The baryonic susceptibility near critical temperature

Shu Lin

Department of Physics and Astronomy, Stony Brook University, Stony Brook NY 11794-3800, USA

Abstract. We discuss the role of quarks and baryons near the QCD phase transition. The former is modelled in the spirit of PNJL model, while the latter is splittted into two classes: “stringy” and “non-stringy” baryons. We represent the non-stringy baryons by a sum over the resonance on equal footing, and obtain the density of states of stringy baryons from string inspired model at finite-\(T\). Our model produce a rise and fall of baryonic contribution to the susceptibility, which is in qualitative agreement with lattice results. We also discuss the chiral effect on the baryonic mass and susceptibility.

Keywords: baryonic susceptibility, string model, chiral transition

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BARYONIC SUSCEPTIBILITY BELOW AND ABOVE \(T_c\)

The baryonic susceptibility is defined as the certain number of derivatives of the QCD partition function with respect to the quark chemical potential: This quantity has been studied extensively by lattice simulations and has the advantage of isolating the degree of freedom with nonvanishing baryonic number, i.e. quark, diquark, and baryon. At low temperature, it is well known that the hadron resonance gas model[1] gives reasonable approximation to the QCD thermodynamical quantities. At very high temperature, \(T > 2.5T_c\), QCD is believed to be in a plasma phase[2], with free quarks and gluons being the elementary degree of freedom. This is nicely illustrated in the plot of kurtosis, defined as \(\frac{\frac{4}{d^2}}{d^2}\), in Fig.1.

Note the baryons contribute to the QCD partition function as \(\cosh(\frac{3\mu}{T})\), while the quarks contribute as \(\cosh(\frac{\mu}{T})\), with \(\mu\) being the quark chemical potential. The kurtosis results from pure baryon gas and quark gas are 9 and 1 respectively. Same argument applied to diquark will yield a kurtosis of 4. Fig.1 is in qualitative agreement with the picture that baryons and quarks dominate at low and high temperature respectively. It is also suggestive that diquarks play a less important role near critical temperature \(T_c\), where the kurtosis changes from 9 to 1. We will assume this is the case, and understand the baryonic susceptibility data in the near \(T_c\) region from the interplay between quark and baryon contributions.

An important point to have in mind is that lattice simulation usually uses large bare quark mass, which needs to be properly taken into account by our model calculation when making comparison with lattice simulation output. The specific lattice data we want to reproduce can be found in the papers by UK-Bielefeld-BNL group[3, 4]. The first set of data is for two flavors \(N_f = 2\) and large pion mass \(m_\pi = 770\,\text{MeV}\). The second set of data has strange quark included, but with a mass larger than the other two flavors. It has a pion mass close to the real world, \(m_\pi = 220\,\text{MeV}\). We will refer to them as set 1 and set 2 for brevity. It is also worth noting the two worlds have different absolute value for the critical temperature, with \(T_c \approx 220\,\text{MeV}\) for set 1 and \(T_c \approx 200\,\text{MeV}\) for set 2.

STRINGY AND NON-STRINGY BARYONS

The quark contribution to thermodynamical quantities is relatively straight forward to treat. According the main idea of the PNJL model[7], the quark contribution is suppressed by the Polyakov loop, which saturates to 1 at high temperature. Our model for the quark is a free quark gas, modulated by the Polyakov loop:

\[
p_q(T, \mu) = d \frac{1}{\pi^{2}} m_q^2 T K_2(m_q/T) \cosh(\mu/T) \langle L(T) \rangle
\]

Eq.(1) is the expression for the pressure due to quark(antiquark). Here \(d\) is the degeneracy factor. \(m_q\) and \(\langle L(T) \rangle\) are thermal mass of the quark and the expectation value of Polyakov loop, both have been measured on the lattice[8, 9].
The temperature dependence of the “kurtosis”, the ratio of susceptibilities $d_4/d_2$. The Set 1 (2 flavor QCD at $m_\pi = 770\text{MeV}$) are shown by black circles, while Set 2 (2+1 flavor QCD at $m_\pi = 220\text{MeV}$) is shown by (brown) solid boxes for $N_\tau = 6$ and open boxes for $N_\tau = 4$). The upper and lower horizontal lines correspond to the values 9 and 1, respectively.

