We investigate the finite-temperature non-perturbative quark–antiquark potential by using the Field Correlator Method (FCM). The $J/\psi$ and $\Upsilon$ binding energies in a quark-gluon plasma are calculated.

Since 1986, the gold-plated signature of deconfinement was thought to be $J/\psi$ suppression. If Debye screening of the Coulomb potential above $T_c$ is strong enough then $J/\psi$ production in A+A collisions will be suppressed.
$m_d(T)$ for $n_f = 0$ (red line) and $n_f = 2$ (blue line)  

N.Agasian: 2007

$$V_C(r) = -\frac{4}{3} \cdot \frac{\alpha_s(r) \cdot e^{-m_d r}}{r}, \quad n \leq \mu_c \int_0^\infty |V_C(r)|r \, dr = \frac{4\alpha_s}{3} \cdot \frac{\mu_c}{m_d} \cdot \frac{1.4 \cdot 0.5}{m_d}$$

If $m_d \geq 0.7$ GeV, there is no $J/\psi$ bound state. Parenthetically, no strange mesons ($\mu_s \sim 500$ MeV) survive. But this is not the full story!
There is a significant change of views on physical properties and underlying dynamics of QGP, produced at RHIC.

Instead of behaving like a gas of free quasiparticles – quarks and gluons, the matter created in RHIC interacts much more strongly than originally expected.

Also, the interaction deduced from lattice studies is strong enough to support multiple bound states.

It is more appropriate to describe the non-perturbative properties of the QCD phase close to $T_c$ in terms remnants of the non–perturbative part of the QCD force rather than a strongly coupled Coulomb force.

In the QCD vacuum, the non–perturbative $Q\bar{Q}$ potential is $V = \sigma r$. At $T \geq T_c$, $\sigma = 0$, but that does not mean that NP potential disappears.
The Field Correlator Method as applied to finite T (Simonov: 1992,2005; NST:2009)

QGP effects on $Q\overline{Q}$ NP potential can be studied through the modification of the correlator functions, which define the quadratic field correlators of the nonperturbative vacuum fields

$$< tr F_{\mu\nu}(x)\Phi(x,0)F_{\lambda\sigma}(0) >= A_{\mu\nu;\lambda\sigma}D(x) + B_{\mu\nu;\lambda\sigma}D_1(x)$$

$\Phi(x,0)$ is the parallel transporter, $x$ Euclidian.

At $T = 0$, the string tension $\sigma$ is expressed only in terms of $D(x)$:

$$\sigma = 2 \int_0^\infty d\lambda \int_0^\infty d\nu \ D(\sqrt{\lambda^2 + \nu^2}), \ x^2 = \nu^2 + \lambda^2$$

At $T \geq T_c$, one should distinguish between electric and magnetic correlators

$$D_E(x), \ D^H(x), \ D_1^E(x), \text{ and } D_1^H(x)$$

Above $T_c$, $D^E(x) = 0$, $\sigma^E = 0$

The color magnetic correlators $D^H(x)$ and $D_1^H(x)$ do not produce static quark–antiquark potentials, they only define the spatial string tension $\sigma_s = \sigma^H$ and the Debye mass $m_d \propto \sqrt{\sigma_s}$ that grows with $T$. 
The main source of the non–perturbative static $Q\overline{Q}$ potential at $T \geq T_c$ originates from the color–electric correlator function $D_1^E(x)$.

The color electric correlator $D_1^E(x)$. Below $T_c$

$$D_1^E(x) = B \frac{\exp(-M_0 \cdot x)}{x}$$

$B = 6\alpha_s^f \sigma_f M_0$, $\alpha_s \sim 0.6$, $\sigma_f = 0.18$, $M_0$ (the gluelump mass)$\sim 1$ GeV,

Above $T_c$ the analytical form of $D_1^E$ should stay unchanged at least up to $T \sim 2T_c$.

The only change is $B \rightarrow B(T) = B(1 - \lambda(T - T_c))$ (DiGiacomo et al. 2007)

$$V_{np}^{Q\overline{Q}}(r, T) = \int_0^{1/T} d\nu (1 - \nu T) \int_0^r \lambda d\lambda D_1^E(x), \quad x^2 = \lambda^2 + \nu^2$$

Integrating over $\lambda$ one obtains

$$V_{np}(r, T) = \frac{B(T)}{M_0} \int_0^{1/T} (1 - \nu T) \left(e^{-\nu M_0} - e^{-\sqrt{\nu^2 + r^2}} M_0\right) d\nu = V(\infty, T) - V(r, T)$$
$V^{np}_{Q\bar{Q}}(\infty, T_c) \approx 0.5$ GeV that agrees with estimate for the free energy obtained from lattice data
$V_{np}(r)$ calculated in the MFC
In the framework of the FCM, the masses of heavy quarkonia are defined as

\[ M_{Q\bar{Q}} = \frac{m_Q^2}{\mu_Q} + \mu_Q + E_0(m_Q, \mu_Q), \]

\[ E_0(m_Q, \mu_Q) \] is an eigenvalue of the Hamiltonian \( H = H_0 + V_{Q\bar{Q}}, \)

\( m_Q \) are the bare quark masses,

\( \mu_Q \) are the auxiliary fields that are introduced to simplify the treatment of relativistic kinematics. The auxiliary fields are treated as c–number variational parameters to be found from the extremum condition

\[ \frac{\partial}{\partial \mu_Q} M_{Q\bar{Q}} = 0. \]

Such an approach allows for a very transparent interpretation of auxiliary fields as the constituent masses that appear due to the interaction.

