Dynamically Generated $J^P = \frac{3}{2}^-$ Resonances from Baryon Decuplet-Meson Octet Interaction

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Plan

I. General discussion on dynamically generated $3/2^-$ resonances

S.S., E. Oset, M.J. Vicente Vacas, NPA 750 (2005) 294 and Eur.Phys.J.A24 (2005) 287

- Introduction
- Unitary coupled channel formalism
- Resonances observed in different channels

II. Detailed study of the $\Lambda(1520)$ resonance

S.S., E. Oset, M.J.Vicente Vacas, PRC72 (2005) 015206 and L.Roca, S.S., V.K.Magas, E Oset (in preparation)

- Improvements; addition of extra channels, etc.
- Determination of unknown parameters and couplings
- Predictions and comparison with experimental data

Introduction

- Chiral Dynamics + Unitary Coupled Channels $\Rightarrow$ Successful prediction of *s-wave* baryon resonances.
- *s-wave* scattering of the pion octet 0^- with the nucleon octet $1/2^+$
  $\Longrightarrow$ Dynamical generation of $J^P = 1/2^-$ baryon resonances

- $N^*(1535)$
- $\Lambda(1405),\ \Lambda(1670),\ \Sigma(1620)$
- $\Xi(1620)$

- *D. Jido et al, NPA 725(03)181;*
- *C. Garcia Recio et al, PLB 582(04)49;*
- *M.F.M. Lutz et al, NPA 700(02)193;*
- *E. Oset et al, NPA 635(98)99*
- *N. Kaiser et al; T. Inoue et al ; etc...*

Introduction

- What about *d-wave* baryon resonances?
  $8 \otimes 8$ in *d-wave* has too many unknown terms in χPT $\Rightarrow$ No predictivity!
- But, these *d-wave* resonances couple in *s-wave* to the $3/2^+$ baryons decuplet and the 0^- mesons octet. If these *s-wave* channels are *dominant* the resonances could appear in the *s-wave* scattering of the baryons decuplet and the mesons octet.

  $\Rightarrow$ *s-wave* implies χPT is more predictive!
- In fact some of these $3/2^-$ *d-wave* resonances ($N^*(1520)$, $N^*(1700)$, $\Delta(1700)$) have large decay branching ratios to $N\pi\pi$ channels even when πN is favoured by phase space!

  Kolomeitsev and Lutz, PLB 585(04)243

Formulation

Lowest order chiral Lagrangian *(Decuplet - Octet interaction)*

$$\mathcal{L} = -i\bar{T}^{\mu}\not{\mathcal{D}}T_{\mu}$$

T^{μ}_{abc} the baryons decuplet and D^{ν} the covariant derivative

$$\mathcal{D}^{\nu}T^{\mu}_{abc} = \partial^{\nu}T^{\mu}_{abc} + (\Gamma^{\nu})^{d}_{a}T^{\mu}_{dbc} + (\Gamma^{\nu})^{d}_{b}T^{\mu}_{adc} + (\Gamma^{\nu})^{d}_{c}T^{\mu}_{abd}$$

$$\Gamma^{\nu} = \frac{1}{2}(\xi\partial^{\nu}\xi^{\dagger} + \xi^{\dagger}\partial^{\nu}\xi); \qquad \xi^{2} = U = e^{i\sqrt{2}\Phi/f}$$

with a single coupling constant f

E. Jenkins et al, PLB 259 (91) 353

We write $T_{\mu} = T u_{\mu}$ where

$$u_{\mu} = \sum_{\lambda,s}\mathcal{C}(1\ \tfrac{1}{2}\ \tfrac{3}{2}\ ;\ \lambda\ s\ s_{\Delta})\ e_{\mu}(p,\lambda)\ u(p,s)$$

Formulation

We will consider only the s-wave part of the interaction and the non-relativistic limit, so that

$$\bar{u}(p', s')\gamma^\nu \, u(p, s) = \delta^{\nu 0}\delta_{ss'} + \mathcal{O}(|\vec{p}|/M)$$

$$\sum_{\lambda', s'} \sum_{\lambda, s} \mathcal{C}(1 \, \tfrac{1}{2} \, \tfrac{3}{2} \, ; \, \lambda' \, s' \, s_\Delta) \, e^*_\mu(p', \lambda') \, \mathcal{C}(1 \, \tfrac{1}{2} \, \tfrac{3}{2} \, ; \, \lambda \, s \, s_\Delta) \, e^\mu(p, \lambda) \, \delta_{ss'} = -1 + \mathcal{O}(|\vec{p}|^2/M^2) \, .$$

$$\mathcal{L} = 3i \; Tr\{\bar{T} \cdot T \;\; \Gamma^{0T}\}$$

$$(\bar{T} \cdot T)^a_d = \sum_{b,c} \bar{T}^{abc} \, T_{dbc}; \qquad \Gamma^\nu = \frac{1}{4f^2}(\Phi\partial^\nu\Phi - \partial^\nu\Phi\Phi).$$

$$T^{111} = \Delta^{++}, T^{112} = \frac{1}{\sqrt{3}}\Delta^{+}, T^{122} = \frac{1}{\sqrt{3}}\Delta^{0}, T^{222} = \Delta^{-}, T^{113} = \frac{1}{\sqrt{3}}\Sigma^{*+},$$
$$T^{123} = \frac{1}{\sqrt{6}}\Sigma^{*0}, T^{223} = \frac{1}{\sqrt{3}}\Sigma^{*-}, T^{133} = \frac{1}{\sqrt{3}}\Xi^{*0}, T^{233} = \frac{1}{\sqrt{3}}\Xi^{*-}, T^{333} = \Omega^{-}.$$

