Charmed Hadron Interactions in Lattice QCD

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Outline

- Introduction
- Lattice setup
- Lüscher’s formula
- Results
In this work, we calculate the scattering lengths of $(J/\Psi, \eta_c) - (\pi, \rho, N)$, $(D, D_s) - (\pi, K)$, $(D, D_s, D^*) - (D, D_s, D^*)$

Studying the interactions of $J/\Psi - N$ is directly related to Charmonium-Nucleon bound states.

The study of $D - K$ interaction is important to understand the state $D_{sJ}(2317)$.

Some new hadronic states $X(3872)$, $Y(4260)$ and $Z^+(4430)$ which can not be accommodated in the conventional quark model have been observed in recent years. Their masses are very close to the thresholds of $D - D^*$, $D^* - D^*$ and $D^* - D_1$. 

Lattice Setup

We use the gauge configurations generated by the MILC Collaboration (C. Aubin et al, Phys. Rev D70, 094505 (2004)).

- Gauge action: one loop tadpole-improved gauge action. \( \mathcal{O}(a^2) \) and \( \mathcal{O}(g^2a^2) \) errors are removed.

- Fermion actions
  - Staggered sea quarks.
  - Domain-wall fermion for light valence quark (u, d, s).
  - Fermilab action for charm quark.
    - No axis-interchange symmetry.
    - Coefficients must be tuned to eliminate lattice artifact for heavy quark.
Heavy quark action


\[ S = S_0 + S_B + S_E, \]

\[ S_0 = \sum_x \bar{q}(x) \left[ m_0 + (\gamma_0 \nabla_0 - \frac{b}{2} \Delta_0) + \nu \sum_i (\gamma_i \nabla_i - \frac{a}{2} \Delta_i) \right] q(x), \]

\[ S_B = -\frac{a}{2} c_B \sum_x \bar{q}(x) (\sum_{i<j} \sigma_{ij} F_{ij}) q(x), \]

\[ S_E = -\frac{b}{2} c_E \sum_x \bar{q}(x) (\sum_i \sigma_{0i} F_{0i}) q(x), \]

- Using the spin average mass of charmonia (\( \eta_c \) and \( J/\Psi \)) to tune the charm quark mass.
- The anisotropy parameter \( \nu \) is tuned to restore the dispersion relations.
- The clover coefficients are evaluated as \( c_B = \frac{\nu}{u_0^3} \)
  \[ c_E = \frac{1}{2} (1 + \nu) \frac{1}{u_0^3}. \]
<table>
<thead>
<tr>
<th>Ensemble</th>
<th>$\beta$</th>
<th>$a m_l$</th>
<th>$a m_s$</th>
<th>$a m_l^{d\text{wf}}$</th>
<th>$a m_s^{d\text{wf}}$</th>
<th>$N_{\text{cfgs}}$</th>
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<tr>
<td>D</td>
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<td>0.030</td>
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<td>0.0478</td>
<td>0.081</td>
<td>563</td>
<td>1689</td>
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</table>

lattice size: $20^3 \times 64$

lattice spacing $\sim 0.125\text{fm}$

$M_\pi \sim 290, 350, 490, 590\text{MeV}$
Lüscher’s Formula

Lüscher has shown that the scattering phase shift is related to the energy shift ($\Delta E$) in the total of two interacting hadrons in a finite box.

\[ p \cot \delta(p) = \frac{1}{\pi L} S\left(\left(\frac{pL}{2\pi}\right)^2\right) \] (1)

If the interaction range is much smaller than the lattice size

\[ p \cot \delta(p) = \frac{1}{a} + O(p^2) \] (2)

From Eq. 1 and Eq. 2, we can get

\[ a = \frac{\pi L}{S\left(\left(\frac{pL}{2\pi}\right)^2\right)} \] (3)
The momentum $p$ is related to energy shift $\Delta E$ by

$$\Delta E = \sqrt{p^2 + m_{h_1}^2} + \sqrt{p^2 + m_{h_2}^2} - m_{h_1} - m_{h_2}$$  \hspace{1cm} (4)$$

