excited states, states of high spin and exotics. As well as highlighting interesting results involving an exotic $1^{-+}$ states and a vector-hybrid candidate, we give comparison with experimental data and models. Some lattice calculations of baryon photocouplings are also mentioned. We conclude with some comments on future prospects.

Keywords: Lattice QCD, radiative transitions, mesons, exotics, nucleons

1. INTRODUCTION

A vital ingredient in understanding the strong interaction is the calculation of the spectrum of QCD, along with the transition rates between different states, and testing these against experimental data. Lattice QCD provides an ab initio method for performing such calculations and some important aims of the Hadron Spectrum Collaboration are to use lattice QCD to extract the meson and baryon spectra and radiative transitions. Recent progress in extracting spectra from lattice QCD has already been reviewed at this conference[1]. Studying the spectrum is an important first step towards understanding the strong interaction, but transitions are needed to probe the structure of hadrons in more detail. Here we concentrate on recent lattice QCD calculations of radiative transitions.

In Section 2 we discuss excited radiative transitions in the charmonium system, giving a brief overview of the method and then highlight some interesting results. In Section 3 we review some similar results in the nucleon sector. We conclude in Section 4 with some comments on future prospects.

2. RADIATIVE TRANSITIONS IN CHARMONIUM

2.1. Introduction

Charmonium is often described as the “hydrogen atom” of meson spectroscopy. It is reasonably nonrelativistic and so fairly well explained by potential models, at least below open charm threshold. It has also been studied using effective field theories and QCD sum rules. There has recently been a resurgence of interest in the charmonium system with BaBar, Belle, CLEO-C, and BES finding missing states, making more accurate measurements of the properties of these states and discovering a number of new resonances that are not easily explained by the quark model. This has caused much theoretical speculation as to whether these states are hybrids or multiquark/molecular mesons. Of particular interest are mesons with exotic quantum numbers, i.e. those with $J^{PC}$ which can not arise from a fermion-antifermion pair. The observation of such an exotic meson would signal physics beyond a simple model of quarks bound in a potential, one possibility being a hybrid meson where the gluonic field is excited. However, to date there has been no observation of a charmonium state having manifestly exotic $J^{PC}$, such as $1^{-+}$, $0^{++}$ or $2^{+-}$. The upcoming GlueX, BES III and PANDA experiments will continue this work, probing the spectroscopy of both light and charm meson sectors.

Radiative transitions in charmonium can have significant branching ratios because there are no OZI allowed strong decays below open-charm threshold, and so there is a profusion of experimental data on these transitions. This, along with the lower computational cost, makes it an ideal place to develop and test techniques for extracting excited meson photocouplings in lattice QCD. To this end, a program of work has been undertaken, first testing techniques for extracting excited spectra[2, 3] and then photocouplings[4, 5]. The aim is to now apply these methods to the light meson spectra where the results will be relevant for, among other things, the GlueX experiment at the JLab 12 GeV
upgrade where exotic mesons will be searched for in photoproduction. Photoproduction is expected in QCD-motivated model to be a favourable method of producing exotic mesons. Progress has already been made in extracting light meson spectral[6] and has been reviewed in another talk at this conference [1].

In the following section we give a short overview of the method used in the first lattice QCD calculation of charmium radiative transitions involving excited states, states of high spin and exotics, referring to Ref. [5] for details.

2.2. Photocouplings on the lattice

The first step is to calculate the spectrum on the lattice[1] using two-point correlation functions in Euclidean time, $C(t) = <0|O_i(t)O_j(0)|0>$. The interpolating operators $O$ are lattice-discretised fermion bilinears containing gamma matrices and gauge-covariant derivatives, $O(x) = \bar{\psi}(x)\Gamma_i D_j \bar{D}_k \ldots \psi(x)$; these operators are chosen to have the quantum numbers of the states under investigation. Inserting a complete set of states, the correlator can be rewritten as