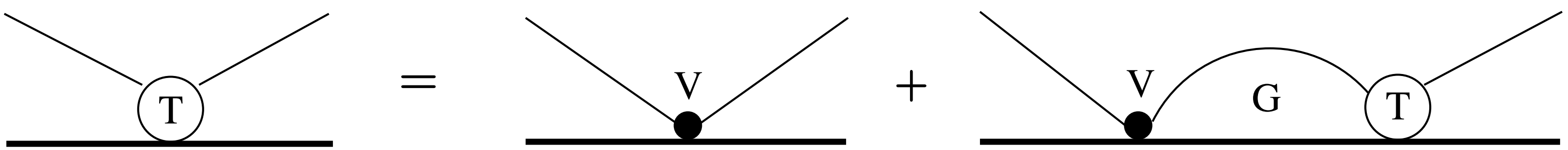
Formulation

For a meson of incoming (outgoing) momenta $k(k')$ we get for the s-$wave$ transition amplitudes,

$$V_{ij} = -\frac{1}{4f^2} C_{ij} (k^0 + k'^0)$$

This V is used as kernel of a coupled channels Bethe Salpeter equation

$$T = (1 - VG)^{-1} V.$$



Formulation

$$G_l = i2M_l \int \frac{d^4q}{(2\pi)^4} \frac{1}{(P-q)^2 - M_l^2 + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon}$$

Using dimensional regularization

$$G_l = \frac{1}{16\pi^2} \left\{ a_l(\mu) + \ln \frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s} \ln \frac{m_l^2}{M_l^2} + \frac{q_l}{\sqrt{s}} \left[\ln(s - (M_l^2 - m_l^2) + 2q_l\sqrt{s}) + \ln(s + (M_l^2 - m_l^2) + 2q_l\sqrt{s}) - \ln(-s + (M_l^2 - m_l^2) + 2q_l\sqrt{s}) - \ln(-s - (M_l^2 - m_l^2) + 2q_l\sqrt{s}) \right] \right\},$$

with unknown parameters a_l for which the ‘natural size‘ is ~ -2 corresponding to a cut-off of $\sim$ 700 MeV.

Formulation

$SU(3)$ decomposition: $8 \otimes 10 = 8 \oplus 10 \oplus 27 \oplus 35$

Projection on to $SU(3)$ basis: $C_{\alpha\beta} = \sum_{i,j} \langle i, \alpha \rangle C_{ij} \langle j, \beta \rangle$

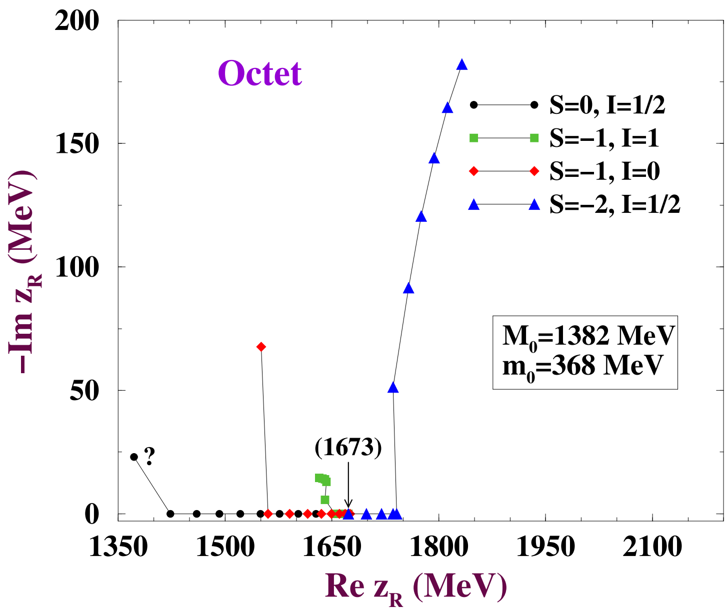
Strength is proportional to: $C_{\alpha\beta} = diag(6, 3, 1, -3)$

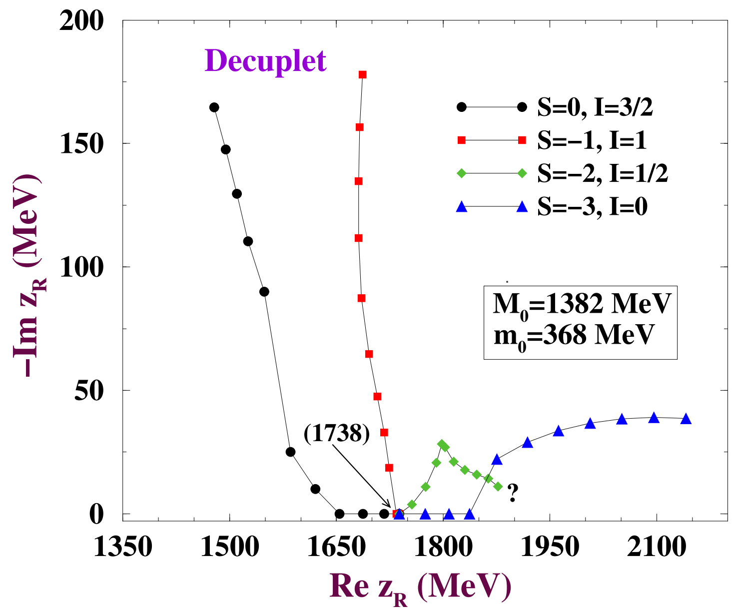
- strong attraction in octet, followed by decuplet
- weak attraction in 27, repulsion in 35

We then solve the BS equation and look for poles in the complex plane. In the $SU(3)$ limit we get two poles, one each for the octet and decuplet representations

