Polarization Observables in the Photoproduction of Two Pseudoscalar Mesons

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Outline

• Introduction and Motivation
• Formalism
• Parity Implications (and Other Relationships)
• Examples
• Conclusions
Introduction and Motivation

Premise:
Polarization measurements are essential for extracting amplitudes.
Introduction and Motivation

Why do we need new observables?
Only alternative in treating a process like $\gamma N \rightarrow N \pi \pi$ is quasi-two-body (QTB) approach
Introduction and Motivation

\[ \gamma N \rightarrow N \pi\pi = \gamma N \rightarrow \Delta \pi \rightarrow N \pi\pi \]
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\[ + \ldots \]
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$$W(\theta, \phi, \Phi) = \frac{3}{4\pi} \left[ \frac{1}{2} (1 - \rho_{00}^0) + \frac{1}{2} (3\rho_{00}^0 - 1) \cos^2 \theta - \sqrt{2}\Re \rho_{10}^0 \sin 2\theta \cos \phi - \rho_{1-1}^0 \sin^2 \theta \cos \phi \right]$$

$$- P_\gamma \cos 2\Phi \left( \rho_{11}^1 \sin^2 \theta + \rho_{00}^1 \cos^2 \theta - \sqrt{2}\Re \rho_{1,0}^1 \sin 2\theta \cos \phi - \rho_{1-1}^1 \sin^2 \theta \cos 2\phi \right)$$

$$- P_\gamma \sin 2\Phi \left( \sqrt{2}\Im \rho_{10}^2 \sin 2\theta \sin \phi + \Im \rho_{1-1}^2 \sin^2 \theta \sin 2\phi \right)$$
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Similar expression needed for each QTB contribution: c’est pas très efficace
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![Diagram](image.png)

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Technique used to obtain new observables: direct calculation
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Technique used to obtain new observables: direct calculation

Valid for $N\pi$, $N\pi\pi$ (and $N(n\pi)$ for that matter)
Formalism

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$$i\mathcal{M}^{\lambda\gamma}_{\lambda N\lambda N'} = \varepsilon_i(\lambda_{\gamma}) \chi^\dagger(\lambda_{N'}) (A_i + \sigma_j B_{ij}) \phi(\lambda_N)$$
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$\varepsilon^\dagger$ = polarization vector of incident photon
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Formalism

Matrix element for a process like $\gamma N \rightarrow N\pi \ldots$ can be written

$$i\mathcal{M}_{\lambda N' \lambda N}^{\lambda\gamma} = \varepsilon_i(\lambda\gamma) \chi^\dagger(\lambda_{N'}) (A_i + \sigma_j B_{ij}) \phi(\lambda_N)$$

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For odd numbers of pions, $\vec{A}$ is an axial vector, $B_{ij}$ are components of a tensor.

For even numbers of pions, $\vec{A}$ is a vector, $B_{ij}$ are components of a pseudotensor.
$$|\mathcal{M}|^2 = \varepsilon_i(\lambda_{\gamma})\varepsilon_l(\lambda_{\gamma})\chi^\dagger(\lambda_{N'}) (A_i + \sigma_j B_{ij} ) \phi(\lambda_N) \phi^\dagger(\lambda_{N}) (A_i^* + \sigma_k B_{ik}^*) \chi(\lambda_{N'})$$
$$|\mathcal{M}|^2 = \varepsilon_i(\lambda_{\gamma})\varepsilon_l(\lambda_{\gamma})\chi^\dagger(\lambda_{N'}) (A_i + \sigma_j B_{ij}) \phi(\lambda_N)\phi^\dagger(\lambda_N) (A^*_l + \sigma_k B^*_{lk}) \chi(\lambda_{N'})$$

