Light scalar glueball possibility
aim on a clean way to identify

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Outline

- Scalar glueball study status
- Mixing between glueball and ordinary scalar meson — study through glueball production
- a theoretical clean way to identify a scalar glueball
- Summary
Scalar Mesons \((J^P=0^+)\)

\[
\begin{align*}
q\bar{q} & & a_0^-(1470) & & a_0(1470) & & a_0^+(1470) \\
K_0^*(1430) & & K_0^*(1430) & & f_0(1500) & & f_0(1370) \\
K_0^*(1430) & & K_0^*(1430) & & f_0(1710) & & f_0(1710) \\
q^2\bar{q}^2 & & a_0^-(980) & & a_0(980) & & a_0^+(980) \\
\kappa(800) & & \kappa(800) & & f_0(980) & & \sigma(600) \\
\kappa(800) & & \kappa(800) & & \sigma(600) & & \sigma(600)
\end{align*}
\]
Glueball study

- Glueballs are allowed by QCD theory
- Many studies have been performed theoretically
- Many candidates have been studied experimentally
- However, up to now no definite evidence for the existence of glueball
Glueball: color-singlet bound state of gluons as gluons have a self coupling

Lightest glueballs:

- $J^{PG}=0^{++}$: $1710\pm50\pm80$ MeV
- $J^{PG}=2^{++}$: $2390\pm30\pm120$ MeV
- $J^{PG}=0^{-+}$: $2560\pm35\pm120$ MeV

Glueball will mix with $qq$ states so that a pure glueball may not exist in nature.
Mixing between glueball and quark states

\[
\begin{pmatrix}
M_G & y & \sqrt{2}y \\
y & M_S & 0 \\
\sqrt{2}y & 0 & M_N
\end{pmatrix}
\]

\( f_0(1710) \): primarily an ss state

\[ |f_0(1710)\rangle = 0.36|G\rangle + 0.09|N\rangle + 0.93|S\rangle \]

\( f_0(1500) \): primarily a glueball

\[ |f_0(1500)\rangle = -0.84|G\rangle - 0.41|N\rangle + 0.35|S\rangle \]

\( f_0(1370) \)

\[ |f_0(1370)\rangle = 0.40|G\rangle - 0.91|N\rangle - 0.07|S\rangle \]

Amsler, Close, Kirk, Zhao, He, Li…

\( M_S > M_G > M_N \)

\( M_G \sim 1500 \text{ MeV}, \quad M_S - M_N \sim 200-300 \text{ MeV} \)
$$f_0(1710)$$ : primarily a glueball

$$f_0(1500)$$ : tends to be an SU(3) octet

$$f_0(1370)$$ : near SU(3) singlet + glueball content ($\sim 13\%$)

$$M_N = 1474\text{ MeV},\quad M_S = 1498\text{ MeV},\quad M_G = 1666\text{ MeV},\quad M_G > M_S > M_N$$

- $M_S - M_N \sim 25\text{ MeV}$ is consistent with LQCD result
  $$\Rightarrow$$ near degeneracy of $a_0(1450), K_0^*(1430), f_0(1500)$
Feynman diagrams of form factors

(a) \( \bar{B} \) \( G \) \( \bar{B} \) \( (a) \)

Ordinary scalar

(b) \( \bar{B} \) \( G \) \( \bar{B} \) \( (c) \)

(c) \( \bar{B} \) \( G \) \( \bar{B} \) \( (c) \)

(d) \( \bar{B} \) \( S \) \( \bar{B} \) \( (d) \)

(e) \( \bar{B} \) \( S \) \( \bar{B} \) \( (e) \)

Glueball production
Feynman diagrams of form factors

Electroweak vertex

Glueball production

Ordinary scalar
Form factors calculated in pQCD

After complicated calculation, we found that the $B \rightarrow$ glueball form factors are not negligible