Results: Trajectories of Poles in the Complex Plane

Two bound states in the $SU(3)$ limit





Break $SU(3)$ symmetry gradually

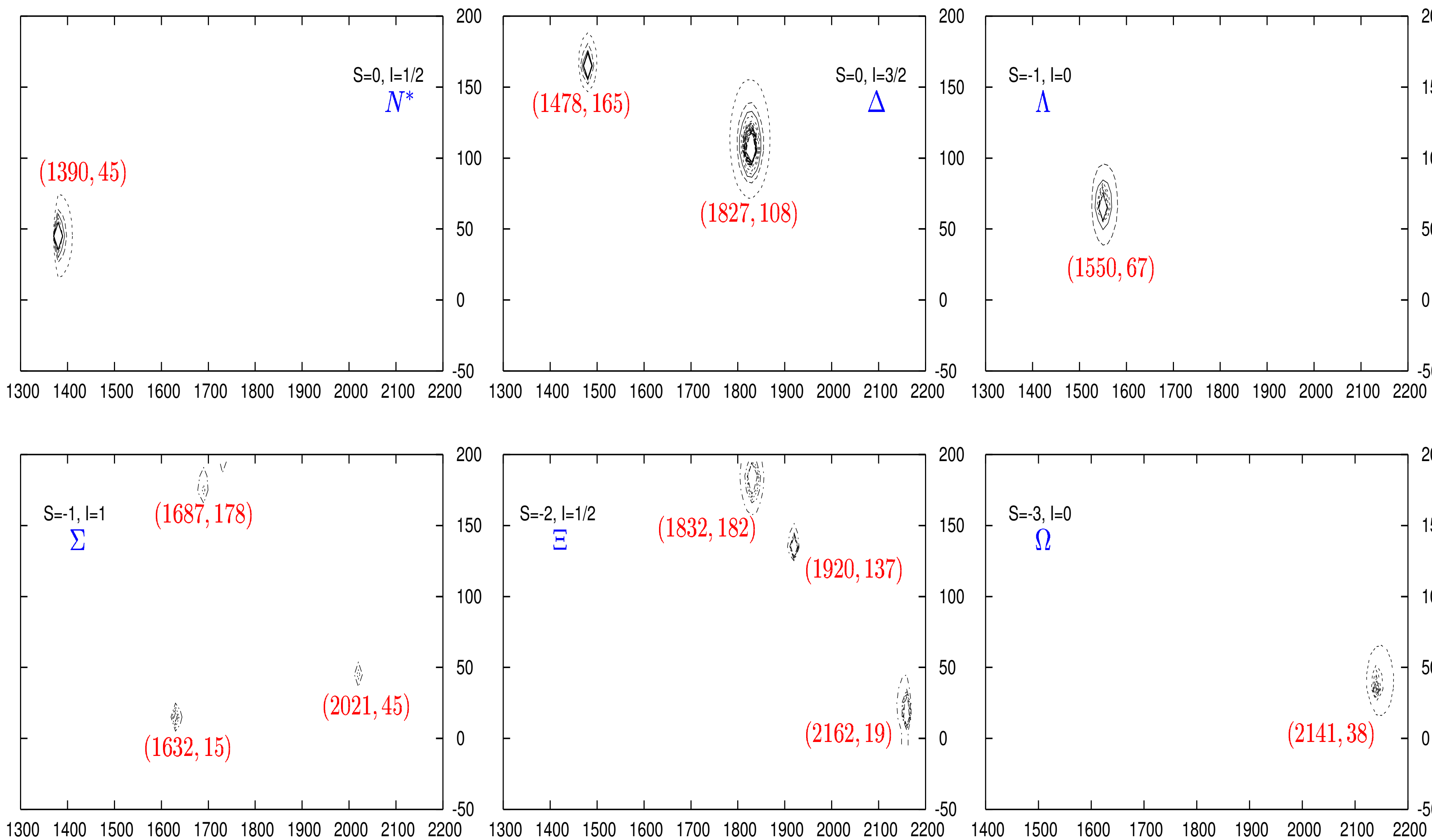
$$\begin{aligned} M_i(x) &= M_0 + x(M_i - M_0) \\ m_i^2(x) &= m_0^2 + x(m_i^2 - m_0^2) \\ & \quad 0 \leq x \leq 1 \end{aligned}$$

Close to the pole (z_R)