The baryon contribution is nontrivial. Given the success of hadron resonance gas model in reproducing the lattice results at low temperature, one may hope similar mechanism can be extrapolated to near $T_c$ region. However, it cannot be true for the following reason[5]: As temperature increases, the baryons will have broader thermal width and overlap with each other, making individual baryon identification impossible. However this does not mean baryon does not exist in near $T_c$ region, a description in terms of density of states may still be valid. We will obtain a density of states from a modified version of string model.

The original string model by Polyakov and Susskind explains the exponential growth of the density of states of hadrons as all possible configuration of the string, which joins the constituent quarks of the hadron. The density of states of a hadron is given by:

$$\rho(L) = e^{S(L)} \sim 5^{L/a}$$

with $L$ the length of the string, $a$ is the string scale, $s$ minimum length of the string that can turn into different spatial directions. In three dimension, there are 5 directions to turn without self-intersecting. Our model for the baryonic density of states is the same but with temperature dependent string tension. The string tension is defined by the linear part of energy associated with quark-antiquark pair as:

$$V = \sigma_V(T)L + V_0$$
$$F = \sigma_F(T)L + V_0$$
$$S = \sigma_S(T)L$$

The three string tensions $\sigma_V, \sigma_F, \sigma_S$ distinguished by the subscripts correspond to internal energy, free energy and entropy respectively. They are related to each other by thermodynamical consistency relations, e.g.

$$\sigma_F = -T \int_{T_0}^T \frac{\sigma_V(T)}{T^2}$$

Fig.2[5] shows the temperature dependence of three string tensions, with the lattice measured internal energy string tension as the input.

Before applying the temperature dependent string model, we want to distinguish two types of baryons. Type one contains all the lowest states in each baryon family: for three flavors the 56-plet, spin 1/2 octet $N(938),\Lambda(1116)$, $\Sigma(1195),\Xi(1317)$ and 3/2 decuplet $\Delta(1232),\Sigma^*(1385),\Xi^*(1530),\Omega(1672)$, and for two flavors, the 20-plet of $N$ and $\Delta$) They are separated from the other baryons in the family by significant mass gaps. They can be described by radial and orbital excitation of quarks. Type two includes the rest of the baryons, which are from excitation of the “string”. The two types also show completely distinct chiral properties[13]: Type one interacts strongly with pion cloud and chiral
condensate while baryons in type two is chirally insensitive and form pair doublet, a signature of partial restoration of chiral symmetry.

In our model, we will interpret the type one baryon as non-stringy, to be described by sum of individual baryon resonances, while treat type two as stringy baryons, with their density of states given by string model [5]. The baryon contribution to the thermodynamics is given explicitly as:

\[ p(T, \mu) \equiv T \frac{\partial \ln Z(T, \mu)}{\partial V} = \sum_i \frac{1}{\pi^2} \frac{m_i^2}{T^2} K_2(m_i/T) \cosh(3\mu/T) + \int \frac{1}{\pi^2} \frac{m^2}{T^2} K_2(m/T) \cosh(3\mu/T) \rho(m) dm \]

\[ d_n(T) = \frac{\partial^n (p/T^n)}{\partial (\mu/T)^n} \bigg|_{\mu=0} \]

To account for the large pion mass used in lattice simulation, we adopt the following parametrization of non-stringy baryon mass:

\[ m_B = km_\pi^2 + N_s m_s + \text{gluonic part} \tag{9} \]

The first term encodes the light quark mass dependence of the baryon mass. The slope \( k \) is known as the “sigma term”, and its value has been studied on the lattice [11]. The second term is obviously from strange quark contribution.

For the stringy baryons, we simply use \( m = m_0 + \sigma_V(T)L(T) \) as their mass formula, with \( m_0 \) the contribution from the quark ends. The density of states of string baryons are given by [5]:

\[ \rho(m)|_{T=0} = ce^{\sigma_S L} = ce^{\sigma/v_s(m-m_0)} \rightarrow \rho(m)|_{T>0} = ce^{\sigma_S(T)/\sigma_V(T)(m-m_0)} \tag{10} \]

The first line in Eq.(10) corresponds to zero temperature case. The normalization constant \( c \) and \( T_0 \) is to be fixed by comparison with particle data book [12]. The second line generalizes to the finite temperature case. The constant factor \( x \) is introduced to account for the tightening effect of the string in the near \( T_c \) region.

