The Influence of Inelastic Channels upon the Pole Structure of PW in the Coupled Channel πN PWA

Status of coupled channel Zagreb PWA

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International Workshop on the Physics of Excited Baryons
Tallahassee (FL), 12^{th}-15^{th} October 2005.

What are we doing ...

... and why coupled channels?

What are we doing?

- Energy dependant PWA, from experimental observables and/or PW data;
- we analysed some I=1/2 PW data in unitary, analytic, coupled channel, and multi resonant model;

Why coupled channel formalism?

- nature opens N* decay channels and we must take care for them (cuts producing cusp effects)
- N* are more visible in some channels (error reduction, noise introduction?)
- coupling effects can be large (transitions through all possible states)
- morning session of BRAG meeting

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Formalism

Coupled channel multi resonance model a'la Cutkosky

Coupled channel, unitary, and analytic formalism

$$T = \sqrt{\mathrm{Im}\Phi}\,\gamma^T G \gamma \sqrt{\mathrm{Im}\Phi}$$

$$\mathrm{Im}\phi = \frac{q^{2L+1}}{\sqrt{s}\cdot(Q_1 + \sqrt{Q_2^2 + q^2})^{2L}}$$

$$G = (s_B - s - \gamma\,\Phi\gamma^T)^{-1}$$

$$\phi(s) = \frac{s - s_o}{\pi}\int_{s_o}^{\infty}\frac{\mathrm{Im}\phi(s')}{(s' - s_o)(s' - s)}$$

Mapping of s plane

$$\phi(s) = \sum_{n=0}^{N} c_n Z^n(s)$$

$$Z(s) = \frac{\alpha + \sqrt{s_o - s}}{\alpha - \sqrt{s_o - s}}$$

Channel propagator

$$\Phi = \begin{pmatrix}\phi_a & 0 & \dots\\ 0 & \phi_b & \dots\\ \vdots & \vdots & \ddots\end{pmatrix}$$

Example (Two channels, one resonance)

$$\underbrace{\begin{pmatrix}\square & \square\\ \square & \square\end{pmatrix}}_{T} = \underbrace{\begin{pmatrix}\square & 0\\ 0 & \square\end{pmatrix}}_{\sqrt{\mathrm{Im}\,\Phi}} \cdot \underbrace{\begin{pmatrix}\square\\ \square\end{pmatrix}}_{\gamma^T} \cdot \underbrace{(\square)}_{G} \cdot \underbrace{(\square \quad \square)}_{\gamma} \cdot \underbrace{\begin{pmatrix}\square & 0\\ 0 & \square\end{pmatrix}}_{\sqrt{\mathrm{Im}\,\Phi}}$$

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Partial Wave Data

What was fitted?

Where does data we fitted come from:

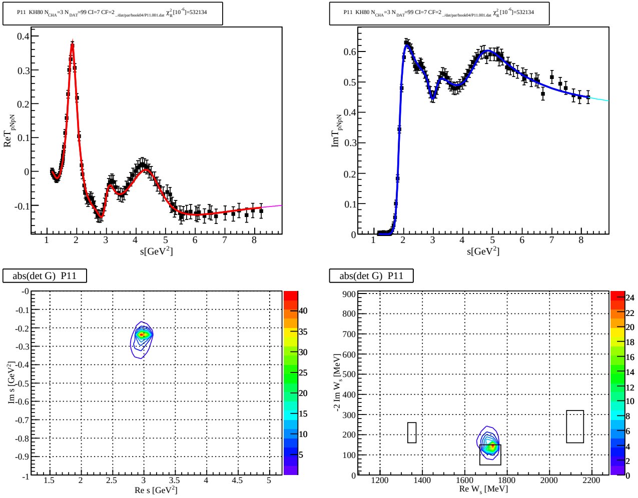
- KH80: $\pi N \rightarrow \pi N$ PW data set
- ZG98: $\pi N \rightarrow \pi N$ and $\pi N \rightarrow \eta N$ PW data
- FA02: $\pi N \rightarrow \pi N$ single energy solutions (SES)
- ZG05: $\pi N \rightarrow K\Lambda$ resonance fit to data

Goodness of the fit criteria:

- the least reduced χ^2
- visual resemblance between data and fitting curves
- bare and dressed pole parameters correspondence (potentialy interesting discrepancies)
- as little resonances as possible to fit is good criteria (higher energy SES PW data are erratic so this can lead to oversimplification)

ZG98 fit results: P11

Elastic channel fit: πN, ηN, EF; 4RB



Fit to elastic ZG98 PWA data

Poles of energy dependant ZG98 PWA vs. poles from our fit to elastic PW ZG98

BRAG homework: S11, P11, and P13 (numbers in MeV)

	Elastic fit		ZG98		PDG	
PW	Re W	-2Im W	Re W	-2Im W	Re W	-2Im W
S11	1525	169	1516	190	1505	170
	1649	199	1645	205	1660	160
	1810	443	1785	420	2150	350
P11	1362	153	1360	160	1350	210
	1679	176	1708	170	-	-
	1733	129	1728	140	1720	250
	2112	358	2113	345	2120	240
P13	1689	226	1685	230	1700	250

