Nucleon shape and electromagnetic form factors in the chiral constituent quark model

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29th November-4th December/ HADRON2009
University of Florida
Outline

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2. Electromagnetic Form Factors
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Fundamental quantities for internal structure of nucleon

- **Structure**: Magnetic moments
  Dirac theory (1.0 $\mu_N$) and experiment (2.5 $\mu_N$).
  Proton is not an elementary Dirac particle but has an inner structure.

- **Size**: Spatial extension.
  Proton charge distribution given by charge radius $r_p$.

- **Shape**: Nonspherical charge distribution.
  Quadrupole moment of the transition $N \rightarrow \Delta$.

- Relation between the properties??
The electromagnetic form factors are fundamental quantities of theoretical and experimental interest to describe the internal structure of nucleon.

The measurements of nucleon form factors at low momentum transfer are sensitive to the pion cloud and provide a test for the nucleon models and effective field theories of the QCD based on the chiral symmetry.

The knowledge of internal structure of nucleon in terms of quarks and gluons degrees of freedom of QCD provide a basis for understanding more complex, strongly interacting matter at the level of quarks and gluons.
The electromagnetic form factors are connected to the spatial charge and current distribution and are important for the determination of strange form factors contribution in the nucleon.

Recent experiment results at Bates, MAMI, JLAB have motivated new parameterizations and a new analysis.
Hadronic current for a spin $\frac{1}{2}$-nucleon with internal structure is

$$\langle B|J_{\text{had}}^\mu(0)|B\rangle = \bar{u}(p') \left( \gamma^\mu F_1(Q^2) + i \frac{\sigma^{\mu\nu}}{2M} q_\nu F_2(Q^2) \right) u(p)$$

$u(p)$ and $u(p')$ are the 4-spinors of the nucleon in the initial and final states.

The Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$ are the only two form factors allowed by relativistic invariance.

They are normalized in such a way that at $Q^2 = 0$, they reduce to electric charge and the anomalous magnetic moment in units of the elementary charge and the nuclear magneton $e/(2m_p)$

$F_1^p(0) = 1$, $F_2^p(0) = \kappa_p = 1.793$

$F_1^n(0) = 0$, $F_2^n(0) = \kappa_n = -1.913$. 
In analogy with non-relativistic physics, the form factors can be associated with the Fourier transforms of the charge and magnetization densities.

However, charge distribution $\rho(\vec{r})$ has to be calculated by a 3-dimensional Fourier transform of the form factor as function of $\vec{q}$, whereas the form factors are generally functions of $Q^2 = \vec{q}^2 - \omega^2$.

In the special Lorentz frame, the Breit frame, the energy of the (space-like) virtual photon vanishes.

We have $J_\mu = \left( G_E(Q^2), i\frac{\vec{\sigma} \times \vec{q}}{2M} G_M(Q^2) \right)$

$G_E(Q^2)$: Time-like component of $J_\mu$ (Fourier transform of the electric charge distribution)

$G_M(Q^2)$: Fourier transform of the magnetization density.
The Sachs form factors $G_E$ and $G_M$ are related to the Dirac form factors by

\[ G_E^N(Q^2) = F_1^N(Q^2) - \tau F_2^N(Q^2) \]
\[ G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2) \]
The Sachs form factors (Fourier transformed in the Breit frame)

\[ G_E(\vec{q}^2) = \int \rho(\vec{r}) e^{i\vec{q} \cdot \vec{r}} d^3\vec{r} = \int \rho(\vec{r}) d^3\vec{r} - \frac{\vec{q}^2}{6} \int \rho(\vec{r}) \vec{r}^2 d^3\vec{r} + \ldots \]

The first integral yields the total charge in units of \( e \), i.e., 1 for the proton and 0 for the neutron, and the second integral defines the square of the electric root-mean-square (rms) radius, \( \langle r^2 \rangle_E \).

The charge density need not be spherically symmetric, i.e., in general \( \rho(\vec{r}) \neq \rho(r) \).

Geometric shape of a baryon is determined by its intrinsic quadrupole moment.
Information on the intrinsic quadrupole moments of the spin 1/2 baryons can be obtained from measurements of electric ($E2$) and Coulomb ($C2$) quadrupole transitions to excited states.

From the multipole expansion at low momentum transfers, \( \rho(q) = e - \frac{q^2}{6} r^2 - \frac{q^2}{6} Q + \cdots \)

The third term characterizes the shape (\(Q\)) of the system.
The charge radius operator can be expressed as a sum of one-, two-, and three-quark terms in spin-flavor space as
\[ r^2 = A \sum_{i=1}^{3} e_i \mathbf{1} + B \sum_{i \neq j}^{3} e_i \sigma_i \cdot \sigma_j + C \sum_{i \neq j \neq k}^{3} e_k \sigma_i \cdot \sigma_j \]

- \[ e_i = \left( 1 + 3 \tau_{i z} \right) / 6 \] and \( \sigma_i \) are the charge and spin of the \( i \)-th quark. \( \tau_{i z} \) denotes the \( z \) component of the Pauli isospin matrix. The constants \( A, B, \) and \( C \) parametrizing the orbital and color matrix elements are determined from experiment.