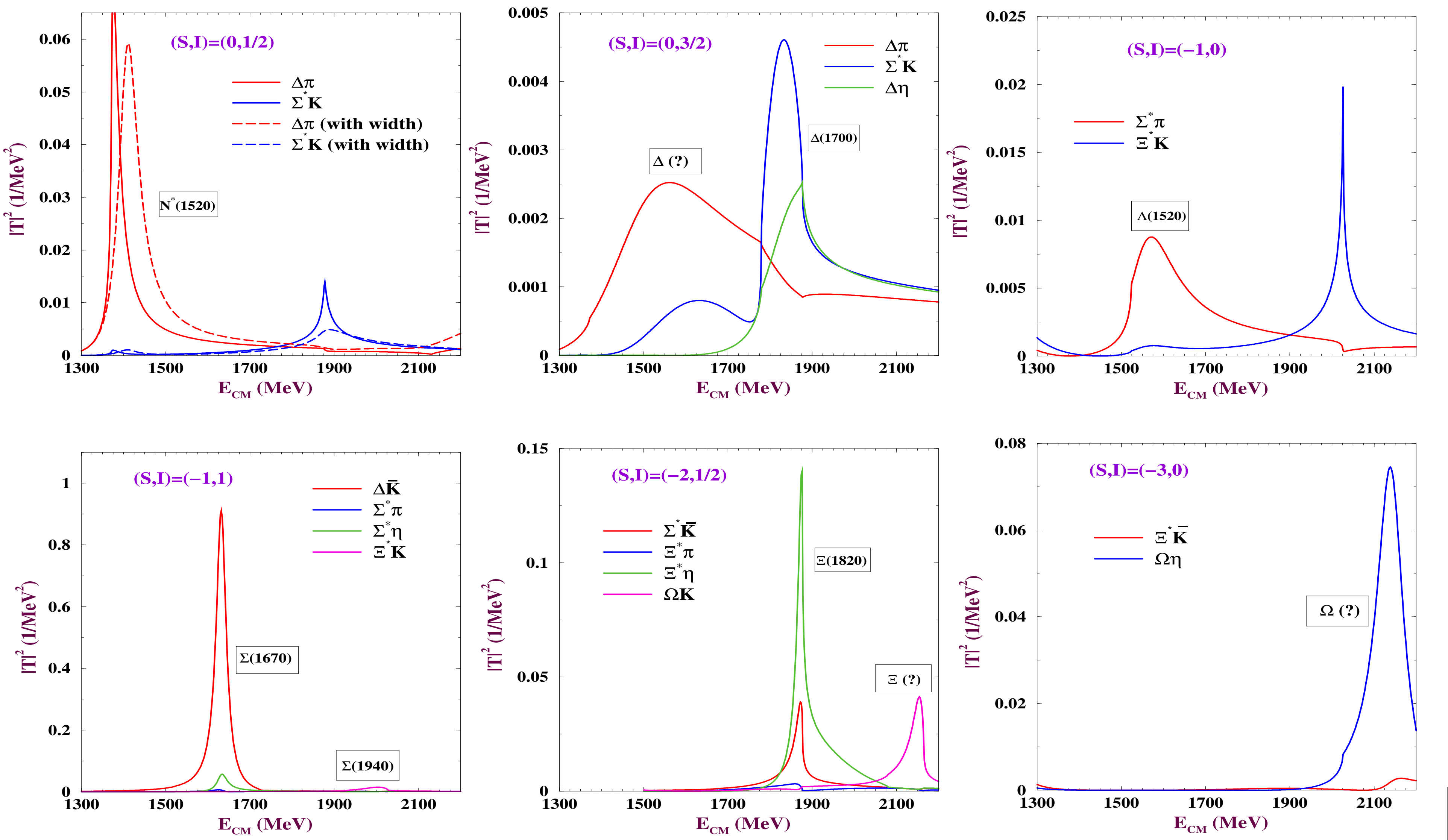
$$T_{ij}(z) = \frac{g_i g_j}{z - z_R}$$

residue $\rightarrow$ couplings g_i

Poles in the Complex Plane



$J^P = \frac{3}{2}^-$ Resonances



$\Lambda(1520)$

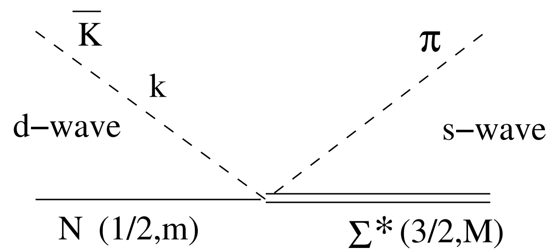
- In our treatment so far with the $\pi\Sigma^*$ and $K\Xi^*$ in coupled channels the $\Lambda(1520)$ is generated dynamically with a large coupling to the $\pi\Sigma^*$ channel.
- It appears about 50 MeV higher in mass and is about 10 times broader than the nominal width which is 15.6 MeV.
- This large width is a consequence of the fact that the pole appears above the $\pi\Sigma^*$ threshold.
- However, the width of the $\Lambda(1520)$ comes essentially from the decay into $\bar{K}N$ (45%) and $\pi\Sigma$ (42%)
- It is mandatory to add these channels to our scheme

$\Lambda(1520)$

- Other channels like $\eta\Lambda$ and $K\Xi$ could also contribute but their influence will be in the mass and not the width. In any case the position (mass) can be fine tuned through the subtraction constants.
- The lowest partial wave in which the channels $\bar{K}N$ and $\pi\Sigma$ can couple to spin parity $3/2^-$ is $L = 2$.
- We will couple these channels in d-wave to the $\pi\Sigma^*$ channel and not to the $K\Xi^*$ which is far away from the region of influence
- We will also introduce the Σ^* width in the meson baryon loop function since the $\pi\Sigma^*$ threshold is very close to the peak of the $\Lambda(1520)$

$\Lambda(1520)$

Coupling of d-wave channels:



$$-it_{\bar{K}N\to\pi\Sigma^*} = -i\gamma'_{\bar{K}N}\;|\vec{k}|^2\left[T^{(2)\dagger}\otimes Y_2(\hat{k})\right]_{0\,0}$$

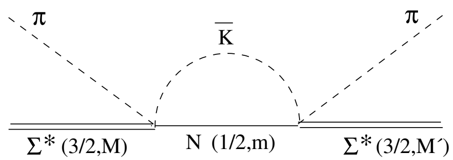
where

$$\langle 3/2\;M|\;T^{(2)\dagger}_{\mu}\;|1/2\;m\rangle = \mathcal{C}(1/2\;2\;3/2;m\;\mu\;M)\;\langle 3/2||\;T^{(2)\dagger}\;||1/2\rangle$$

so that

$$-it_{\bar{K}N\to\pi\Sigma^*} = -i\gamma_{\bar{K}N}\;|\vec{k}|^2\;\mathcal{C}(\tfrac{1}{2}\;2\;\tfrac{3}{2};m,M-m)Y_{2,m-M}(\hat{k})(-1)^{M-m}\sqrt{4\pi}$$

$\Lambda(1520)$



$$
\begin{aligned}
T_2 &= i \int \frac{d^4 q}{(2\pi)^4}\, G_N\, D_{\bar K}\, 4\pi \\
&\quad \gamma_{\bar K N} |\vec q|^2 \sum_m \mathcal{C}(1/2\ 2\ 3/2; m, M' - m) Y_{2,m-M'}(\hat q)(-1)^{M'-m} \\
&\quad \gamma_{\bar K N} |\vec q|^2\ \mathcal{C}(1/2\ 2\ 3/2; m, M - m) Y^*_{2,m-M}(\hat q)(-1)^{M-m}
\end{aligned}
$$

- Angular integration of the two $Y \longrightarrow \delta_{MM'}$
- Orthogonality of the Clebsch Gordan coefficients
- On-shell factorization of vertex

$$T_2 = [\gamma_{\bar K N} q_{on}]^2 \times G_{\bar K N} = V_{\pi\Sigma^* \to \bar K N}\ G_{\bar K N}\ V_{\bar K N \to \pi\Sigma^*}$$

$\Lambda(1520)$

With the V matrix given by

$$V = \begin{vmatrix} C_{11}(k_1^0 + k_1^0) & C_{12}(k_1^0 + k_2^0) & \gamma_{13}\, q_3^2 & \gamma_{14}\, q_4^2 \\ C_{21}(k_2^0 + k_1^0) & C_{22}(k_2^0 + k_2^0) & 0 & 0 \\ \gamma_{13}\, q_3^2 & 0 & \gamma_{33}\, q_3^4 & \gamma_{34}\, q_3^2\, q_4^2 \\ \gamma_{14}\, q_4^2 & 0 & \gamma_{34}\, q_3^2\, q_4^2 & \gamma_{44}\, q_4^4 \end{vmatrix}$$

where $C_{11} = 4$, $C_{22} = 3$ and $C_{12} = C_{21} = -\sqrt{6}$

$$q_i = \frac{1}{2\sqrt{s}} \sqrt{[s - (M_i + m_i)^2][s - (M_i - m_i)^2]}$$

$$k_i^0 = \frac{s - M_i^2 + m_i^2}{2\sqrt{s}}$$

we can continue with the formalism as in ordinary s-wave scattering.