$$= \varepsilon_i\varepsilon^*_l \text{Tr} \left[ \frac{1}{2} \left(1 + \vec{\sigma} \cdot \vec{\Lambda}_f \right) (A_i + \sigma_j B_{ij}) \frac{1}{2} \left(1 + \vec{\sigma} \cdot \vec{\Lambda}_i \right) (A^*_l + \sigma_k B^*_{lk}) \right]$$
\[ |\mathcal{M}|^2 = \varepsilon_i(\lambda_\gamma)\varepsilon_l(\lambda_\gamma)\chi^\dagger(\lambda_{N'}) (A_i + \sigma_j B_{i,j}) \phi(\lambda_N)\phi^\dagger(\lambda_N) (A_i^* + \sigma_k B_{i,k}^*) \chi(\lambda_{N'}) \]

\[ = \varepsilon_i\varepsilon_l^* \text{Tr} \left[ \frac{1}{2} \left(1 + \vec{\sigma} \cdot \vec{\Lambda}_f \right) (A_i + \sigma_j B_{i,j}) \frac{1}{2} \left(1 + \vec{\sigma} \cdot \vec{\Lambda}_i \right) (A_i^* + \sigma_k B_{i,k}^*) \right] \]

\( \Lambda_i, \Lambda_f \) are the polarizations of the initial and final nucleons, respectively.
\[ |M|^2 = \epsilon_i(\lambda_\gamma)\epsilon_i(\lambda_\gamma)\chi^\dagger(\lambda_{N'}) (A_i + \sigma_j B_{ij}) \phi(\lambda_N)\phi^\dagger(\lambda_N) (A_i^* + \sigma_k B_{ik}^*) \chi(\lambda_{N'}) \]

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For a beam of \( N \) circularly polarized photons with momentum \( \vec{k} \) along the \( z \)-axis, with \( \frac{1+\delta_\odot}{2} \) photons polarized along the positive \( z \) axis, and \( \frac{1-\delta_\odot}{2} \) photons polarized along the negative \( z \) axis (corresponding to degree of circular polarization \( \delta_\odot \))

\[ \frac{1}{N} \sum_{\text{photons}} \epsilon' \cdot \vec{a} \epsilon^* \cdot \vec{b} = \vec{a} \cdot \vec{b} - \hat{\vec{k}} \cdot \vec{a} \hat{\vec{k}} \cdot \vec{b} - i\delta_\odot \hat{\vec{k}} \cdot \vec{a} \times \vec{b} \]
For $N$ linearly polarized photons, with $\frac{1+\delta_\ell}{2}$ polarized along the $x'$ axis, and $\frac{1-\delta_\ell}{2}$ along the $y'$ axis ($\delta_\ell$ is the degree of linear polarization)

\[ \frac{1}{N} \sum_{\text{photons}} \bar{\epsilon}' \cdot \bar{a} \bar{\epsilon}^{*} \cdot \vec{b} = \bar{a} \cdot \vec{b} - \hat{k} \cdot \hat{a} \hat{k} \cdot \vec{b} \]

\[ + \delta_\ell \left[ \cos 2\beta (a_x b_x - a_y b_y) + \sin 2\beta (a_x b_y + a_y b_x) \right] \]
After some manipulation, the cross section can be written

\[
\rho_f I = I_0 \left\{ \left( 1 + \vec{\Lambda}_i \cdot \vec{P} + \vec{\sigma} \cdot \vec{P}' + \Lambda_i^\alpha \sigma^{\beta'} O_{\alpha\beta'} \right) \\
+ \delta_\circ \left( I_1^\circ + \vec{\Lambda}_i \cdot \vec{P}^\circ + \vec{\sigma} \cdot \vec{P}'^\circ + \Lambda_i^\alpha \sigma^{\beta'} O_{\alpha\beta'}^\circ \right) \\
+ \delta_\ell \left[ \sin 2\beta \left( I^s + \vec{\Lambda}_i \cdot \vec{P}^s + \vec{\sigma} \cdot \vec{P}'^s + \Lambda_i^\alpha \sigma^{\beta'} O_{\alpha\beta'}^s \right) \\
+ \cos 2\beta \left( I^c + \vec{\Lambda}_i \cdot \vec{P}^c + \vec{\sigma} \cdot \vec{P}'^c + \Lambda_i^\alpha \sigma^{\beta'} O_{\alpha\beta'}^c \right) \right] \right\},
\]
In terms of helicity amplitudes,

