The Three-Body structure of the $x(2175)$ and the $Y(4260)$ states

Alberto Martínez Torres
IFIC-Instituto de Física Corpuscular
Centro mixto -CSIC Universidad de Valencia
HADRON 2009, Florida
Why should we study three meson systems?

- Until recently, meson-meson and meson-baryon structure of meson and baryon resonances has been explored extensively using chiral dynamics, e.g.:

  \[ \sigma(600), f_0(980), a_0(980), \kappa, \Lambda(1405), \Lambda(1520), \text{etc.} \]

- We have extend that formalism to systems made of two mesons and one baryon like, for example, \( \pi\pi N, \pi K N, \pi \pi \Sigma, \pi \pi \Lambda, \text{etc.} \),

---

Why should we study three meson systems?

• Until recently, meson-meson and meson-baryon structure of meson and baryon resonances has been explored extensively using chiral dynamics, e.g.:

\[ \sigma(600), f_0(980), a_0(980), \kappa, \Lambda(1405), \Lambda(1520), etc. \]

• We have extended that formalism to systems made of two mesons and one baryon like, for example, \( \pi \pi N, \pi K N, \pi \pi \Sigma, \pi \pi \Lambda \), etc.,

Dynamical generation of all the low-lying \( 1/2^+ \) \( \Sigma \) and \( \Lambda \).
Why should we study three meson systems?

- Until recently, meson-meson and meson-baryon structure of meson and baryon resonances has been explored extensively using chiral dynamics, e.g.:

  \[ \sigma(600), f_0(980), a_0(980), \kappa, \Lambda(1405), \Lambda(1520), \text{etc.} \]

- We have extended that formalism to systems made of two mesons and one baryon like, for example, \( \pi\pi N, \pi K N, \pi \pi \Sigma, \pi \pi \Lambda \), etc.,

Dynamical generation of \( N^*(1710), N^*(2100), \Delta(1910) \) and a new \( N^*(1920) \)

• What about three-meson states?

• Many new states found with a three-meson structure, for example:

  1) $X(2175)$ in $\Phi f_0(980)$  \[ \text{BABAR Collaboration, Phys.Rev.D74,091103 (2006), Phys.Rev.D76,012008 (2007).} \]

  2) $Y(4260)$ in $J/\Psi\pi\pi$  \[ \text{BES Collaboration, Phys.Rev.Lett.100,102003 (2008).} \]

  3) $X(1576)$ in $K^*K\pi$

  4) $Y(4660)$ in $J/\Psi(2s)\pi\pi$
• What about three-meson states?

• Many new states found with a three-meson structure, for example:

  1) $X(2175)$ in $\Phi f_0(980)$  

  2) $Y(4260)$ in $J/\Psi \pi \pi$  
     **Belle Collaboration**, e-Print: *hep-ex/0612006*

  3) $X(1576)$ in $K^*K\pi$

  4) $Y(4660)$ in $J/\Psi(2s)\pi\pi$
• What about three-meson states?

• Many new states found with a three-meson structure, for example:

1) $X(2175)$ in $\Phi f_0(980)$
   

2) $Y(4260)$ in $J/\Psi \pi\pi$
   
   Belle Collaboration, e-Print: hep-ex/0612006

3) $X(1576)$ in $K^*K\pi$
   

4) $Y(4660)$ in $J/\Psi(2s)\pi\pi$
• What about three-meson states?

• Many new states found with a three-meson structure, for example:

  1) $X(2175)$ in $\Phi f_0(980)$


  2) $Y(4260)$ in $J/\Psi \pi\pi$

  $\textit{Belle Collaboration}$, e-Print: hep-ex/0612006

  3) $X(1576)$ in $K^*K\pi$


  4) $Y(4660)$ in $J/\Psi(2s)\pi\pi$


• The study of three meson systems like $J/\Psi\pi\pi$, $K^*K\pi$, etc., could lead to some of these states.