$\Lambda(1520)$

To take the $\pi\Sigma^*$ width into account we fold G with the spectral function of the Σ^*:

$$G_{\pi\Sigma^*}(\sqrt{s}, M_{\Sigma^*}, m_\pi) \quad \rightarrow \quad \int_{M_{\Sigma^*}-2\Gamma_0}^{M_{\Sigma^*}+2\Gamma_0} d\sqrt{s'} \ \frac{-1}{\pi} \, \text{Im} \left[\frac{1}{\sqrt{s'} - M_{\Sigma^*} + i\Gamma_{\Sigma^*}(s')/2} \right] \times G_{\pi\Sigma^*}(\sqrt{s}, \sqrt{s'}, m_\pi)$$

where

$$\Gamma_{\Sigma^*}(s') = \Gamma_0 \left(0.88 \frac{q^3(s', M_\Lambda^2, m_\pi^2)}{q^3(M_\Sigma^{*2}, M_\Lambda^2, m_\pi^2)} \Theta(\sqrt{s'} - M_\Lambda - m_\pi) + 0.12 \frac{q^3(s', M_\Sigma^2, m_\pi^2)}{q^3(M_\Sigma^{*2}, M_\Sigma^2, m_\pi^2)} \Theta(\sqrt{s'} - M_\Sigma - m_\pi) \right),$$

$\Lambda(1520)$

Using V as the kernel and the loop function G we use the coupled channel BS equation to get the amplitudes T_{ij} for $\bar{K}N \rightarrow \bar{K}N$ and $\bar{K}N \rightarrow \pi\Sigma$

From a fit to the experimental amplitudes $\tilde{T}_{ij}$ we find the unknown parameters $\gamma_{13}, \gamma_{14}, \gamma_{33}, \gamma_{34}, \gamma_{44}$ in the V matrix and the subtraction constants a_0, a_2 in G

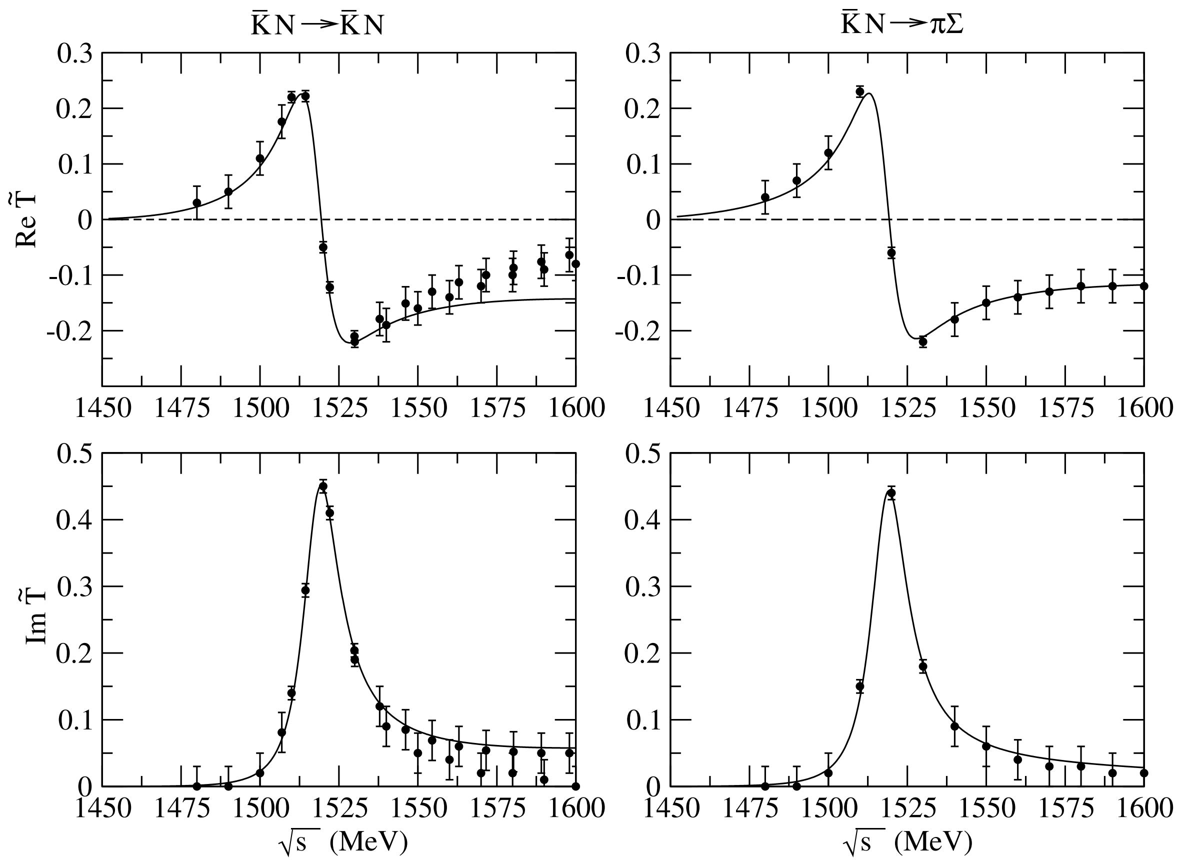
$$\tilde{T}_{ij}(\sqrt{s}) = -\sqrt{\frac{M_i q_i}{4\pi\sqrt{s}}}\sqrt{\frac{M_j q_j}{4\pi\sqrt{s}}}\,T_{ij}(\sqrt{s})$$

$$B_i = \frac{\Gamma_i}{\Gamma} = Im\tilde{T}_{ii}(\sqrt{s} = M_{\Lambda(1520)})$$