<table>
<thead>
<tr>
<th>transitions</th>
<th>$F_0(0) = F_1(0)$</th>
<th>$F_T(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \rightarrow G$</td>
<td>$0.11^{+0.02}_{-0.02}$</td>
<td>$0.05^{+0.01}_{-0.01}$</td>
</tr>
<tr>
<td>$B \rightarrow \pi$</td>
<td>$0.22^{+0.04}_{-0.05}$</td>
<td>$0.27^{+0.05}_{-0.06}$</td>
</tr>
<tr>
<td>$B \rightarrow f_0$ Scenario I</td>
<td>$-0.30^{+0.08}_{-0.09}$</td>
<td>$-0.39^{+0.10}_{-0.11}$</td>
</tr>
<tr>
<td>$B \rightarrow f_0$ Scenario II</td>
<td>$0.63^{+0.23}_{-0.14}$</td>
<td>$0.76^{+0.37}_{-0.17}$</td>
</tr>
</tbody>
</table>
Estimated branching ratios for pure glueball

\[ \mathcal{B}(B \rightarrow Gl\bar{\nu}) = (0.73^{+0.30+0.14+0.10}_{-0.23-0.13-0.07}[0.44^{+0.18+0.09+0.06}_{-0.14-0.08-0.04}]) \times 10^{-5} \]

\[ \mathcal{B}(B \rightarrow G\tau\bar{\nu}_{\tau}) = (0.36^{+0.15+0.07+0.05}_{-0.11-0.06-0.04}[0.23^{+0.09+0.04+0.03}_{-0.07-0.04-0.03}]) \times 10^{-5} \]

Compared with the measured result of \( B \rightarrow \eta l\bar{\nu} \) decay [15]

\[ \mathcal{B}(B^{-} \rightarrow \eta l^{-}\bar{\nu}) = (3.1 \pm 0.6 \pm 0.8) \times 10^{-5} \]

\[ \mathcal{B}(B \rightarrow Gl^{+}l^{-}) = (1.2^{+0.5+0.2}_{-0.5-0.2}[0.73^{+0.30+0.14}_{-0.22-0.13}]) \times 10^{-9} \]

\[ \mathcal{B}(B \rightarrow G\tau^{+}\tau^{-}) = (3.0^{+1.2+0.5}_{-0.9-0.5}[2.0^{+0.8+0.4}_{-0.6-0.4}]) \times 10^{-11} \]

\[ \mathcal{B}(B_{s} \rightarrow Gl^{+}l^{-}) = (2.3^{+0.8+0.4}_{-0.6-0.4}[1.4^{+0.7+0.3}_{-0.4-0.3}]) \times 10^{-8} \]

\[ \mathcal{B}(B_{s} \rightarrow G\tau^{+}\tau^{-}) = (1.0^{+0.3+0.2}_{-0.3-0.2}[0.66^{+0.24+0.15}_{-0.16-0.11}]) \times 10^{-9} \]
Determination of mixing parameters

- The semileptonic $B_s \rightarrow f_0 l^+ l^-$ channel only receive contributions from the $SS$ component but without $nn$ component.
Determination of mixing parameters

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- It can also receive gluon component contributions.
Determination of mixing parameters

- The semileptonic $B \rightarrow f_0 l \nu$ decays receive contributions from the $nn$ component but without $ss$ component (at least negligible)
Determination of mixing parameters

- The semileptonic $B \rightarrow f_0 \ell^- \nu$ decays receive contributions from the $nn$ component but without $ss$ component (at least negligible).

- It can also receive gluon component contributions. Thus the two independent mixing parameters can be fitted from the above two experimental measurements, in principle.
For the three kinds of $f_0$'s, we have altogether 6 experimental measurements, but only three real parameters (unitary) to be fixed, if form factors provided.

Since the branching fraction of $B_s \rightarrow f_0 \ell^+ \ell^-$ is expected to have the order of $10^{-8}$ or even smaller, one needs to accumulate a large number of $B$ decay events.

