Problem of Exotic Hadrons:
view from complex angular momenta

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based on papers with R.A. Arndt,
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The problem of exotic hadrons

«Why are there no strongly bound exotic states…, like those of two quarks and two antiquarks or four quarks and one antiquark?»

H.J. Lipkin (1973)

Contents

- Exotics : brief overview
- CAM and exotics
- Summary and conclusion

Why may we think that exotic hadrons <u>do</u> exist?

Experimental status

<u>Summary</u> of Lepton-Photon2005:

«The Θ-pentaquark is *not in good health*, but it is *still alive*.»

V. Burkert

Why may we think that exotic hadrons do exist?

Theoretical reasons

- **No** general arguments against exotics!
- QCD suggests **no** veto for exotic hadrons
- Any hadron may be viewed as a multi-quark system (*e.g.*, in hard processes). Why could not it have exotic quantum numbers?

Why may we think that exotic hadrons do exist?

Theoretical reasons (cont.)

- Calculations in various approaches, as a rule, provide exotic states, though with properties strongly model-dependent (bag model, soliton model, sum rules, lattice, ...)
- Complex angular momenta (CAM) may suggest one more (indirect) argument for existence of exotic hadrons

CAM and exotics

Preliminaries

Take a $2 \to 2$ process
(begin, for simplicity, with no spins).
The amplitude A has 2 independent variables
(*e.g.*, W - c.m. energy, θ - c.m. angle),
or 3 invariant variables
($s=W^2$, $t \sim z=\cos\theta$, $u \sim -z$; $s+t+u=\mathrm{const}$).
Decompose $A(s, z)$ in z into partial waves.
Physical partial-wave amplitudes $f_l(s)$ have
integer values of the orbital momentum l .

Assumptions :

► Amplitudes $f_l(s)$ admit unambiguous analytical continuation in l from integer physical points.

Fulfilled, if the amplitude $A(s, z)$ satisfies dispersion relation (DR) in the momentum transfers t, u

(Gribov-Froissart formula, 1961).

DR provides <u>sufficient</u> condition for the continuation .

<u>Necessary</u> conditions are essentially weaker .

DR's are not formally proved

(neither in general QFT, nor in QCD),

but are widely used in phenomenology of strong interactions.

Assumptions :

► There are no massless hadrons
(and no massless exchanges).

Ensures a finite range of interactions
and threshold behavior $\sim k^{2l}$ for elastic amplitudes $f_l(s)$
at physical (integer) l and $s \to s_{\rm th}$
($k \to 0$; k is the c.m. relative momentum) .

GF formula (where it is applicable)
provides the same behavior for continued $f_l(s)$.

For physical amplitudes the elastic unitarity condition is

$$f_l(s) - f_l^*(s) = 2ik\, f_l(s)\, f_l^*(s)\ .$$

For continued amplitudes $f_l(s)$ it takes the form

$$f_l(s) - [\, f_{l^*}(s)]^* = 2ik\, f_l(s)\, [\, f_{l^*}(s)\ ]^*\ ,$$

<u>inconsistent</u> with the k^{2l} -behavior at $\mathrm{Re}\, l < -1/2$

(the left-hand side terms $\sim |k|^{2\mathrm{Re}\, l}$,

the right-hand side $\sim |k|^{4\mathrm{Re}\, l + 1}$) .

The problem was ***first presented***

and ***solved*** by Gribov and Pomeranchuk (1962) .

Near threshold, reggeons condense to the point $l = -1/2$, and invalidate the k^{2l}-behavior at $\mathrm{Re}\, l < -1/2$.

With R being the effective interaction radius, the condensing trajectories are

$$l_n(s) \approx -1/2 + 2\, i\pi\, n/\ln(-k^2R^2),$$
$$k^2 \to 0, \quad n = \pm 1, \pm 2, \ldots, \pm \infty .$$

When accounting for ***spins***, the orbital momentum l changes by the total angular momentum j .

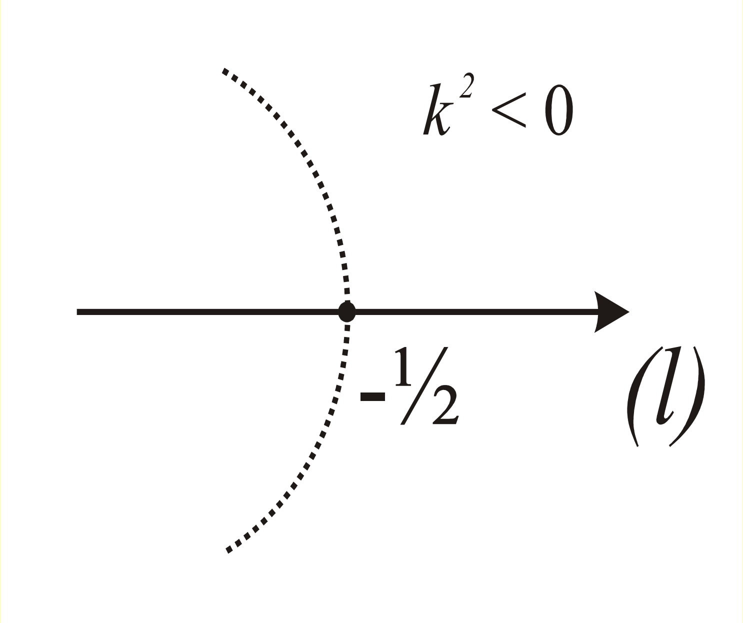
The threshold condensation of reggeons still exists, with the same structure, but shifted limiting point (Azimov, 1962)

