Gauge-Invariant Approach to Meson Photoproduction Including the Final-State Interaction

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Example: $\gamma N \rightarrow \pi N$
Attaching a Photon to the $\pi NN$ Vertex

The total photoproduction amplitude is given by:

$$M^\mu = M_s^\mu + M_u^\mu + M_t^\mu + M_{int}^\mu$$
Pions, Nucleons, and Photons

Hadronic scattering: \( \pi N \rightarrow \pi N \)

\[
\begin{align*}
\text{Pion} & = \text{Neutron} + \text{Pion} \\
\text{Neutron} & = \text{Proton} + \text{Pion} \\
\text{Pion} & = \text{Proton} + \text{Pion}
\end{align*}
\]

Photoproduction: \( \gamma N \rightarrow \pi N \)

Amplitude:

\[
\begin{align*}
\text{S-channel} & = \text{Proton} + \text{Pion} \\
\text{U-channel} & = \text{Proton} + \text{Pion} \\
\text{T-channel} & = \text{Proton} + \text{Pion}
\end{align*}
\]

Interaction Current \( M^\mu_{\text{int}} \)

Theory:

PRC 56, 2041 (1997)
Hadronic Driving Terms

\[ U = \ \pi + \Delta + \ \ldots \]

\[ \pi = \sigma, \rho + \Delta + \ldots \]

\[ \Delta = \Delta + \ldots \]

Electromagnetic Couplings to Driving Terms

\[ U = \ \pi + \Delta + \ \ldots \]

\[ \pi = 1 + \chi \]

\[ \chi = \chi + 1 \]

\[ \chi = \chi + \chi + \chi + \ldots \]

Dressed Nucleon Current

\[ = \ \chi + \chi + \chi + \ldots \]
This is **VERY** complicated!

- The full theory is gauge-invariant as a matter of course.
- In practice, truncations are necessary.
- Truncations usually destroy gauge invariance.

Prescriptions must be found that restore gauge invariance in practical applications.
New Theoretical Tools for Nucleon Resonance Analysis
Old Theoretical Tool

New Theoretical Tools for Nucleon Resonance Analysis

Gauge Invariance

Gauge Invariance

\[ p_2 p_1 n p_2 - n p_1 m p' - p_1 m' - p_2 m' \]

\[ k \mu M^{\mu} \]

\( \Leftarrow \) Time \( \Rightarrow \)

Physical condition:

Current Conservation: \( k_\mu M^{\mu} = 0 \) all hadrons on-shell
Gauge Invariance

$\leftarrow$ Time $\rightarrow$

Physical condition:

Current Conservation: $k_\mu M^\mu = 0$  all hadrons on-shell

Recipe for Phenomenological Currents: $M^\mu = \tilde{M}^\mu - a^\mu \frac{k \cdot \tilde{M}}{k \cdot a}$  ($a^\mu$ arbitrary)
Gauge Invariance

\[ \begin{align*}
M^\mu \quad \Leftarrow \text{Time} \quad \Leftarrow
\end{align*} \]

Physical condition:

Current Conservation: \( k_\mu M^\mu = 0 \) \quad all hadrons on-shell

Recipe for Phenomenological Currents:

\[ M^\mu = \tilde{M}^\mu - a^\mu \frac{k \cdot \tilde{M}}{k \cdot a} \quad (a^\mu \text{ arbitrary}) \]

Not good enough for microscopic approaches!
Q: So why is it not okay to simply subtract terms to create a transverse current?

A: Because such currents are always transverse — on- and off-shell. This creates unacceptable inconsistencies in microscopic approaches.

To show this consider

and attach photon in all places of the internal 3-point function...

Details, details... → Proof
Example: Two-pion production
Basic Two-pion Production Mechanisms

(a)

(b)

M
N

CNS
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H. Haberzettl — N*2005, 12Oct05 — twopion1.tex
Ward–Takahashi Identities for the Nucleon and the Pion Currents

\[ k_\mu \Gamma^\mu_N(p', p) = S_N^{-1}(p') Q_N - Q_N S_N^{-1}(p) \quad \text{nucleon} \]

Form factors: \( f^{ij}_N = f^{ij}_N(p'^2, p^2; k^2) \)

\[ \Gamma^\nu_N(p', p) = \gamma^\nu Q_N + \left[ t^\nu f^{00}_1 + \frac{i\sigma^{\nu\mu} k_\mu}{2m} f^{00}_2 \right] + \frac{p' - m}{2m} \left[ t^\nu f^{10}_1 + \frac{i\sigma^{\nu\mu} k_\mu}{2m} f^{10}_2 \right] \]

\[ + \left[ t^\nu f^{01}_1 + \frac{i\sigma^{\nu\mu} k_\mu}{2m} f^{01}_2 \right] \frac{p - m}{2m} + \frac{p' - m}{2m} \left[ t^\nu f^{11}_1 + \frac{i\sigma^{\nu\mu} k_\mu}{2m} f^{11}_2 \right] \frac{p - m}{2m} \]

\[ t^\nu = \gamma^\nu k^2 - k^\nu k \]