The exponential growth of the density of states will win over the Boltzmann suppression factor when temperature crosses \( T_0 \) from below (the free energy tension vanishes), allowing the string to become infinitely long. We argue that the string length should be limited by the presence of quarks. Assuming random walk of the string end points, the end points are confined in the domain of influence of the quarks attached to them, otherwise switch to other quarks can occur. This gives us a string length cutoff:

\[ L_{\text{max}} = y^2 L_q^2 / a \tag{11} \]
FIGURE 3. (color online) Various contributions to the susceptibility $d_2(T)$ as a function of the temperature. The left figure shows the lattice data of the Set 1, with quark contribution subtracted, are shown by (black) circles. The unsubtracted data are also included, as (brown) boxes for comparison. The (red) dashed line corresponds to “stringy” baryons. The (blue) dash-dotted line corresponds to the sum of non-stringy baryons. The (black) solid line is the overall baryonic contribution. The parameters used are $x = 0.62, y = 0.08$. The right plot is for Set 2. The lattice data on susceptibility with quark contribution subtracted are shown as black circles(solid ones for $N_\tau = 6$ and open ones for $N_\tau = 4$). The unsubtracted data are also included as brown boxes (solid ones for $N_\tau = 6$ and open ones for $N_\tau = 4$) for comparison. The parameters used are $x = 0.51, y = 0.08$.

with $L_q$ the quark inter-distance, and $y$ some constant of order 1.

A comparison with lattice data on baryonic susceptibility is readily obtainable. In Fig.3[5], we show the results of our model calculation, together with lattice data. We have indicated relative contributions of non-stringy and stringy baryons, with the latter dominant in near $T_c$ region. Our model produces the rise of baryons, which is in nice agreement with lattice results. We have also observed the fall of baryons due to the proliferation of quarks above $T_c$.

CHIRAL EFFECT ON DENSITY OF STATES

The last ingredient we want to add to the model is the chiral effect on the baryonic density of states. We have identified two types of baryons: The stringy baryons partially respect chiral symmetry and barely interact with pion cloud, while the lowest non-stringy baryons are chirally sensitive. Our assumption of the chiral effect is it only enters the density of states through non-stringy baryons. The term $km_2^2$ in the mass formula (9) is proportional to the chiral condensate $m_q \langle B|\bar{\psi}\psi|B\rangle$, evaluated for specific baryon state. Therefore we have the following chirally modified mass formula for non-stringy baryons[5]:

$$m(T, \mu) = km_2^2 \frac{\langle B|\bar{\psi}\psi|B\rangle(T, \mu)}{\langle B|\bar{\psi}\psi|B\rangle|_{\mu=0}} + N_s m_s + \text{gluonic part}$$ (12)

The chiral condensate for certain baryon state is assumed to be the same as vacuum chiral condensate. The temperature dependence of the latter has been measured on the lattice[14]. Its $\mu$ dependence is obtained though the replacement: $T \rightarrow \sqrt{(T/T_c)^2 + (\mu/\mu_c)^2}$, with $\mu_c$ a free parameter. It has been pointed out early that the $\mu$ dependence of the density of states is essential for the deviation of kurtosis from 9[6, 15]. Fig.4[5] shows the effect of chiral condensate on the susceptibility results. We see the inclusion of this effect hardly change the $d_2$ plot, but shifts the peak upwards in $d_4$ and $d_6$ plots. Smaller values of $\mu_c$ result in higher peaks. However the peaks of $d_4$ and $d_6$ are over predicted either with ($\mu_c = 1.33, 2T_c$) or without ($\mu_c = \infty$) $\mu$ dependence of the baryon mass. The shift of the peak is more prominent near $T_c$, where the chiral condensate drops rapidly, thus the masses of the lowest states change rapidly as well.

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FIGURE 4. (color online) Three figures display the temperature dependence of the second, fourth and sixth susceptibilities. The points are the Set 1 lattice data with the quark contribution subtracted. The curves are for the model discussed in the text, with parameters values: \( x = 0.6, y = 0.08 \), and \( \mu_c = \infty \) (black solid line), \( \mu_c = 2T_c \) (red dashed line) and \( \mu_c = 1.33T_c \) (blue dash-dotted line).

FIGURE 5. (color online) The temperature dependence of the second and fourth susceptibilities, for Set 2 lattice data, with the quark contribution subtracted. The (black) solid circles correspond to \( N_T = 6 \) and open circles to \( N_T = 4 \). The curves are for the model discussed in the text, with parameters values: \( x = 0.51, y = 0.08 \), and \( \mu_c = \infty \) (black solid line), \( \mu_c = 2T_c \) (red dashed line) and \( \mu_c = 1.33T_c \) (blue dash-dotted line).

REFERENCES

11. F. X. Lee, “Excited baryon spectrum from lattice QCD,”