How to proceed?
The model

- We solve the Faddeev equations

\[ T = T^1 + T^2 + T^3 \]

\[ T^i = t^i \delta^3(\vec{k}'_i - \vec{k}_i) + T^{ij}_R + T^{ik}_R \]

- The $T^{ij}_R$ matrices contain all the possible diagrams where the last two successive interactions are $t^i$ and $t^j$
• And they satisfy the equations:

\[
T_{R}^{12} = t^{1}g^{12}t^{2} + t^{1}\left[G^{121}T_{R}^{21} + G^{123}T_{R}^{23}\right]
\]

\[
T_{R}^{13} = t^{1}g^{13}t^{3} + t^{1}\left[G^{131}T_{R}^{31} + G^{132}T_{R}^{32}\right]
\]

\[
T_{R}^{21} = t^{2}g^{21}t^{1} + t^{2}\left[G^{212}T_{R}^{12} + G^{213}T_{R}^{13}\right]
\]

\[
T_{R}^{23} = t^{2}g^{23}t^{3} + t^{2}\left[G^{231}T_{R}^{31} + G^{232}T_{R}^{32}\right]
\]

\[
T_{R}^{31} = t^{3}g^{31}t^{1} + t^{3}\left[G^{312}T_{R}^{12} + G^{313}T_{R}^{13}\right]
\]

\[
T_{R}^{32} = t^{3}g^{32}t^{2} + t^{3}\left[G^{321}T_{R}^{21} + G^{323}T_{R}^{23}\right]
\]
\( t^i \) is the two-body \( t \)-matrix

\[ t = V + V \tilde{g} t \]

\( g^{ij} \) is the three-body green function

\[
g^{ij}(\vec{k}_i', \vec{k}_j) = \left( \prod_{r=1}^{D} \frac{N_r}{2E_r} \right) \frac{1}{\sqrt{s - E_i(\vec{k}_i') - E_l(\vec{k}_i' + \vec{k}_j) - E_j(\vec{k}_j)}}
\]

\( N_r = \begin{cases} 
1 & \text{meson-meson interaction} \\
2M_r & \text{meson-baryon interaction}
\end{cases} \)
• $t^i$ is the two-body t-matrix

\[ t = V + V \tilde{g} t \]

• $g^{ij}$ is the three-body green function
- $t^i$ is the two-body t-matrix

\[ t = V + V\tilde{g}t \]

- $g^{ij}$ is the three-body green function

Chiral amplitudes

On-shell part

Off-shell part
• $t^i$ is the two-body t-matrix

\[ t = V + V \tilde{g} t \]

• $g^{ij}$ is the three-body green function

Chiral amplitudes

On-shell part

Off-shell part
- $t^i$ is the two-body t-matrix
  $$t = V + V \tilde{g}t$$

- $g^{ij}$ is the three-body green function
• $t^j$ is the two-body t-matrix

\[ t = V + V \tilde{g} t \]

• $g^{ij}$ is the three-body green function

---

• $t^i$ is the two-body t-matrix $\Rightarrow t = V + V \tilde{g}t$

• $g^{ij}$ is the three-body green function

• $t^j$ is the two-body $t$-matrix

\[ t = V + V \tilde{g} t \]

• $g^{ij}$ is the three-body green function

\[ g_{ij} \]

\[ \text{On-shell part} \]

\[ \text{Off-shell part} \]

---

- $t^j$ is the two-body t-matrix

$$ t = V + V \tilde{g} t $$

- $g^{ij}$ is the three-body green function

• \( t^i \) is the two-body t-matrix

\[ t = V + V \tilde{g} t \]

• \( g^{ij} \) is the three-body green function

\[ \mathbf{cancellation in the SU(3) limit} \]

\[ \bullet \text{On-shell part} \]

\[ \bullet \text{Off-shell part} \]

\[ \begin{array}{l}
\mathbf{Chiral amplitudes} \\
\mathbf{On-shell part} \\
\mathbf{Off-shell part} \\
\mathbf{cancellation in the SU(3) limit}
\end{array} \]
• $t^i$ is the two-body t-matrix

\[ t = V + V \tilde{g}t \]