SES from Arndt et al. (SAID)

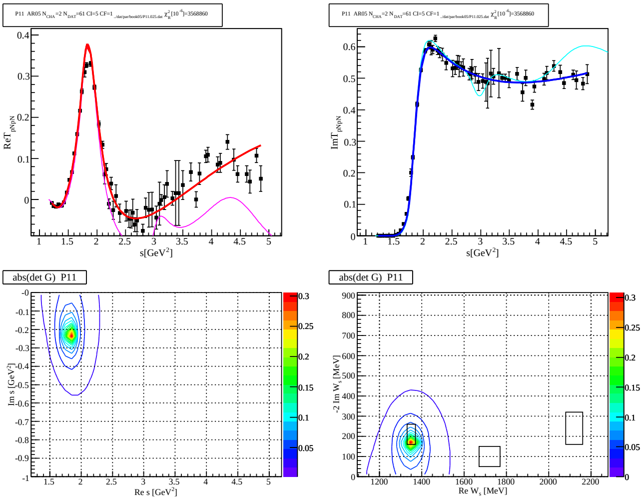
Single Energy Solutions - representation of experimental data

Characteristics of our fits of SES

- less structure than in the KH80 PW data
- large error bars in some (maybe interesting) parts of fitted region
- we need less poles in measured region to fit the data
- very hard to fit (little data, large errors, looks nonresonant)

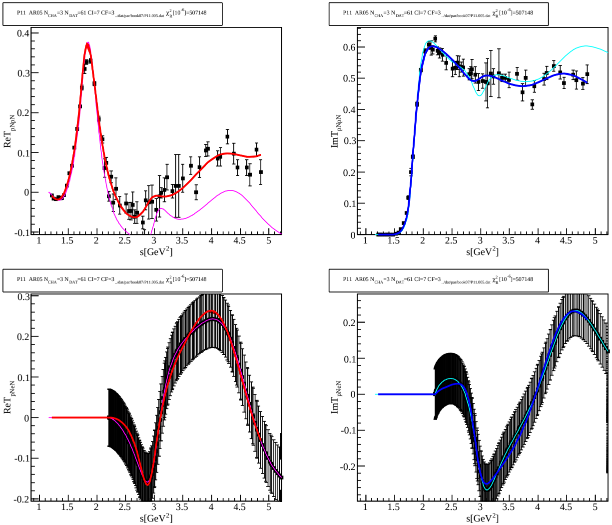
SES and P11: 1 Resonance + Background

Without inelastic channel, save the unitarity channel



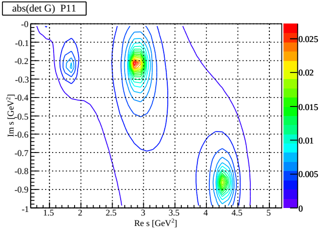
SES and P11 fit - more resonances is a must have

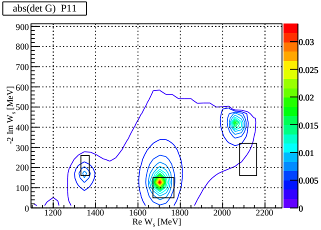
With inelastic channel - PW

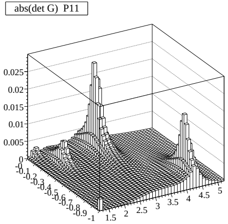


SES and P11

With inelastic channel - poles







Conclusion
Closing

Conclusions

- we report poles for ZG98 I=1/2 PWA
- PW fits have been done to test the influence of inelastic channels to pole positions
- the most of the poles can be noticed in elastic fits (depending on dataset)
- many poles can be seen even in fits of just inelastic data
- method and formalism are multichannel in nature, and need inelastic data (in addition to elastic) to "sharpen" pole positions
- choice of inelastic channel is important - ηN is not the best choice for P11 (is it the KΛ?)
- SES dataset is hard to fit, and fits do not seem reliable (problems with continuum ambiguities?)
- good πN elastic PWA is very welcome

Conclusions

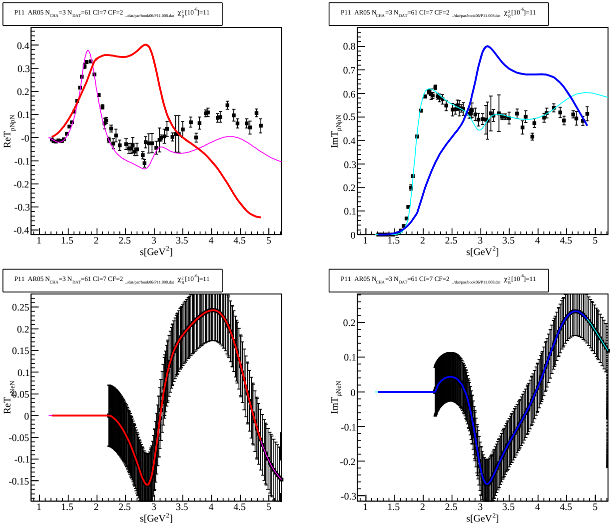
Things yet to come

Next steps

- fitting of various PW data ηN, $K\Lambda$, $K\Sigma$, ωN, $\pi\pi N$, γN, ...
- inelastic experimental data fits (σ, $d\sigma/d\Omega$, polarization, ...)
- clarification of formalism (new background treatment, analytic improvements, ...)