The energy shift is obtained by fitting the ratio

$$R^{h_1-h_2}(t) = \frac{G^{h_1-h_2}(t,0)}{G^{h_1}(t,0)G^{h_2}(t,0)} \rightarrow \exp(-\Delta E \cdot t)$$  \hspace{1cm} (5)$$

to a simple exponential, where

$$G^{h_1-h_2}(t) = \langle \mathcal{O}^{h_1}(t)\mathcal{O}^{h_2}(t)(\mathcal{O}^{h_1}(0)\mathcal{O}^{h_2}(0))^\dagger \rangle.$$
\( \eta_c - \pi, \ J/\Psi - \pi, \ D_s - \pi \)

\[
a^{h-\pi} = -(1 + \frac{m_\pi}{m_h})^{-1} \frac{m_\pi}{8\pi f_\pi^2} [l(l + 1) - l_h(l_h + 1) - 2] + \mathcal{O}(m_\pi^2)
\]


The extrapolation formula for these channels is

\[
a \frac{m^2_\pi}{\mu} = c_3 \frac{m^3_\pi}{f^3_\pi} + c_4 \frac{m^4_\pi}{f^4_\pi}
\]
Next to the leading order in chiral perturbation theory, the s-wave scattering length of $D - \pi$ are

$$a^{(3/2)} = \frac{1}{8\pi} (1 + \frac{m_\pi}{M_D})^{-1} (-\frac{m_\pi}{f_\pi} + C_1 \frac{m_\pi^2}{f_\pi^2}),$$

$$a^{(1/2)} = \frac{1}{8\pi} (1 + \frac{m_\pi}{M_D})^{-1} (2\frac{m_\pi}{f_\pi} + C_1 \frac{m_\pi^2}{f_\pi^2}),$$

The extrapolation formula for $l=3/2$ channel is

$$a \frac{m_\pi^2}{\mu} = -\frac{1}{8\pi} \frac{m_\pi^2}{f_\pi^2} + \frac{1}{8\pi} C_1 \frac{m_\pi^3}{f_\pi^3},$$

$$a^{(3/2)} = -0.073(2), \quad a^{(1/2)} = 0.104(2)$$
$D - (K, \bar{K})$

Extrapolation formula for $D - K (I = 1)$ channel:

$$a \frac{m_{K}^{2}}{\mu} = c_{3} \frac{m_{K}^{3}}{f_{K}^{3}} + c_{4} \frac{m_{K}^{4}}{f_{K}^{4}}$$

Extrapolation formula for $D - \bar{K}$ channel:

$$a(\bar{I}=1) \frac{m_{K}^{2}}{\mu} = - \frac{1}{8\pi} \frac{m_{K}^{2}}{f_{K}^{2}} + c'_{3} \frac{m_{K}^{3}}{f_{K}^{3}} + c'_{4} \frac{m_{K}^{4}}{f_{K}^{4}}$$

$$a(I=0) \frac{m_{K}^{2}}{\mu} = \frac{1}{8\pi} \frac{m_{K}^{2}}{f_{K}^{2}} + c''_{3} \frac{m_{K}^{3}}{f_{K}^{3}} + c''_{4} \frac{m_{K}^{4}}{f_{K}^{4}}$$
\[ \frac{J}{\psi} - N \]

\[ am_\pi = c_2 \frac{m_\pi^2}{f_\pi^2} + c_3 \frac{m_\pi^3}{f_\pi^3} \]
$D - \bar{D}^*$

![Graph showing $am_\pi(D - \bar{D}^*, I=1)$ vs. $m_\pi / f_\pi$.]
Conclusions and Outlook

- In $D - \bar{K}$ isoscalar channel, we see relatively strong attractive interaction.
- In $J/\Psi - N$ channel, there is attractive interaction, but it’s hard to say it is bound state or not.
- In $D - D^*$ channel, the sign of the scattering length changed.
- Perform the light quark extrapolation using $\chi$PT formula.
- Mixed channel problem.
Singly and Doubly Charmed J=1/2 Baryon Spectrum I
Singly and Doubly Charmed J=1/2 Baryon Spectrum II
Singly and Doubly Charmed J=1/2 Baryon Spectrum III

\[ M_{\Omega_{cc}} = 3763 \pm 19 \pm 26^{+13}_{-79} \text{MeV} \quad M_{\Xi_{cc}} = 3665 \pm 17 \pm 14^{+0}_{-78} \text{MeV} \]