\[ I_0 = \left| \mathcal{M}^{++}_{++} \right|^2 + \left| \mathcal{M}^{++}_{+-} \right|^2 + \left| \mathcal{M}^{++}_{-+} \right|^2 + \left| \mathcal{M}^{++}_{--} \right|^2 
+ \left| \mathcal{M}^{+-}_{++} \right|^2 + \left| \mathcal{M}^{+-}_{+-} \right|^2 + \left| \mathcal{M}^{+-}_{-+} \right|^2 + \left| \mathcal{M}^{+-}_{--} \right|^2 \]
Parity Implications (and Other Relationships)

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\[ + \left| \mathcal{M}_{++}^- \right|^2 + \left| \mathcal{M}_{+-}^- \right|^2 + \left| \mathcal{M}_{-+}^+ \right|^2 + \left| \mathcal{M}_{--}^+ \right|^2 \]

\[ I^\odot = \left| \mathcal{M}_{++}^+ \right|^2 + \left| \mathcal{M}_{+-}^+ \right|^2 + \left| \mathcal{M}_{-+}^- \right|^2 + \left| \mathcal{M}_{--}^- \right|^2 \]

\[ - \left| \mathcal{M}_{++}^- \right|^2 - \left| \mathcal{M}_{+-}^- \right|^2 - \left| \mathcal{M}_{-+}^+ \right|^2 - \left| \mathcal{M}_{--}^+ \right|^2 \]
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+ \left| \mathcal{M}_{++}^- \right|^2 + \left| \mathcal{M}_{+-}^- \right|^2 + \left| \mathcal{M}_{-+}^+ \right|^2 + \left| \mathcal{M}_{--}^+ \right|^2 \\
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\]

Can we use parity invariance to simplify these expressions?
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If we examine \( I^\odot \), \( M_{++}^+ \) and \( M_{--}^- \) must be related by parity invariance, so \( I^\odot \) would vanish.
In terms of helicity amplitudes,

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I_0 = \left| \mathcal{M}_{++}^+ \right|^2 + \left| \mathcal{M}_{+-}^+ \right|^2 + \left| \mathcal{M}_{-+}^+ \right|^2 + \left| \mathcal{M}_{--}^+ \right|^2 \\
+ \left| \mathcal{M}_{--}^- \right|^2 + \left| \mathcal{M}_{+-}^- \right|^2 + \left| \mathcal{M}_{-+}^- \right|^2 + \left| \mathcal{M}_{++}^- \right|^2
\]

\[
I^\oplus = \left| \mathcal{M}_{++}^+ \right|^2 + \left| \mathcal{M}_{+-}^+ \right|^2 + \left| \mathcal{M}_{-+}^+ \right|^2 + \left| \mathcal{M}_{--}^+ \right|^2 \\
- \left| \mathcal{M}_{--}^- \right|^2 - \left| \mathcal{M}_{+-}^- \right|^2 - \left| \mathcal{M}_{-+}^- \right|^2 - \left| \mathcal{M}_{++}^- \right|^2
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If we examine \( I^\oplus \), \( \mathcal{M}_{++}^+ \) and \( \mathcal{M}_{--}^- \) must be related by parity invariance, so \( I^\oplus \) would vanish.