$$C(t) = \sum_n e^{-E_n t} <0|O_i(0)|n> <n|O_j(0)|0>, \tag{1}$$

where the sum is over all states which overlap on to the operators, $E_n$ are the energies of the states and $Z_n = <n|O_i(0)|0>$ are the matrix elements (overlaps of operators on to states). The ground state dominates at large $t$ and so its energy and overlap ($Z$) can be extracted by fitting to an exponential for large $t$. To extract excited states, a better alternative than trying to fit multiple exponentials to a single correlator is to use a large basis of interpolating operators to form a matrix of correlators and a variational method, as implemented in Ref. [2]. In essence, the result is a generalised eigenvector problem with the eigenvalues giving $E_n$ and the eigenvectors related to the overlaps $Z_n$. This method also gives the linear combination of operators, $\Omega^{(n)}$, which have the ‘best’ overlap on to a particular state $n$. In addition, the overlaps can give information on about the nature of that state[2, 3].

Transition form factors are then calculated from three-point correlation functions with a source operator at timeslice $t_i$, a local vector current insertion at $t$ and a sink operator at $t_f$:

$$C_{ij}(\vec{p}_i, \vec{p}_j; t_i, t_f) = <0| \sum_\vec{z} \sum_\vec{y} e^{-i\vec{p}_i \cdot \vec{z}} O_i(\vec{z}, t_f) \times \sum_\vec{y} e^{i\vec{p}_j \cdot \vec{y}} \bar{\psi}(\vec{y}, t)\gamma^\mu \psi(\vec{y}, t) \times O_j(\vec{0}, t_i) |0> \tag{2}.$$ 

The sums over $\vec{z}$ and $\vec{y}$ project on to definite momenta $\vec{p}_i$ and $\vec{p}_j$, momentum at the source being enforced by momentum conservation. Inserting a complete set of states the three-point correlator can be written as

$$C_{ij}(\vec{p}_i, \vec{p}_j; t_i, t_f) = \sum \sum e^{-E_n(t_f-t_i)} e^{-E_m(t-t_i)} \times <0|O_i(\vec{0}, 0)|n(\vec{p}_j)> <n(\vec{p}_j)|\bar{\psi}(\vec{0}, 0)\gamma^\mu \psi(\vec{0}, 0)|m(\vec{p}_i)> <m(\vec{p}_i)|O_j(\vec{0}, 0)|0>. \tag{3}$$

The energies and overlaps are known from the two-point correlation function spectrum analysis and so the only unknown in above expression is the transition amplitude, $<n(\vec{p}_j)|\vec{p}^\mu (\vec{0}, 0)|m(\vec{p}_i)>$ with $\vec{p}^\mu (\vec{0}, 0) = \bar{\psi}(\vec{0}, 0)\gamma^\mu \psi(\vec{0}, 0)$. This transition amplitude can be parametrised in terms of known Lorentz covariant factors and multipole amplitudes[5]. These multipoles amplitudes are extracted experimentally at $Q^2 = 0$ from measured angular distributions; the sum of the squares of the multipoles amplitudes is proportional to the radiative width.

In Ref. [5], the ‘best’ operator $\Omega^{(n)}$, only overlapping on to the particular state being considering, was used at the sink, giving access to excited states at the sink. At the source, a simple local operator was used and large $t - t_i$ considered so that the only contribution at the source is from the ground state.

2.3. Results

Ref. [5] used three-point correlation functions and a previous spectrum analysis[2, 3] to extract radiative transition rates and multipole amplitudes from quenched anisotropic lattices; only connected diagrams were considered. The anisotropy (ratio of spatial to temporal lattice spacings) is $\xi = a_s/a_t = 3$, inverse temporal lattice spacing $a_t^{-1} = \ldots$
6.05 GeV and spatial extent $L_s \approx 1.2$ fm. The use of anisotropic lattices allows a finer temporal lattice spacing, necessary for extracting excited states, without the additional cost (for a given volume) of also having a finer lattice spacing in the spatial directions. The quenched approximation, where the light-quark degrees of freedom have been neglected, allows rather direct comparison to simple quark potential models. We refer to Ref. [5] for calculational details, results, systematic uncertainties, more detailed phenomenological discussion and comparison with models.

One technicality to note here is that on a finite lattice we are constrained to discrete values of momenta and so can not in general compute the multipole amplitudes at the physical point, $Q^2 = 0$. Therefore the $Q^2$ dependence of the lattice form factor must be fit to a suitable form and interpolated/extrapolated to the physical point as described in Ref. [5].