\[ k_\mu \Gamma^\mu_\pi(q', q) = \Delta^{-1}_\pi(q') Q_\pi - Q_\pi \Delta^{-1}_\pi(q) \quad \text{pion} \]

\[ \Gamma^\nu_\pi(q', q) = (q' + q)^\nu Q_\pi + \left[(q' + q)^\nu k^2 - k^\nu k \cdot (q' + q)\right] Q_\pi f(q'^2, q^2; k^2) \]
Generalized Ward–Takahashi Identity for the Pion-Production Current

\[ k_\mu M^\mu = -[F_s \tau] S_{p+k} Q_N S_{p'}^{-1} + S_{p'}^{-1} Q_N S_{p-k} [F_u \tau] + \Delta_{p-p'+k}^{-1} Q_\pi \Delta_{p-p'}^{-1} [F_t \tau] \]

Equivalently:

\[ k_\mu M^\mu_{\text{int}} = -[F_s \tau] Q_N + Q_N [F_u \tau] + Q_\pi [F_t \tau] \]

Note:

Charge conservation:

\[ -\tau Q_N + Q_N \tau + Q_\pi \tau = 0 \]
Prescriptions to Restore Gauge Invariance

**Gross/Riska**
Absorb hadronic form factors in propagator and construct single-hadron currents to satisfy the appropriate WT identities.

**Ohta**
Construct contact current based on minimal substitution of $\pi NN$ form factor. Basically removes hadronic structure from non-transverse contributions. Not applicable to explicit final-state interactions.

**This work**
Generalizes Ohta by introducing common form factor for non-transverse contributions only. Allows inclusion of explicit final-state interactions.
Replace by phenomenological contact term such that WTI for interaction current is satisfied.
Lowest-Order Strategy

Replace \(
\begin{array}{c}
\text{circle} + \text{rectangle} \text{bullet}
\end{array}
\)
by phenomenological contact term such that WTI for interaction current is satisfied.

Result

\[
\begin{align*}
M_{\mu}^{\text{int}} &= M_{\mu}^{\text{c}} + T^{\mu} + XG_0 \left[ \left( M_{\mu}^{u} - m_{\mu}^{u} \right) + \left( M_{\mu}^{t} - m_{\mu}^{t} \right) \right] \\
&= M_{\mu}^{\text{c}} + T^{\mu} + XG_0 \left[ \text{transverse} \right] + XG_0 \left[ \text{transverse} \right]
\end{align*}
\]

\( T^{\mu} \): undetermined transverse current

with

\[
k_{\mu}M_{\mu}^{\text{c}} = -F_s e_i + F_u e_f + F_t e_\pi .
\]
Phenomenological Choice for $M^\mu_c$

\[
M^\mu_c = g\pi\gamma_5 \left\{ \left[ \lambda + (1 - \lambda) \frac{q - \beta k}{m' + m} \right] C^\mu - (1 - \lambda) \frac{\gamma^\mu}{m' + m} \left[ e_\pi f_t - \beta k_p C^\rho \right] \right\}
\]

$\lambda$, $\beta$: free parameters

Non-singular auxiliary current:

\[
C^\mu = -e_\pi \frac{(2q - k)^\mu}{t - q^2} (f_t - \hat{F}) - e_f \frac{(2p' - k)^\mu}{u - p'^2} (f_u - \hat{F}) - e_i \frac{(2p + k)^\mu}{s - p^2} (f_s - \hat{F}),
\]

with

\[
k^\mu C^\mu = e_\pi f_t + e_f f_u - e_i f_s
\]

Subtraction function:

\[
\hat{F} = 1 - \hat{h} \left( 1 - \delta_s f_s \right) \left( 1 - \delta_u f_u \right) \left( 1 - \delta_t f_t \right)
\]

$\delta_x = 0, 1$

The function $\hat{h} = \hat{h}(s, u)$ is free fit function.

[Ohta: $\hat{h} = 0$, $X = 0$, $T^\mu = 0$]
First Application

(straight out of the ‘box’ — not optimized)

\[ d\sigma/d\Omega (\mu b/sr) \]

\[ \gamma + p \rightarrow \pi^0 + p \]
\[ \gamma + p \rightarrow \pi^+ + n \]
\[ \gamma + n \rightarrow \pi^- + p \]

\[ T_\gamma = 390 \text{ MeV} \]
\[ T_\gamma = 340 \text{ MeV} \]
\[ T_\gamma = 220 \text{ MeV} \]
\[ T_\gamma = 180 \text{ MeV} \]

\[ \theta (\text{deg}) \]

Includes

- $\rho$, $\omega$, and $a_1$ exchanges in the $t$-channel;
- $\Delta$ in $s$- and $u$-channels;
- FSI with Jülich $\pi N$ $T$-matrix.
Next Order

Instead of replacing all of \( \rightarrow \) \( \rightarrow \) \( \rightarrow \) , take into account explicitly.

\[ = E^\mu \text{ exchange current} \]
Instead of replacing all of \( U \), take into account explicitly.