Similar questions arise for many observables.
The upshot of this is that... would indeed vanish for...
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The upshot of this is that

\[ M_{\lambda_{N}, \lambda_{N}'}^{\lambda_{\gamma}}(\theta, \Theta) = \pm M_{-\lambda_{N}, -\lambda_{N}'}^{\lambda_{\gamma}}(\theta, \Theta), \]

and \( I^\circ \) would indeed vanish for \( \gamma N \rightarrow N \pi \).
The upshot is that, under a parity transformation, helicity amplitudes are related to each other at different kinematic points.
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\[ M_{\lambda_{N},\lambda_{N'}}^{\lambda\gamma} (\theta, \Theta, \Phi) = \pm M_{-\lambda_{N},-\lambda_{N'}}^{\lambda\gamma} (\theta, \Theta, 2\pi - \Phi) \]
The upshot is that, under a parity transformation, helicity amplitudes are related to each other \textit{at different kinematic points}

\[ M_{\lambda N, \lambda N'}^{\gamma} (\theta, \Theta, \Phi) = \pm M_{-\lambda N, -\lambda N'}^{\gamma} (\theta, \Theta, 2\pi - \Phi) \]

\[ \rightarrow \text{Polarization observables are either even or odd under the transformation} \]
\[ \Phi \leftrightarrow 2\pi - \Phi \]
The upshot is that, under a parity transformation, helicity amplitudes are related to each other at different kinematic points.

\[
\mathcal{M}^{\lambda_\gamma}_{\lambda_N,\lambda_{N'}} (\theta, \Theta, \Phi) = \pm \mathcal{M}^{\lambda_\gamma}_{-\lambda_N,-\lambda_{N'}} (\theta, \Theta, 2\pi - \Phi)
\]

⇒ Polarization observables are either even or odd under the transformation \( \Phi \leftrightarrow 2\pi - \Phi \).

If we compare \( N\pi \) and \( N\pi\pi \) final states, \( N\pi\pi \) observables that are odd have analogs that vanish in \( N\pi \), while \( N\pi\pi \) observables that are even are non-vanishing in \( N\pi \).
From parity,

\[
\begin{align*}
I_0 &= -O_{yy}'^c & P_y &= -P_y'^c \\
P_{y'} &= -P_y^c & O_{xx}' &= -O_{zz}'^c \\
O_{xz}' &= O_{zx}'^c & O_{yy}' &= -I_c^c \\
O_{z zij}' &= O_{xz}'^c & O_{zz}' &= -O_{xx}'^c \\
P_x^c &= O_{zy}'^s & P_z^c &= -O_{xy}'^s \\
P_{x'} &= -O_{yz}'^s & P_z' &= O_{yx}'^s \\
P_x^s &= -O_{zy}'^c & P_z &= O_{xy}'^c \\
P_{x'} &= O_{yz}' & P_z' &= -O_{yx}'
\end{align*}
\]

all at \( \Phi = 0, \Phi = \pi, \Phi = 2\pi \).
From parity,
\[
I_0 = -\mathcal{O}_{yy'}^c, \quad P_y = -P_y^c,
\]
\[
P_{y'} = -P_y^c, \quad \mathcal{O}_{xx'} = -\mathcal{O}_{zz'}^c,
\]
\[
\mathcal{O}_{xz'} = \mathcal{O}_{zx'}^c, \quad \mathcal{O}_{yy'} = -I^c
\]
\[
\mathcal{O}_{zx'} = \mathcal{O}_{xz'}^c, \quad \mathcal{O}_{zz'} = -\mathcal{O}_{xx'}^c
\]
\[
P_{x}^c = \mathcal{O}_{zy'}^s, \quad P_{z}^c = -\mathcal{O}_{xy'}^s
\]
\[
P_{x'}^c = -\mathcal{O}_{yz'}^s, \quad P_{z'}^c = \mathcal{O}_{yx'}^s
\]
\[
P_{x}^s = -\mathcal{O}_{zy'}^\circ, \quad P_{z}^s = \mathcal{O}_{xy'}^\circ
\]
\[
P_{x'}^s = \mathcal{O}_{yz'}^\circ, \quad P_{z'}^s = -\mathcal{O}_{yx'}^\circ
\]

all at $\Phi = 0, \Phi = \pi, \Phi = 2\pi$.

Note that because $I_0 = -\mathcal{O}_{yy'}^c$, $\mathcal{O}_{yy'}^c = -1$ at $\Phi = 0, \Phi = \pi, \Phi = 2\pi$. 