We get the following values

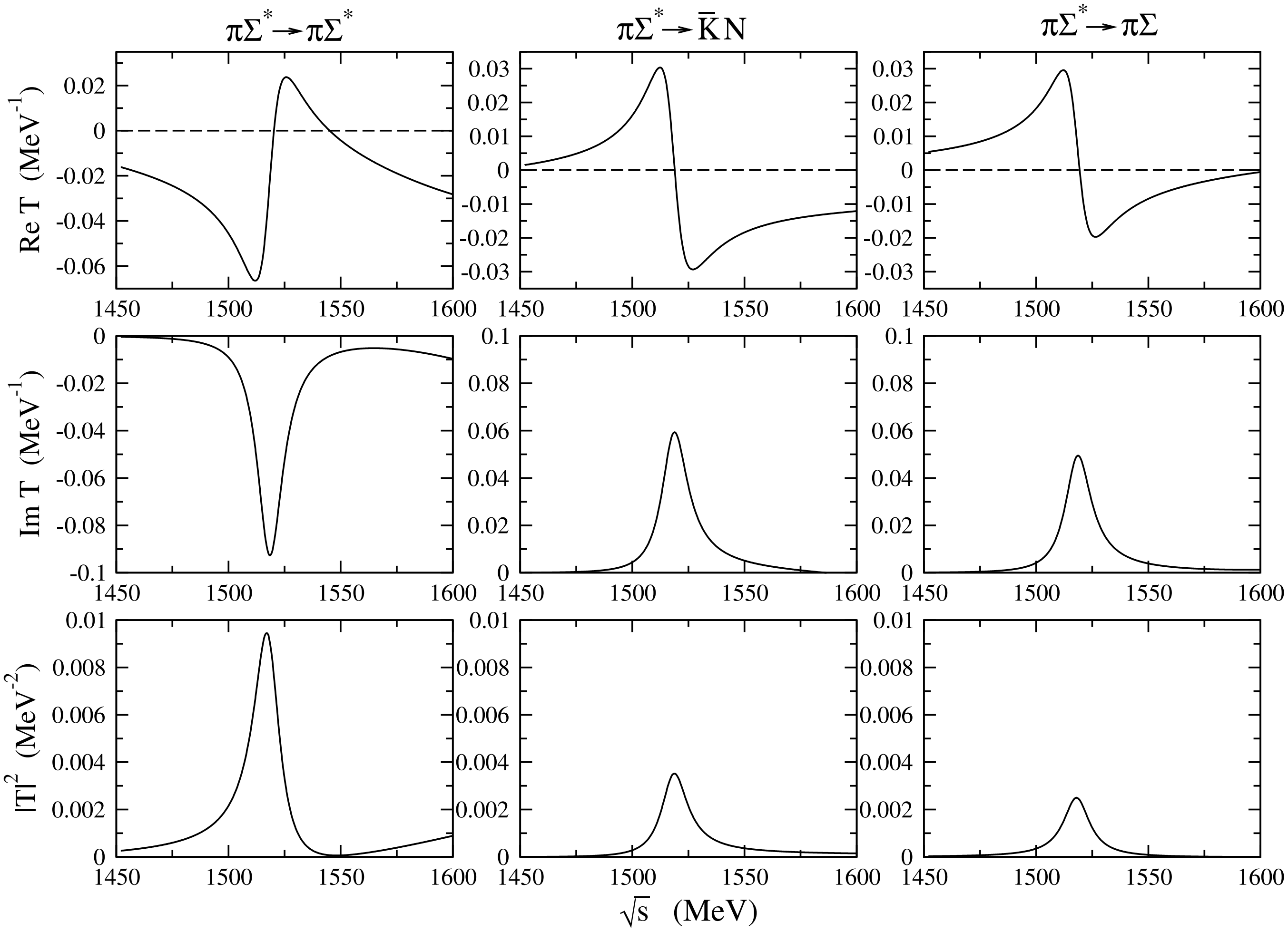
a_0	a_2	γ_{13} (MeV^{-3})	γ_{14} (MeV^{-3})	γ_{33} (MeV^{-5})	γ_{44} (MeV^{-5})	γ_{34} (MeV^{-5})
-1.8	-8.1	0.98×10^{-7}	1.1×10^{-7}	-1.7×10^{-12}	-0.7×10^{-12}	-1.1×10^{-12}

$\Lambda(1520)$



Fit to the experimental amplitudes. Left column: $\bar{K}N \to \bar{K}N$; right column: $\bar{K}N \to \pi\Sigma$. Experimental data from G. P. Gopal *et al.* NPB119 (1977) 362 and M. Alston-Garnjost *et al.* PRD18 (1978) 182

$\Lambda(1520)$



From left to right: Unitary amplitudes for $\pi\Sigma^* \to \pi\Sigma^*$, $\pi\Sigma^* \to \bar{K}N$ and $\pi\Sigma^* \to \pi\Sigma$.

$\Lambda(1520)$

Close to the peak of the $\Lambda(1520)$ the amplitudes are given by

$$T_{ij}(\sqrt{s}) = \frac{g_i g_j}{\sqrt{s} - M_{\Lambda(1520)} + i\Gamma_{\Lambda(1520)}/2}$$

from where we calculate the couplings of the $\Lambda(1520)$ to the different channels:

$$g_i g_j = -\frac{\Gamma_{\Lambda(1520)}}{2} \frac{|T_{ij}(M_{\Lambda(1520)})|^2}{Im[T_{ij}(M_{\Lambda(1520)})]},$$