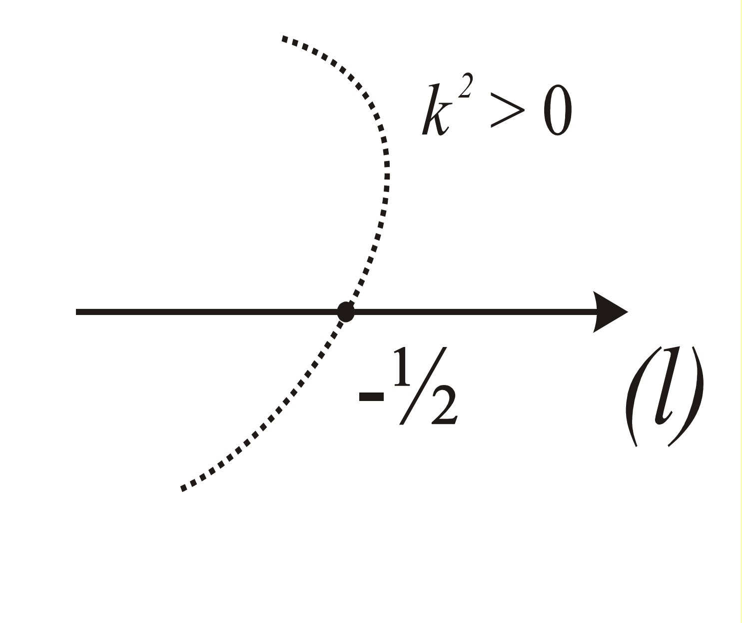
$$-1/2 \to -1/2 + \sigma_1 + \sigma_2 .$$

Thus, there are infinite number of reggeons.

Schematic structure of the threshold condensation of reggeons, as seen for the non-relativistic Yukawa potential

(Azimov, Anselm, and Shekhter, 1963)





Reggeon trajectories solve an equation of the form

$$F(j, s) = 0 \, .$$

Every pole of the partial-wave amplitude may be considered in two ways:

- ***either*** as the reggeon, *i.e.*, the pole in j , with position (and residue) dependent on energy s ,
- ***or*** as the energy-plane pole in s , with position (and residue) dependent on angular momentum j .

one-to-one correspondence

between reggeons and energy-plane poles

Infinite number of reggeons

$\Downarrow$

Infinite number of energy-plane poles

There is an **infinite "reservoir"** of poles .

Bound state is a pole at the physical sheet of the energy plane.

Resonance is a pole near the physical region of the energy plane .

Main part of energy-plane poles are "hidden" at far Riemann sheets of the energy plane .

Investigation of the non-relativistic Yukawa potential $e^{-\mu r}/r$ shows that the reggeons producing the Gribov-Pomeranchuk condensation, on one side, and bound-state (or resonance) poles, on the other, have the same nature .

They come from the same "reservoir" and, moreover, may be interchanged .

(Azimov, Anselm, and Shekhter, 1963)

The limiting transition $\mu \to 0$ visualizes the infinite set of Yukawa poles as the infinite set of Coulomb levels .

Gribov-Pomeranchuk threshold condensations are independent of quantum numbers.

Therefore, the strong interaction ***S***-matrix should contain infinite number of energy-plane poles with any quantum numbers,

both exotic *and* non-exotic.

The necessary condition for existence of exotics, existence of exotic energy-plane poles,

is satisfied.

It is now a problem of more detailed dynamics, which of the poles may appear near the physical region, to reveal bound states or resonances.

Note : CAM are used here differently from traditional usage.

- Usually: begin in the t-channel, construct t-channel partial-wave amplitudes, continue them in j, then obtain result (high-energy asymptotics) for the crossed s-channel.
- Here: begin in the s-channel, construct s-channel partial-wave amplitudes, continue them in j, then obtain result (energy-plane poles) for the same s-channel.

Summary

- Under familiar assumption of analyticity, hadronic amplitudes have infinite number of energy-plane poles with ***any*** quantum numbers, both exotic and non-exotic.
- Can one constrain dynamics so, that ***no exotic pole*** may approach the physical region?

Conclusion

«…<u>either</u> these states will
be *found* by experimentalists
<u>or</u> our confined, quark-gluon
theory of hadrons is as yet *lacking*
in some fundamental ingredient...»

R.L. Jaffe, K. Johnson (1976)