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I_0 &= -\mathcal{O}_{yy'}^c \\
\mathcal{O}_{yy'} &= -I^c \\
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\end{align*}
\]

all at \( \Phi = 0, \Phi = \pi, \Phi = 2\pi \).

Note that because \( I_0 = -\mathcal{O}_{yy'}^c, \mathcal{O}_{yy'}^c = -1 \) at \( \Phi = 0, \Phi = \pi, \Phi = 2\pi \).

All other observables vanish at \( \Phi = 0, \Phi = \pi, \Phi = 2\pi \).
In $N\pi\pi$, not all 64 observables are independent.
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Relationships among observables, derived from amplitudes of helicity (or transversity) amplitudes, look like

$$\left[ P_{x'} + \xi \mathcal{O}_{yx'} + \zeta \left( P_{x'} + \xi \mathcal{O}^{\circ}_{yx'} \right) \right]^2 + \left[ P_{z'} + \xi \mathcal{O}_{yz'} + \zeta \left( P_{z'} + \xi \mathcal{O}^{\circ}_{yz'} \right) \right]^2$$

$$= \left[ 1 + \xi P_y + \zeta \left( I^{\circ} + \xi P^\circ_y \right) \right]^2 - \left[ P_{y'} + \xi \mathcal{O}_{yy'} + \zeta \left( P_{y'} + \xi \mathcal{O}^{\circ}_{yy'} \right) \right]^2$$
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$$= \left[1 + \xi P_y + \zeta (I^\circ + \xi P_y^\circ)\right]^2 - \left[P_{y'} + \xi O_{yy'} + \zeta \left(P_{y'}^\circ + \xi O_{yy'}^\circ\right)\right]^2$$

$\zeta, \xi = \pm 1$, independently: this expression represents 4 relationships.
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$$= \left[ 1 + \xi P_y + \zeta \left( I^{\circ} + \xi P_y^{\circ} \right) \right]^2 - \left[ P_{y'} + \xi \mathcal{O}_{yy'} + \zeta \left( P_{y'}^{\circ} + \xi \mathcal{O}_{yy'}^{\circ} \right) \right]^2$$

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There are 28 relations that arise from consideration of the amplitudes of the (helicity or transversity) amplitudes.
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\left[ P_{x'} + \xi \mathcal{O}_{yx'} + \zeta \left( P_{x'}^\circ + \xi \mathcal{O}_{yx'}^\circ \right) \right]^2 + \left[ P_{z'} + \xi \mathcal{O}_{yz'} + \zeta \left( P_{z'}^\circ + \xi \mathcal{O}_{yz'}^\circ \right) \right]^2
\]

\[
= \left[ 1 + \xi P_y + \zeta \left( I^\circ + \xi P_y^\circ \right) \right]^2 - \left[ P_{y'} + \xi \mathcal{O}_{yy'} + \zeta \left( P_{y'}^\circ + \xi \mathcal{O}_{yy'}^\circ \right) \right]^2
\]

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There are a further 21 relations that arise from consideration of their phases
In $N\pi\pi$, not all 64 observables are independent.

Relationships among observables, derived from amplitudes of helicity (or transversity) amplitudes, look like

\[ \left[ P_{x'} + \xi O_{yx'} + \zeta \left( P_{x'} + \xi O_{yx} \right) \right]^2 + \left[ P_{z'} + \xi O_{yz'} + \zeta \left( P_{z'} + \xi O_{yz} \right) \right]^2 \]

\[ = \left[ 1 + \xi P_y + \zeta (I^\odot + \xi P_{y'}^\odot) \right]^2 - \left[ P_{y'} + \xi O_{yy'} + \zeta \left( P_{y'} + \xi O_{yy} \right) \right]^2 \]

$\zeta, \xi = \pm 1$, independently: this expression represents 4 relationships.

There are 28 relations that arise from consideration of the amplitudes of the (helicity or transversity) amplitudes.

There are a further 21 relations that arise from consideration of their phases.