• $g^{ij}$ is the three-body green function

\[
g^{ij}(\vec{k}_i', \vec{k}_j) = \left( \prod_{r=1}^{D} \frac{N_r}{2E_r} \right) \frac{1}{\sqrt{s} - E_i(\vec{k}_i') - E_l(\vec{k}_i' + \vec{k}_j) - E_j(\vec{k}_j)}
\]

\[ N_r = \begin{cases} 
1 & \text{meson-meson interaction} \\
2M_r & \text{meson-baryon interaction}
\end{cases} \]

• $G^{ijk}$ is the loop function for diagrams involving three t-matrices

\[
G^{121} = \int \frac{d^3q_1}{(2\pi)^3} g^{12} t^2(q_1^2) g^{21} [g^{21}(\vec{k}_2', \vec{k}_1)]^{-1} t^2(\sqrt{s_{13}})^{-1}
\]

\[
t^1(\sqrt{s_{23}}) G^{121} t^2(\sqrt{s_{13}}) g^{21}(\vec{k}_2', \vec{k}_1) t^1(\sqrt{s_{23}})
\]
The $X(2175)$ resonance

**BaBar**

\[ e^+ e^- \rightarrow \phi f_0(980) \]

**BES**

\[ J/\Psi \rightarrow \eta \phi f_0(980) \]
• A study of the reaction $e^+e^- \rightarrow \phi f_0(980)$ using some mechanism involving loops reproduce the background, but not the peak.

• In the chiral models the $f_0(980)$ gets dynamically generated in the $\pi\pi$ and $K\bar{K}$ channels.

Then we need to calculate the $\phi K, \phi \bar{K}, \phi \pi, \pi \pi, K\bar{K}$ amplitudes.
• In the chiral models the $f_0(980)$ gets dynamically generated in the $\pi\pi$ and $K\bar{K}$ channels

[Diagram showing $K\bar{K}$ and $\phi\pi\pi$ structures]

• Then we need to calculate the $\phi K, \phi \bar{K}, \phi \pi, \pi \pi, K\bar{K}$ amplitudes.

We solve the Bethe-Salpeter equations

---


• In the chiral models the $f_0(980)$ gets dynamically generated in the $\pi\pi$ and $K\bar{K}$ channels

- Then we need to calculate the amplitudes $\phi K$, $\phi \bar{K}$, $\phi \pi$, $\pi \pi$, $K\bar{K}$.
• In the chiral models the $f_0(980)$ gets dynamically generated in the $\pi\pi$ and $K\bar{K}$ channels.

• Then we need to calculate the $\phi K$, $\phi \bar{K}$, $\phi \pi$, $\pi \pi$, $K\bar{K}$ amplitudes.
• In the chiral models the $f_0(980)$ gets dynamically generated in the $\pi\pi$ and $K\bar{K}$ channels.

\begin{align*}
&K \quad \bar{K} \\
&\phi
\end{align*}

\begin{align*}
&\pi \quad \pi \\
&\phi
\end{align*}

• Then we need to calculate the $\phi K, \phi\bar{K}, \phi\pi, \pi\pi, K\bar{K}$ amplitudes.

\begin{align*}
&\pi\pi, K\bar{K}, \pi\eta \\
&\sigma, f_0(980)
\end{align*}

• In the chiral models the \( f_0(980) \) gets dynamically generated in the \( \pi\pi \) and \( K\bar{K} \) channels

\[ \begin{array}{c}
\phi K, \phi \bar{K}, \phi \pi, \pi\pi, K\bar{K}
\end{array} \]

• Then we need to calculate the \( \phi K, \phi \bar{K}, \phi \pi, \pi\pi, K\bar{K} \) amplitudes.

\[ a_0(980) \]

\[ \begin{array}{c}
\pi\pi, K\bar{K}, \pi\eta
\end{array} \]

• In the chiral models the $f_0(980)$ gets dynamically generated in the $\pi\pi$ and $K\bar{K}$ channels

• Then we need to calculate the $\phi K$, $\phi \bar{K}$, $\phi \pi$, $\pi \pi$, $K\bar{K}$ amplitudes.
• In the chiral models the $f_0(980)$ gets dynamically generated in the $\pi\pi$ and $K\bar{K}$ channels

• Then we need to calculate the $\phi K$, $\phi \bar{K}$, $\phi \pi$, $\pi\pi$, $K\bar{K}$ amplitudes.