2.3.1. Exotic $\eta_{c1}(1^{-+})$ decay

One highlight of this study is the exotic $\eta_{c1}(1^{-+})$ radiative decay to $J/\psi \gamma$. This state was found in the spectrum analysis with a mass of 4300(50) MeV. We note that in a quenched lattice calculation, where there are no light-quark degrees of freedom, a state at this mass can not be a multiquark or molecular state. This strongly suggests that this state is a hybrid.

The extracted multipole amplitudes for $\eta_{c1} \rightarrow J/\psi \gamma$ are shown in Fig. 1. The decay, dominated by the magnetic dipole ($M_1$) amplitude, was found to be significant: $\Gamma(\eta_{c1} \rightarrow J/\psi \gamma) = 115(16)$ keV. This is of the same scale as many measured conventional charmonium transitions but is significantly larger than conventional magnetic dipole transitions in charmonium (c.f. $\Gamma(J/\psi \rightarrow \eta_c \gamma) \sim 2$ keV). Magnetic dipole transitions between conventional mesons require spin-flip (e.g. spin-triplet vector to a spin-singlet pseudoscalar) and in quark models this generically leads to suppression by the inverse quark mass, $\sim 1/m_c$. However, a transition from a spin-triplet hybrid to a vector requires no spin-flip, the gluonic field provides an additional degree of freedom that can be excited, and so there is no quark mass suppression, supporting the identification of this state as a spin-triplet hybrid. We note that if heavy quark physics is any guide to light meson physics, then GlueX would be expected to produce lots of exotic $1^{-+}$ mesons.

2.3.2. Vector – Pseudoscalar transitions and a vector-hybrid candidate

Another set of interesting results involve excited vector mesons and results of the spectrum analysis are summarised in Table 1. The radiative transition rates calculated on the lattice for the ground and first excited state decays to $\eta_c$ are given in Table 2, along with the experimental results and some model calculations. In these transitions only a magnetic dipole amplitude is allowed and the widths are small, being spin-flip transitions and so suppressed by the inverse quark
mass $\sim 1/m_c$. The CLEO-c measurement of $\Gamma(J/\psi \to \eta_c \gamma)$[7] was motivated by the discrepancy between an earlier lattice calculation of this rate[4] and the single experimental measurement from Crystal Ball[8]. The $\psi'$ transition is further suppressed because it is effectively the overlap of two orthogonal states broken only by the small recoil factor; this makes it even harder to calculate reliably in a model.

From considering the operator overlaps, the spectrum analysis hinted that one vector state (‘Y’), with mass around 4400 MeV, is a non-exotic hybrid. Fig. 2 shows the extracted magnetic dipole amplitude for the decay of this state to $\eta_c$. The resulting radiative width of $\Gamma(Y \to \eta_c \gamma) = 42(18)$ keV is significantly larger than conventional magnetic dipole transitions in charmonium. This supports the identification of this state as a spin-singlet hybrid for analogous reasons to those discussed above with regard to the $1^{-+}$ exotic: the transition of a spin-singlet hybrid to a pseudoscalar does not require spin-flip and so there is no quark mass suppression. We note that this radiative width is compatible with flux tube model calculations ($30 - 60$ keV). Further support for this assignment is provided by the scalar - vector transitions[5].

### 2.3.3. Tensor – Vector Transitions

Another phenomenologically interesting set of results is provided by the excited tensor transitions to a ground state vector $J/\psi$. Three tensor states were found: $\chi_{c2}$, $\chi_{c2}'$ and $\chi_{c2}''$ with masses 3545(7), 4115(28) and 4165(30) MeV respectively. The extracted multipole amplitudes are shown in Figs. 3 and 4.

The ground state multipole amplitudes have the hierarchy expected in models and that observed experimentally: $|E_1(0)| > |M_2(0)| > |E_3(0)|$, although the ratio $M_2/E_1$ is considerably larger than experiment. The calculated radiative transition width $\Gamma(\chi_{c2} \to J/\psi \gamma) = 380(50)$ keV is compatible with experiment $406(31)$ keV[9] and quark model expectations for a $1^3P_2$ state$^1$ ($\sim 290 - 420$ keV).