The number of independent observables is $64-28-21=15$. This is equal to the minimum number of measurements needed at each kinematic point for an unambiguous extraction of the amplitudes (up to quadrant ambiguities in their phases).
In $N\pi\pi$, not all 64 observables are independent.

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The analogous count for $N\pi$ gives 7 independent observables, 7 observables that must be measured for extraction of amplitudes (up to quadrant ambiguities in their phases).
Relationships can be manipulated to give two sets of inequalities
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\[ |1 + \xi P_y + \zeta (I^\circ + \xi P_y^\circ)| \geq \left\{ |P_{y'} + \xi O_{yy'} + \zeta (P_{y'} + \xi O_{yy'})|, \right. \]
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\]

\[
\left| P_{x'} + \xi O_{yx'} + \zeta \left( P_{x'}^\circ + \xi O_{yx'}^\circ \right) \right|, \left| P_{z'} + \xi O_{yz'} + \zeta \left( P_{z'}^\circ + \xi O_{yz'}^\circ \right) \right| \right\}
\]

\[
1 + P_y^2 + (I^\circ)^2 + (P_y^\circ)^2 \geq \left\{ P_{y'}^2 + O_{yy'}^2 + \left( P_{y'}^\circ \right)^2 + \left( O_{yy'}^\circ \right)^2, \right.
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What observables need to be measured?

To obtain the amplitudes of the (transversity) amplitudes, we MUST measure differential cross section, along with $\rho$, $\phi_1$, $\phi_2$, and $\phi_3$ (angular distributions and mass distributions only probe 8 phases of transversity amplitudes mean that there are 7 independent phase differences that can be extracted, and 7 measurements are needed for this. For instance, four of these phase differences can be extracted by measuring any 4 of the 8 observables $\rho$, $\phi_1$, $\phi_2$, and $\phi_3$ of the remaining phases can be extracted from measuring any 2 of the 8 observables $\phi_4$, $\phi_5$, $\phi_6$, and $\phi_7$, along with use of identities among the phase differences (such as). The remaining independent phase difference then can be extracted from one of the observables requiring linearly polarized photons. Complete’set of experiments requires measurement of single, double and triple polarization observables (including observables with both linearly and circularly polarized photons), along with the differential cross section.
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What observables need to be measured?

To obtain the amplitudes of the (transversity) amplitudes, we **MUST** measure differential cross section, along with $P_y$, $P_y'$, $O_{yy'}$, $I$, $P_y^\circ$, $P_y'^\circ$, and $O_{yy'}^\circ$ (angular distributions and mass distributions only probe $I_0 = \left| M_{++}^+ \right|^2 + \left| M_{+-}^+ \right|^2 + \left| M_{-+}^+ \right|^2 + \left| M_{--}^+ \right|^2 + \left| M_{++}^- \right|^2 + \left| M_{+-}^- \right|^2 + \left| M_{-+}^- \right|^2 + \left| M_{--}^- \right|^2$).

8 phases of transversity amplitudes mean that there are 7 independent phase differences that can be extracted, and 7 measurements are needed for this.
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What observables need to be measured?

To obtain the amplitudes of the (transversity) amplitudes, we must measure differential cross section, along with $P_y$, $P_y'$, $O_{yy'}$, $I^0$, $P_y^0$, $P_y'^0$, and $O_{yy'}^0$ (angular distributions and mass distributions only probe $I_0 = |M_{++}^+|^2 + |M_{+-}^+|^2 + |M_{-+}^+|^2 + |M_{--}^+|^2$)

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2 of the remaining phases can be extracted from measuring any 2 of the 8 observables $P_x$, $P_z$, $O_{xy}$, $O_{yz}$, $P_x^0$, $P_z^0$, $O_{xy}^0$, and $O_{yz}^0$, along with use of identities among the phase differences (such as $\phi_1 - \phi_4 = \phi_1 - \phi_2 + \phi_2 - \phi_3 + \phi_3 - \phi_4$).
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'Complete' set of experiments requires measurement of single, double and triple polarization observables (including observables with both linearly and circularly polarized photons), along with the differential cross section.
Examples

In $\gamma N \rightarrow N\pi\pi$ (or $\gamma N \rightarrow NK\bar{K}$), observables are 5-fold differential, and so can be shown in a variety of ways (even Dalitz plots, for observables that are even under $\Phi \leftrightarrow 2\pi - \Phi$).