**General gauge-invariance condition for** \( U^\mu \)

\[
k^\mu U^\mu(p', q', p, q) = Q'_\pi U(p', q' - k, p, q) + Q'_N U(p'^{-k}, q', p, q) - U(p', q', p, q + k)Q_\pi - U(p', q', p + k, q)Q_N
\]

Holds also true for **every subset** of \( U^\mu \) that originates from attaching a photon in all possible ways to a single two-body irreducible hadron graph \( \Rightarrow \) also true for \( E^\mu \)
Next-Order Result

Previously

\[ M_{\text{int}}^\mu = M_c^\mu + T^\mu + XG_0 \left[ (M_u^\mu - m_u^\mu) + (M_t^\mu - m_t^\mu) + T^\mu \right] \]

transverse \hspace{1cm} transverse

\[ T^\mu: \text{undetermined transverse current} \]

Now:  \( T^\mu \) is replaced by

\[ T^\mu = \left[ E^\mu - \tilde{E}^\mu \right] G_0 [F \tau] + T''^\mu \]

\[ T''^\mu: \text{undetermined transverse current} \]
Phenomenological Subtraction Current

\[ \tilde{E}^\mu = g_\pi^2 \gamma_5 \left[ \lambda - (1 - \lambda) \frac{\dot{q}}{m' + m_N} \right] S_N \gamma_5 \left[ \lambda + (1 - \lambda) \frac{\dot{q}'}{m + m_N} \right] D^\mu \]

\[ + (1 - \lambda) g_\pi^2 f_\pi f_1 e_\pi \frac{\gamma_5 \gamma^\mu}{m' + m_N} S_N \gamma_5 \left[ \lambda + (1 - \lambda) \frac{\dot{q}'}{m + m_N} \right] \]

\[ - (1 - \lambda) g_\pi^2 f_2 f_\pi' \gamma_5 \left[ \lambda - (1 - \lambda) \frac{\dot{q}}{m' + m_N} \right] S_N e'_\pi \frac{\gamma_5 \gamma^\mu}{m + m_N} \]

Non-singular auxiliary current:

\[ D^\mu = e'_\pi \frac{(2q' - k)^\mu}{(q' - k)^2 - q'^2} (f_2 f_\pi' - \hat{G}) + e'_N \frac{(2p' - k)^\mu}{(p' - k)^2 - p'^2} (f_N f_1 - \hat{G}) \]

\[ + e_\pi \frac{(2q + k)^\mu}{(q + k)^2 - q^2} (f_\pi f_1 - \hat{G}) + e_N \frac{(2p + k)^\mu}{(p + k)^2 - p^2} (f_2 f_N - \hat{G}) \]

with \( \hat{G} = 1 - \dot{g} (1 - f_\pi f_1)(1 - f_2 f_N)(1 - f_2 f_\pi')(1 - f_N f_1) \)

\( \dot{g} \): free fit function

Note: \( k_\mu D^\mu = e_\pi f_\pi f_1 + e_N f_2 f_N - e'_\pi f_2 f_\pi' - e'_N f_N f_1 \)
Transition Currents

- Gauge invariance demands that transition currents be transverse. Outline of proof:

\[
\gamma
\]

Coupling the photon to \( MB_1 B_2 \) vertex, produces

First line produces correct off-shell relation. Terms in second line must vanish individually. QED

- In a consistent microscopic approach, this should be ensured dynamically. There should be no need to adjust the transversality manually.
Example: $\gamma N \rightarrow \Delta$

Transversality of this current is ensured by the fact that $N \rightarrow \Delta$ is not possible as a physical process, i.e.

$$\begin{array}{c}
\Delta \\
\gamma N
\end{array} = \begin{array}{c}
\Delta \\
\gamma N
\end{array} + \begin{array}{c}
\Delta \\
\gamma N
\end{array} + \begin{array}{c}
\Delta \\
\gamma N
\end{array}
$$

With consistent dynamics, this current satisfies

$$\Gamma^{\beta \mu} = G_1 \gamma_5 \left(k^\beta \gamma^\mu - g^{\beta \mu} k\right) + G_2 \gamma_5 \left(k^\beta P^\mu - g^{\beta \mu} k \cdot P\right) + G_3 \gamma_5 \left(k^\beta k^\mu - g^{\beta \mu} k^2\right)$$

as a matter of course, where $P = (p + p')/2 = (2p + k)/2$. There is no need for subtractions.
Summary

• Gauge invariance is a fundamental symmetry — without it results become arbitrary.

• Current conservation \( k_\mu M^\mu = 0 \) necessary, but not sufficient, for gauge invariance of microscopic theories.

• Subtractions \( M^\mu = \tilde{M}^\mu - a^\mu \frac{k \cdot \tilde{M}}{k \cdot a} \) in general do not ensure gauge invariance.

• Most of the popular dynamical models are not gauge-invariant; despite claims to the contrary.

• Generalized Ward–Takahashi identities are necessary and sufficient.

• Real-world calculations require truncations of dynamical mechanisms that destroy gauge invariance.

• Prescriptions required to restore gauge invariance.

• Transition currents are transverse and this property must be ensured dynamically.

• Formalism allows inclusion of final-state interaction.

• FSI dynamics can be refined step-by-step in controlled manner.