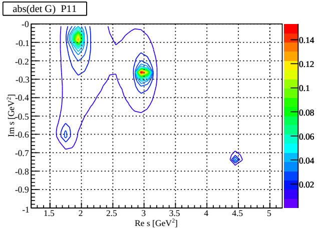
Inelastic P11 fit

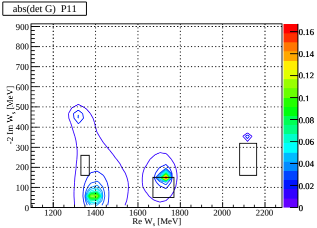
Just inelastic channel:- PW

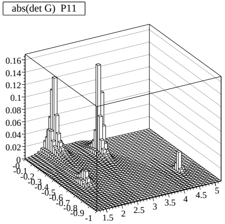


Fit $\pi N \longrightarrow \eta N$: 4 R + B

Just inelastic channel: POLES

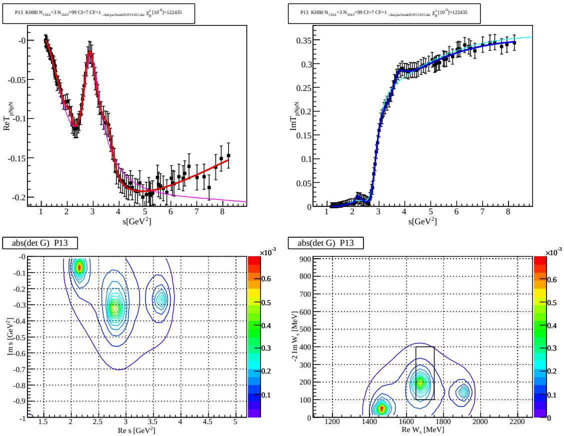






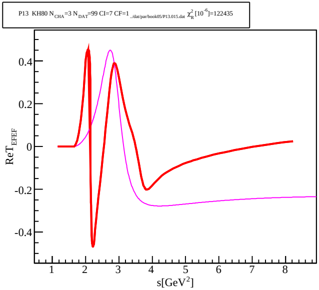
KH80 elastic fits

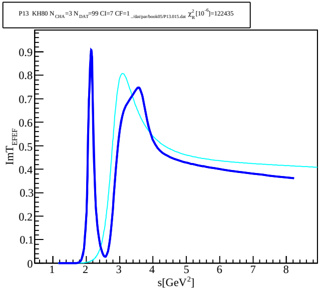
Surplus of resonances in P13 PW: is it justified?



KH80 elastic fits

Surplus of resonances in P13 PW: resolution



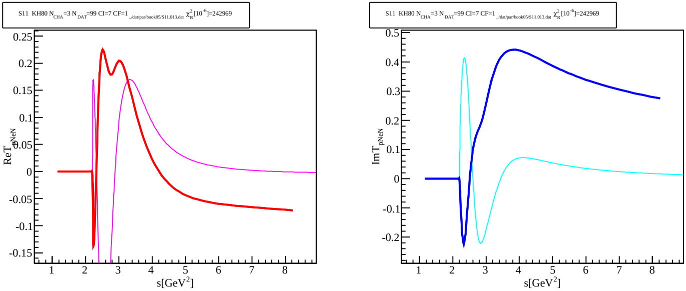


At least one of the three is dubious

The first "resonance" escaped almost completely to the effective channel (the one that takes care of non ηN inelasticity), and is very narrow. There is no experimental need for this one - but that fact is not obvious from considering just elastic channel.

KH80 fits: S11

Inelastic channel



KH80: $\pi N \longrightarrow \eta N$

Channel $\pi N \longrightarrow \eta N$:
Thin lines are result of ZG98 PWA (KH80 have been fitted there, along with $\pi N \longrightarrow \eta N$ experimental data!).
Discrepancy is obvious - do we have an explanation?

Explanation of ...

... inelastic channel discrepancy

Breit-Wigner approximation

$$T_{aa} = \frac{\Gamma_a/2}{M - W - i(\Gamma_a + \Gamma_b + \Gamma_c)/2} \tag{1}$$

The best that fit of T_{aa} can fix is $\Gamma_b + \Gamma_c$; we need additional information from inelastic channels.

Small print

In our model widths depend on W, but the variation is very slow in the resonances region so we would need extremely large number of precise data in the wide W region to determine parameters of all open channels from just one channel fit. Fitting the area around channel opening cusps could help with that.