We get

g_1	g_2	g_3	g_4
0.91	-0.29	-0.54	-0.45

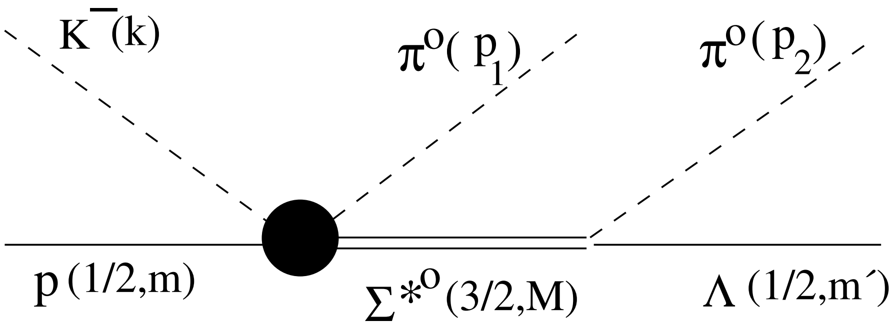
The partial decay widths can be obtained from

$$\Gamma_i = \frac{g_i^2}{2\pi} \frac{M_i}{M_{\Lambda(1520)}} q_i$$

$\Lambda(1520)$

Predictions:

The reaction $K^- p \to \pi^0 \Sigma^{*0}(1385) \to \pi^0 \pi^0 \Lambda(1116)$

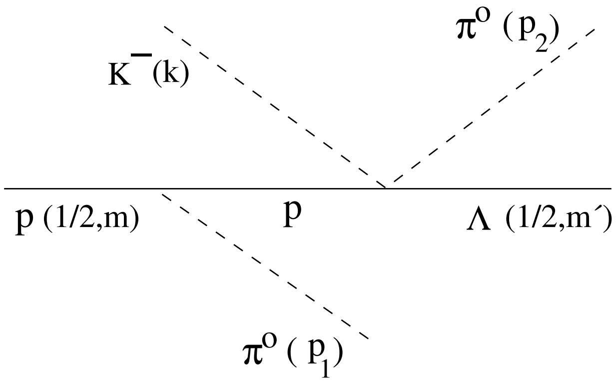


$$-it(\vec{p}_1, \vec{p}_2) = \frac{-iT_{\bar{K}N \to \pi\Sigma^*}}{3\sqrt{2}} \, \frac{f_{\Sigma^*\pi\Lambda}/m_\pi}{M_R - M_{\Sigma^*} + i\Gamma_{\Sigma^*}(M_R)/2} \left\{ \begin{array}{ll} -2p'_{2z} & m' = +1/2 \\ p'_{2x} + ip'_{2y} & m' = -1/2 \end{array} \right\}$$

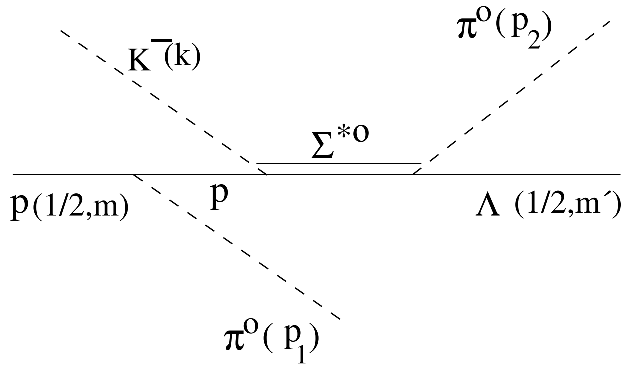
Symmetrize the amplitude $\to$ three-body phase space $\to$ cross-section

$\Lambda(1520)$

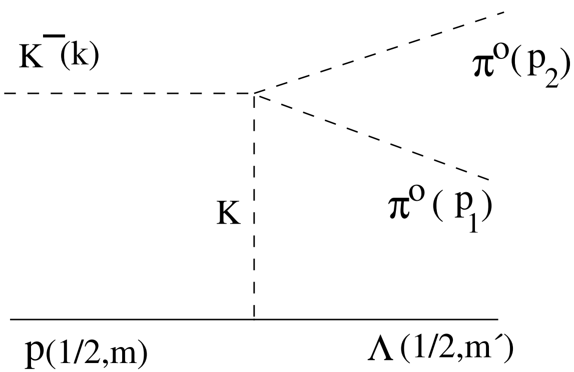
We also add the following conventional diagrams



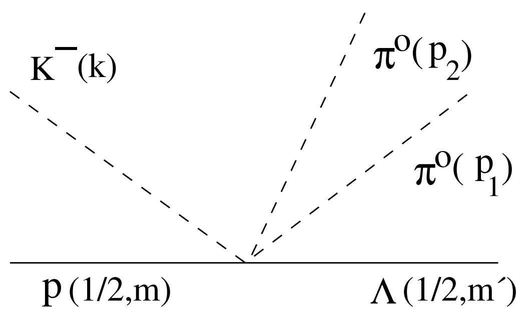
(a)



(b)