• In the chiral models the $f_0(980)$ gets dynamically generated in the $\pi\pi$ and $K\bar{K}$ channels.

Then we need to calculate the $\phi K, \phi\bar{K}, \phi\pi, \pi\pi, K\bar{K}$ amplitudes.

$\phi\bar{K}, \omega\bar{K}, \rho\bar{K}, \bar{K}^*\eta, \bar{K}^*\pi$
• In the chiral models the $f_0(980)$ gets dynamically generated in the $\pi\pi$ and $K\bar{K}$ channels.

\[ K, \bar{K}, \phi \]
\[ \pi, \bar{\pi}, \phi \]

• Then we need to calculate the $\phi K, \phi \bar{K}, \phi \pi, \pi \pi, K\bar{K}$ amplitudes.

$K^* \bar{K}, \rho \pi, \rho \eta, \omega \pi, \omega \eta, \phi \pi, \phi \eta, \bar{K}^* K$

$b_1(1235), h_1(1170), h_1(1380), a_1(1260), f_1(1285)$

• First, we solve the few-body equations with $\phi K \bar{K}$ as coupled channels.
\[ T_{pw}^{\phi f_0} [1 + G_{\phi f_0} T_{\phi f_0}] \]
• Uncertainties:

  1) $\phi\pi\pi$: 2% change in the peak position.
  2) Changes in the two-body cut-off of 20 MeV: shift of 8 MeV.

• No $\Phi a_0(980)$ resonance is found in that region.
The Y(4260) resonance

- Observed in the reaction $e^+e^- \rightarrow \pi^+\pi^- J/\psi$.
• Enhancement near 1 GeV in the ππ invariant mass.

• Analogy with X(2175) :

$$\pi\pi + J/\psi \rightarrow Y(4260)$$

$$f_0(980)$$

• We consider $J/\psi \pi\pi, J/\psi K\bar{K}$ as coupled channels.
systems. We find a resonance in both the systems at $\sqrt{s} = 4.50\text{MeV}$ with a full width at half maximum of 90 MeV. The peak appears when the invariant mass of two pseudoscalars is around that of the $f_0(980)$, indicating that the resonance has a strong coupling to the $J/\psi f_0(980)$ channel. Both the $J/\psi \pi\pi$ and the $J/\psi K\bar{K}$ amplitudes are similar in this energy region, with a difference in their magnitudes. We find the $J/\psi K\bar{K}$ amplitude to be much larger in magnitude as compared to that of the $J/\psi \pi\pi$ system. This reveals the strong coupling of the three-body resonance to $J/\psi f_0(980)$, since the $f_0(980)$ couples most strongly to $K\bar{K}$ \[34, 44, 45\].

In Fig. 4 we show the $|T_R|^2$ amplitude as a function of the total energy of the three body system and the invariant mass of the $K\bar{K}$ system. We have also studied the invariant mass spectrum of the two pions.
Summary and Future plans

- We have studied the three-meson systems, $\phi K \bar{K}$, $\phi \pi \pi$, where we got the resonance $X(2175)$.

- In the $J/\psi K K$, $J/\psi \pi \pi$ systems we obtain the $Y(4260)$.

- We have obtained four $\Sigma$'s and two $\Lambda$'s resonances in the $\pi \bar{K} N$, which correspond to all the $1/2^+ \Sigma$ and $\Lambda$ states in the energy region 1500-1870.

- We observed the $N^*(1710)$, $N^*(2100)$, $\Delta(1910)$ in the $\pi \pi N$ system and coupled channels and a possible $N^*(1910)$ with $J^P=1/2^+$ in the $K \bar{K} N$ system.

- A broad bump is obtained in the study of the $\pi K N$ system around 1700 MeV.

- Study of the systems $\omega \pi \pi$, $\rho \pi \pi$, $K^* \pi K$, etc., to get the low-lying vector resonances as $\omega(1420)$, $\omega(1650)$, etc.

and many more!!