This could be achieved on the future experiments such as the Super $B$ factory.
Semileptonic B decays are clean but in $B \to f_0 l^+ l^-$, the neutrino is identified as missing energy thus the efficiency is limited.

The $B \to J/\psi f_0$ has a small branching ratio.

The lepton pair can also be replaced by a charmonium state such as $J/\psi$, since $J/\psi$ does not carry any light flavor either.

$B \to J/\psi (\eta_c) f_0$ decays are also ideal channels.
The branching fraction is expected to have the order

\[ \mathcal{B}(B \rightarrow f_0 J/\psi) \sim \mathcal{B}(\bar{B}^0 \rightarrow \rho^0 J/\psi) \frac{\mathcal{B}(D_s \rightarrow f_0 l\bar{\nu})}{\mathcal{B}(D_s \rightarrow \phi l\bar{\nu})} \]

\[ \sim 10^{-5} \times \frac{10^{-3}}{10^{-2}} = 10^{-6}. \quad (22) \]

On experimental side, the \( J/\psi \) is easily detected through a lepton pair \( l^+l^- \) and thus this mode may be more useful. If the \( J/\psi \) meson is replaced by \( \eta_c \) in eq.(20,21), one can get the similar sum rules.
Brangching ratios estimated in naïve factorization

Using \( \mathcal{B}(\bar{B}^0 \to J/\psi \bar{K}^0) = (8.71 \pm 0.32) \times 10^{-4} \).

The branching ratios are roughly predicted as

\[
\begin{align*}
\mathcal{B}(\bar{B}^0 \to J/\psi f_0(\bar{n}n)) & \simeq \begin{cases} 
(23^{+12}_{-14}) \times 10^{-6} & \text{S1} \\
(10^{+7}_{-5}) \times 10^{-5} & \text{S2}
\end{cases} \\
\mathcal{B}(\bar{B}^0 \to J/\psi G) & \simeq (6.2 \pm 2.2) \times 10^{-6},
\end{align*}
\]

\[
\begin{align*}
\mathcal{B}(\bar{B}_s \to J/\psi f_0(\bar{s}s)) & \simeq \begin{cases} 
(6.5^{+4.0}_{-4.5}) \times 10^{-4} & \text{S1} \\
(3.5^{+2.3}_{-1.4}) \times 10^{-3} & \text{S2}
\end{cases} \\
\mathcal{B}(\bar{B}_s \to J/\psi G) & \simeq (9.7 \pm 3.9) \times 10^{-5}.
\end{align*}
\]
$B \to f_0 D$ can also be used, if the power-suppressed annihilation diagrams are neglected,

Only $uu$ and $G$ contribute

Only $ss$ and $G$ contribute
$B_c \rightarrow f_0 l \bar{\nu}$ can only occur when $f_0$ is a glueball

- Light scalar mesons are flavor SU(3) singlet
- While glueballs are SU(6) singlet, so the coupling to $c\bar{c}$ is the same as $u\bar{u}$

- The CKM here is $V_{cb}$, which is large
- $\text{Br} \sim 10^{-4}$

CD Lu
Both $B_c$ and $f_0$ need to be reconstructed

Hadronic machine like LHCb is difficult, although branching ratios are large

$Z^0$ factory?
$B_c \rightarrow f_0 \pi^-$ can be another channel for glueball study

- But in this mode, the $nn$ component also contributes through the annihilation diagrams, which is expected to be suppressed.

- $B_c \rightarrow f_0 \pi^-$ also filters out the gluon component of the scalar meson, only when reconstruction of $B_c$ and $f_0$ together
Summary

- We propose various B decay channels to measure the mixing matrix of $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$

- more theoretical study of form factors are needed

- In $B_c$ decays, there is a theoretical clean way to identify the existence of a scalar glueball, however, no experiments can do it up to now
Thank you!
The difference from $J/\psi$ radiative decays

More suppression in $\alpha_s$ and also loop factor $i/16\pi^2$