To illustrate these observables, I use a ‘simple’ model
Examples

[Diagrams of polarization observables in the photoproduction of two pseudoscalar mesons]
Model includes $s -$ (and $u -$) channel hyperons: $\Lambda(1405), \Lambda(1520), \Lambda(1600), \Lambda(1670), \Lambda(1690), \Lambda(1800), \Lambda(1810), \Lambda(1890), \Sigma(1385), \Sigma(1580), \Sigma(1620), \Sigma(1660), \Sigma(1670), \Sigma(1750), \Sigma(1880), \Sigma(1940)$;
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meson production: $\phi(1020)$;

exotic baryons: $\Theta^+(1540)$. 
Model includes $s-$ (and $u-$) channel hyperons: $\Lambda(1405), \Lambda(1520), \Lambda(1600), \Lambda(1670),$ $\Lambda(1690), \Lambda(1800), \Lambda(1810), \Lambda(1890),$ $\Sigma(1385), \Sigma(1580), \Sigma(1620), \Sigma(1660),$ $\Sigma(1670),$ $\Sigma(1750), \Sigma(1880), \Sigma(1940);$ 

$t-$channel mesons: $K^*, K, \pi, \eta;$ 

meson production: $\phi(1020);$ 

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Missing: more mesons ($f_0, f_2$, etc.);
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$N^*, \Delta^*$ that couple to hyperons.

Nevertheless, should be sufficient to illustrate the salient points (W. Roberts, Phys. Rev. C 70, 065201 (2004) for more details.)
Figure (b) shows the polarization observables in the photoproduction of two pseudoscalar mesons as a function of the mass difference $M_{NK}$ (GeV). The graph includes lines for different values of $\Phi^*$, with $\Phi^* = 30$, $\Phi^* = 45$, $\Phi^* = 60$, and $\Phi^* = 90$. The y-axis represents the polarization observable $O_{n\ell}^s$. The x-axis represents the mass difference $M_{NK}$ in GeV.
Polarization Observables in the Photoproduction of Two Pseudoscalar Mesons – p.23

(a)

\[ P_z^0 \]

\[
\begin{align*}
\phi^* &= 30 \\
\phi^* &= 45 \\
\phi^* &= 60 \\
\phi^* &= 90 
\end{align*}
\]
The figure shows polarization observables in the photoproduction of two pseudoscalar mesons as a function of the invariant mass $M_{NK} (GeV)$.

Different curves represent different values of $\Phi^*$:
- $\Phi^* = 30$
- $\Phi^* = 45$
- $\Phi^* = 60$
- $\Phi^* = 90$

The y-axis represents the polarization observable $I^0$, with values ranging from -0.1 to 0.4.

The x-axis represents $M_{NK}$ (GeV), ranging from 1.4 to 2.0.
(c)
Conclusions

Polarization observables are essential for extracting amplitudes of processes like $\gamma N \rightarrow N\pi$ and $\gamma N \rightarrow N\pi\pi$. 
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Single, double and triple polarization measurements, along with the measurements using both circularly and linearly polarized photons, are needed to 'unambiguously' extract amplitudes.
Conclusions

Polarization observables are essential for extracting amplitudes of processes like $\gamma N \rightarrow N\pi$ and $\gamma N \rightarrow N\pi\pi$.

Such observables are very sensitive to underlying dynamics.

Single, double and triple polarization measurements, along with the measurements using both circularly and linearly polarized photons, are needed to 'unambiguously' extract amplitudes.

Facilities are poised to make a number of measurements that will challenge (existing and future) models of such processes.