However, the first excited state multipole amplitudes have a completely different hierarchy: $|E_3(0)| > |M_2(0)|, |E_1(0)|$ with $\Gamma(\chi_{c2}' \to J/\psi \gamma) = 20(13)$ keV. The second excited state multipole amplitudes revert to the original hierarchy and the width, $\Gamma(\chi_{c2}'' \to J/\psi \gamma) = 88(13)$ keV, is compatible with quark model expectations for a $2^3P_2$ state ($\sim 50 - 80$ keV).

It turns out that these different hierarchies can be understand rather simply in terms of the general assumption that only a single quark is involved in the transition. In general, for a transition from a meson with spin $J_f$ to one with spin $J_i$, the allowed multipoles $F_k$ are those which satisfy $J_i = J_f \otimes k$ and $k > 0$. (Whether $F_k$ is a electric multipole, $E_k$, or a magnetic multipole, $M_k$, depends on the relative parities of the initial and final mesons.) Hence in the tensor to vector transitions we are considering here, $k = 1, 2, 3$ and the allowed multipole are $E_1, M_2$ and $E_3$.

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1 we use the standard $n^{3S+1}L_J$ spectroscopic notation

### Table 1.

<table>
<thead>
<tr>
<th>level</th>
<th>mass / MeV</th>
<th>suggested state</th>
<th>model assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3106(2)</td>
<td>$J/\psi$</td>
<td>$1^3S_1$</td>
</tr>
<tr>
<td>1</td>
<td>3746(18)</td>
<td>$\psi'(3686)$</td>
<td>$2^3S_1$</td>
</tr>
<tr>
<td>3</td>
<td>3864(19)</td>
<td>$\psi''(3770)$</td>
<td>$1^3D_1$</td>
</tr>
<tr>
<td>5</td>
<td>4400(60)</td>
<td>$Y$ ?</td>
<td>hybrid</td>
</tr>
</tbody>
</table>

### Table 2.

<table>
<thead>
<tr>
<th>$\Gamma$ / keV</th>
<th>Lattice</th>
<th>Exp.</th>
<th>Barnes, Godfrey, Swanson ‘NR’</th>
<th>Eichten et. al. ‘GI’</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J/\psi \to \eta_c \gamma$</td>
<td>2.51(8)</td>
<td>1.85(29) (CLEO-c)</td>
<td>2.9</td>
<td>2.4</td>
</tr>
<tr>
<td>$\psi' \to \eta_c \gamma$</td>
<td>0.4(8)</td>
<td>0.95(16) (PDG08)</td>
<td>4.6, 9.7</td>
<td>9.6</td>
</tr>
</tbody>
</table>
However, if only a single quark is involved in the transition there are further restrictions on \( k \). For the transition of a \( ^3P_2 \) tensor state (where one quark has angular momentum \( j_i = 3/2 \) relative to the other) to a \( ^3S_1 \) vector state (with \( j_f = 1/2 \), \( j_i = j_f \otimes k \) and so \( k = 1, 2 \) only and hence \( E_3 = 0 \) independent of any model details. Also, in general, quark models predict that \( E_1 > M_2 \), although the exact degree of this suppression is model dependent. This discussion explains the hierarchies observed in the \( \chi_{c1} \) and \( \chi'_{c1} \) transitions.

In contrast, by a similar argument, for the transition of a \( ^3F_2 \) tensor state \( (j_i = 5/2) \) to a \( ^3S_1 \) vector state \( (j_f = 1/2) \), the only allowed values of \( k \) are \( k = 2, 3 \) and hence \( E_1 = 0 \) independent of any model details. Again, in general, quark models predict that \( E_3 > M_2 \), although the exact degree of this suppression is model dependent. This discussion explains the hierarchy observed in the \( \chi'_{c1} \) transition if the \( \chi'_{c1} \) is identified as the \( 1^3F_2 \) state, an assignment supported by the spectrum analysis.