Once \( m_Q \) is fixed, the quarkonia spectrum is described. We take \( m_c = 1.4 \text{ GeV}, \)

\( m_b = 4.8 \text{ GeV}. \) The dissociation points are defined as those temperature values for which the energy gap between \( V(\infty, T) \) and \( E_0 \) disappears.
Table 1: 1S $b\bar{b}$ state above the deconfinement region. $V_{Q\bar{Q}}(\infty, T)$ is the continuum threshold (a constant shift in the potential). The dissociation point is defined as such the temperature value for which the energy gap between $V_{Q\bar{Q}}(\infty, T)$ and $E_0$ disappears. Units are GeV or GeV$^{-1}$

<table>
<thead>
<tr>
<th>(T/T_c)</th>
<th>(V_{Q\bar{Q}}(\infty, T))</th>
<th>(\mu_b)</th>
<th>(E_0(T) - V_{Q\bar{Q}}(\infty, T))</th>
<th>(\sqrt{\langle r^2 \rangle})</th>
<th>(M_{b\bar{b}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.445</td>
<td>4.948</td>
<td>-0.255</td>
<td>1.39</td>
<td>9.796</td>
</tr>
<tr>
<td>1.3</td>
<td>0.332</td>
<td>4.922</td>
<td>-0.158</td>
<td>1.69</td>
<td>9.777</td>
</tr>
<tr>
<td>1.6</td>
<td>0.237</td>
<td>4.894</td>
<td>-0.084</td>
<td>2.23</td>
<td>9.755</td>
</tr>
<tr>
<td>2.0</td>
<td>0.134</td>
<td>4.854</td>
<td>-0.022</td>
<td>4.23</td>
<td>9.712</td>
</tr>
<tr>
<td>2.2</td>
<td>0.090</td>
<td>4.831</td>
<td>-0.006</td>
<td>6.77</td>
<td>9.684</td>
</tr>
<tr>
<td>2.3</td>
<td>0.070</td>
<td>4.821</td>
<td>-0.002</td>
<td>8.32</td>
<td>9.668</td>
</tr>
</tbody>
</table>

At $T = T_c$ the $b\bar{b}$ separation $r_0$ for the $\Upsilon$ is 0.25 fm that compatible with $r_0 = 0.28$ fm at $T = 0$. The 1S bottomium undergo very little modification till $T \sim 2T_c$. 
Table 2: $J/\psi$ above the deconfinement region.

<table>
<thead>
<tr>
<th>$T/T_c$</th>
<th>$V_{Q\overline{Q}}(\infty)$</th>
<th>$\mu_b$</th>
<th>$E_0 - V_{Q\overline{Q}}(\infty)$</th>
<th>$\sqrt{\langle r^2 \rangle}$</th>
<th>$M_{J/\psi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.445</td>
<td>1.443</td>
<td>-0.011</td>
<td>8.23</td>
<td>3.235</td>
</tr>
<tr>
<td>1.2</td>
<td>0.368</td>
<td>1.423</td>
<td>-0.003</td>
<td>10.07</td>
<td>3.171</td>
</tr>
</tbody>
</table>
The three quark potential is given by

\[ V_{QQQ} = \frac{1}{2} \sum_{i<j} V_{QQ}(r_{ij}, T), \]

where \( \frac{1}{2} \) is the color factor. We solve the three quark Schrödinger equation using the hyperspherical method.

\[
\frac{d^2 u(R, T)}{dR^2} + 2 \left[ E_0 - \frac{15}{8R^2} - \frac{3}{2} \xi(T) (V_C(R, T) + V_{np}(R, T)) \right] u(R, T) = 0,
\]

\[
R^2 = \frac{\mu_Q}{3} \left( r_{12}^2 + r_{23}^2 + r_{31}^2 \right), \quad V_C(R, T) = -\frac{4\alpha_s}{3R} a_C(R),
\]

\[
a_C(R) = \frac{16}{\pi} \sqrt{\frac{\mu_Q}{2}} \int_0^{\pi/2} \exp \left( -\frac{m_D R \sin \theta}{\sqrt{\mu_Q/2}} \right) \sin \theta \cos^2 \theta d\theta,
\]

\[
V_{np}(R, T) = V_{np}(\infty, T) - \frac{16B}{\pi M_0} \sqrt{\frac{2}{\mu_Q}} \left( \int_0^{\pi/2} K_1 \left( \frac{M_0 R \sin \theta}{\sqrt{\mu_Q/2}} \right) \sin^3 \theta \cos^2 \theta d\theta \right) R.
\]
Table 3: The melting points (in units of $T_c$) for $J/\psi$, $\Upsilon$, $\Omega_c$, and $\Omega_b$

<table>
<thead>
<tr>
<th></th>
<th>$J/\psi$</th>
<th>$\Upsilon$</th>
<th>$\Omega_c$</th>
<th>$\Omega_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work</td>
<td>1.3</td>
<td>2.3</td>
<td>1.2</td>
<td>2.3</td>
</tr>
<tr>
<td>Blaschke et al.</td>
<td>1.2</td>
<td>2.3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Alberigo et al.</td>
<td>2</td>
<td>$(4 - 6)$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The results for the $c\bar{c}$ and $c\bar{c}$ bound state qualitatively agree with those obtained from the calculations based on phenomenological $Q\bar{Q}$ potentials identified with the free energy measured on the lattice (Blaschke et.al: 2005). However, our melting temperature for $1S(\Upsilon)$ is much smaller than $T \sim (4 - 6) T_c$ found in Alberico et al: 2007)