- The matrix element can be calculated for the operator \( \mathcal{O} \) (in the present case for the electric quadrupole \( Q \) and charge radius operator) and the wavefunction for the baryon state.
Similarly, the charge quadrupole operator is composed of a two- and three-body term in spin-flavor space

\[ Q = B' \sum_{i \neq j} e_i \left( 3\sigma_i z \sigma_j z - \sigma_i \cdot \sigma_j \right) + C' \sum_{i \neq j \neq k} e_k \left( 3\sigma_i z \sigma_j z - \sigma_i \cdot \sigma_j \right). \]

Baryon decuplet quadrupole moments \( Q_{B^*} \) and octet-decuplet transition quadrupole moments \( Q_{B \rightarrow B^*} \) are obtained by calculating the matrix elements of the quadrupole operator between the three-quark spin-flavor wave functions

\[ Q_{B^*} = \langle B^* | Q | B^* \rangle \]
\[ Q_{B \rightarrow B^*} = \langle B^* | Q | B \rangle \]

\( B \) denotes a spin 1/2 octet baryon and \( B^* \) a member of the spin 3/2 baryon decuplet.
Chiral Constituent Quark Model

- $\chi$CQM initiated by Weinberg and developed by Manohar and Georgi to explain the successes of NRQM.
- "Quark sea" generation $q_{\pm} \rightarrow GB^0 + q'_{\mp} \rightarrow (q\bar{q}') + q'_\mp$
- Incorporates confinement and chiral symmetry breaking.
- "Justifies" the idea of constituent quarks and scope of the model extended in the context of "proton spin crisis"
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Methodology

- "Quark sea" generation \( q_\pm \rightarrow GB^0 + q'_\mp \rightarrow (q\bar{q}') + q'_\mp \)
- \( \mathcal{L} = g_8 \bar{q} \Phi q + g_1 \bar{q} \frac{n'}{\sqrt{3}} q = g_8 \bar{q} (\Phi + \zeta \frac{n'}{\sqrt{3}} I) q \)

\[
\Phi = \begin{pmatrix} 
\frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{n'}{\sqrt{3}} & \pi^+ & \alpha K^+ \\
\pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{n'}{\sqrt{3}} & \alpha K^0 \\
\alpha K^- & \alpha \bar{K}^0 & -\beta \frac{2\eta}{\sqrt{6}} + \zeta \frac{n'}{\sqrt{3}} 
\end{pmatrix}
\]

- The parameter \( a(=|g_8|^2) \) denotes the transition probability of chiral fluctuation of the splittings \( u(d) \rightarrow d(u) + \pi^+(\pm) \), whereas \( \alpha^2 a, \beta^2 a \) and \( \zeta^2 a \) respectively denote the probabilities of transitions of \( u(d) \rightarrow s + K^{-(o)}, u(d, s) \rightarrow u(d, s) + \eta, \) and \( u(d, s) \rightarrow u(d, s) + \eta' \).
Methodology

- "Quark sea" generation $q_{\pm} \rightarrow GB^0 + q'_{\mp} \rightarrow (q\bar{q}') + q'_{\mp}$
- $\mathcal{L} = g_8\bar{q}\Phi q + g_1\bar{q}'\frac{\eta'}{\sqrt{3}}q = g_8\bar{q}\left(\Phi + \zeta\frac{\eta'}{\sqrt{3}}I\right)q$

$$\Phi = \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} + \zeta\frac{\eta'}{\sqrt{3}} & \pi^+ & \alpha K^+ \\
\pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} + \zeta\frac{\eta'}{\sqrt{3}} & \alpha K^0 \\
\alpha K^- & \alpha \bar{K}^0 & -\beta\frac{2\eta}{\sqrt{6}} + \zeta\frac{\eta'}{\sqrt{3}}
\end{pmatrix}$$

- The parameter $a(=|g_8|^2)$ denotes the transition probability of chiral fluctuation of the splittings $u(d) \rightarrow d(u) + \pi^+(-)$, whereas $\alpha^2 a$, $\beta^2 a$ and $\zeta^2 a$ respectively denote the probabilities of transitions of $u(d) \rightarrow s + K^{-}(o)$, $u(d, s) \rightarrow u(d, s) + \eta$, and $u(d, s) \rightarrow u(d, s) + \eta'$. 
Results

- **Experiment**
  1. $Q_{p \rightarrow \Delta^+} = -0.108 \, fm^2$ [Blanpied et al. (2001)]
  2. $Q_{p \rightarrow \Delta^+} = -0.0846 \, fm^2$ [Taitor et al. (2003)]

- $N \rightarrow \Delta$ quadrupole moment is related to the neutron charge radius as
  
  $$Q_{p \rightarrow \Delta^+} = Q_{n \rightarrow \Delta^0} = \frac{1}{\sqrt{2}} \, r_n^2 = -0.08$$
  which is experimentally well satisfied.

- The neutron charge radius plays an important role for the size of quadrupole moments and the corresponding intrinsic baryon deformation.
SU(6) symmetry breaking results better as compared to SU(6) symmetry results.

Substantiated by a measurement of the baryon charge radius and other transition quadrupole moments.

At leading order, the model envisages constituent quarks, the Goldstone bosons ($\pi$, $K$, $\eta$ mesons) as appropriate degrees of freedom.