As we have shown, these calculations of multipole amplitudes have allowed the interpretation of the three tensors as \( 1^3P_1, 1^3F_2 \) and \( 2^3P_2 \) states. In general we’d expect the \( 1F \) and \( 2P \) states to mix due to the tensor potential, although this is suppressed by the inverse quark mass squared \( \sim 1/m_c^2 \) and the multipole hierarchies show no evidence for such
a mixing\(^2\). We note that the degeneracy of the 1\(F\) and 2\(P\) states may be an artefact of ‘squeezing’ these states in to a finite box in this lattice calculation. A discussion of excited tensor states is timely given the observation by Belle of a \(\chi_{c2}'\) candidate at 3930 MeV in \(\gamma\gamma \rightarrow D\bar{D}\)[12]. This calls for an extension of the lattice calculation of two-photon couplings[13] to excited states.

In summary, Ref. [5] has shown that these techniques can be used to successfully extract excited meson radiative transition rates and multipole amplitudes from lattice QCD. Many more results are given in that reference. In the following section we move on to the baryon sector.

### 3. EXCITED BARYON TRANSITION FORM FACTORS

The spectrum of excited baryons and their transition form factors is another active area, both experimentally and theoretically. The Roper resonance has been of particular interest since its discovery and there has been much discussion as to whether or not it is the first radially excited state of the nucleon. An understanding of the nature of this resonance may be gained by studying the Roper-nucleon transition form factors and these form factors are, among other things, being measured in experiments at Jefferson Lab (CLAS), MIT-Bates, LEGS, Mainz, Bonn, GRAAL and Spring-8. Although comparison with models is useful in understanding the Roper, many models fail to predict the value of its form factors and it is important to perform an \textit{ab initio} lattice calculation of these form factors.

Recently, the Roper-nucleon transition form factors have been calculated in the first study of excited nucleon transitions using lattice QCD[14, 15]. Quenched anisotropic lattices were used with similar techniques to those described above in Section 2.2 and only connected diagrams were considered; we refer to Refs. [14, 15] for details. An additional complication when studying light hadrons is that it is computationally very expensive to calculate with light quarks and so it isn’t currently possible to perform calculations at the physical pion mass. Instead three pion masses (480, 720 and 1100 MeV) were used in this study.

Preliminary lattice results from Ref. [15] on the Roper-proton transition form factors \(F_1\) and \(F_2\) are shown in Fig. 5. Other results, including Roper-neutron transition form factors are given in that reference. It can be seen that there is relatively mild dependence on pion mass although the results for the lightest pion are very noisy. In the \(F_2\) form factor there is good agreement with data except in the low \(Q^2\) region. Pion-cloud effects are though to be important in this region and would not be accounted for in such a quenched lattice calculation.

In summary, the first study of excited nucleon transition form factors has been successful and shown that there is relatively mild pion mass dependence. Future dynamical (unquenched) calculations are called for, especially to re-examine the low \(Q^2\) region, along with a larger basis of operators to study more excited states.

\(^2\) the short distance tensor interaction may not be reliable in these quenched lattice calculations[5]
4. CONCLUSIONS

We have discussed recent progress in calculating excited radiative transitions using lattice QCD. We have reviewed the results of Ref. [5] where the first calculation of excited meson radiative transitions using lattice QCD was performed and showed that the techniques developed can be applied successfully. Of particular interest are the exotic $1^{-+}$ hybrid candidate with a large radiative width and the identification of a non-exotic vector hybrid candidate. We have also shown how the pattern of multipole amplitudes can be used to shed light on the structure of states, such as the excited tensor charmonia. We highlighted related results in the baryon sector where the Roper-nucleon transition form factors have been studied for the first time in lattice QCD.

Although the numerical results presented are subject to lattice systematic errors, such as those arising from a finite volume, a finite lattice spacing, extrapolation in momentum, ignoring disconnected diagrams and the quenched approximation, systematic improvements can be made to overcome these. The results here are just the start of ongoing work which will go on to address some of these systematics, for example by using dynamical (unquenched) lattices on different volumes.

In the meson sector, having developed and tested techniques in the charmonium system, work is now underway towards calculating photocouplings of light mesons using unquenched lattices. Among other things, such photocouplings will be important inputs for the GlueX experiment at the JLab 12 GeV upgrade, with particular interest in exotic states.

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