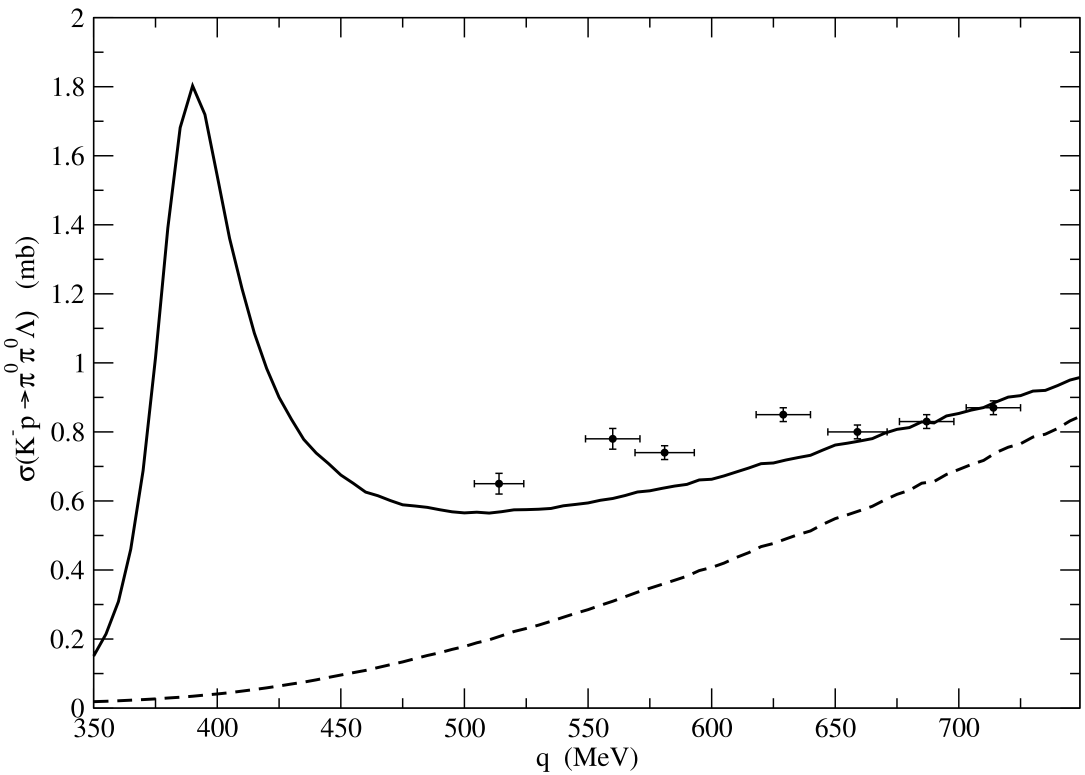


(a)



(b)

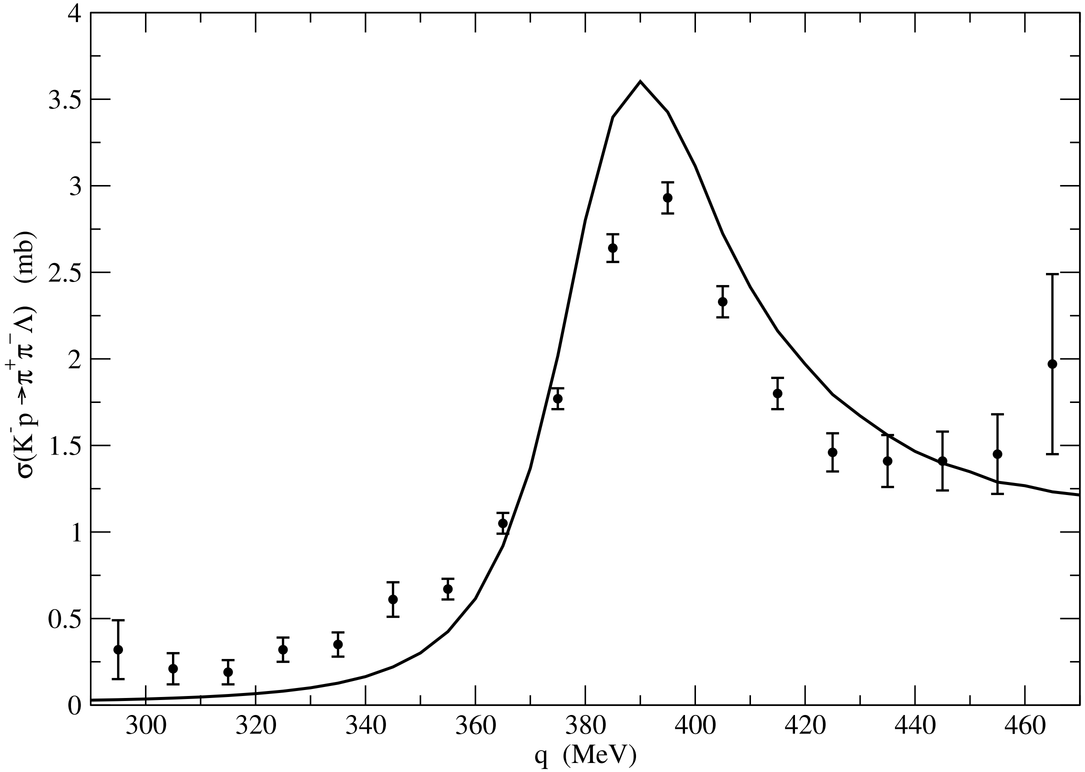
$\Lambda(1520)$



$K^- p \to \pi^0 \pi^0 \Lambda$ cross section (mb) vs p_{lab} (MeV) of K^-

Experimental data from S. Prakhov *et al.*, PRC69 (2004) 042202

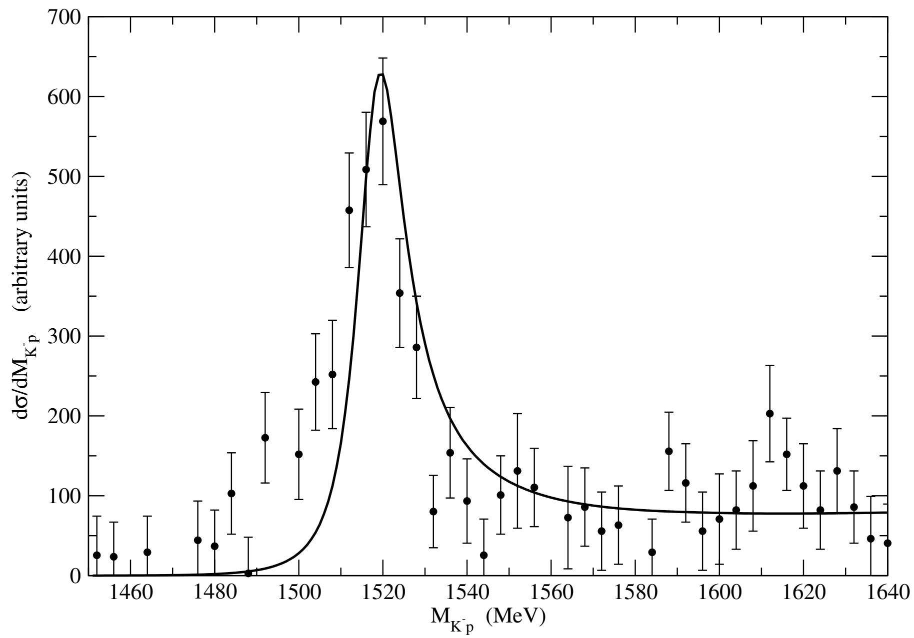
$\Lambda(1520)$



$K^- p \to \pi^+ \pi^- \Lambda$ cross section (mb) vs p_{lab} (MeV) of K^-

Experimental data from T. S. Mast *et al.*, PRD7 (1973) 5

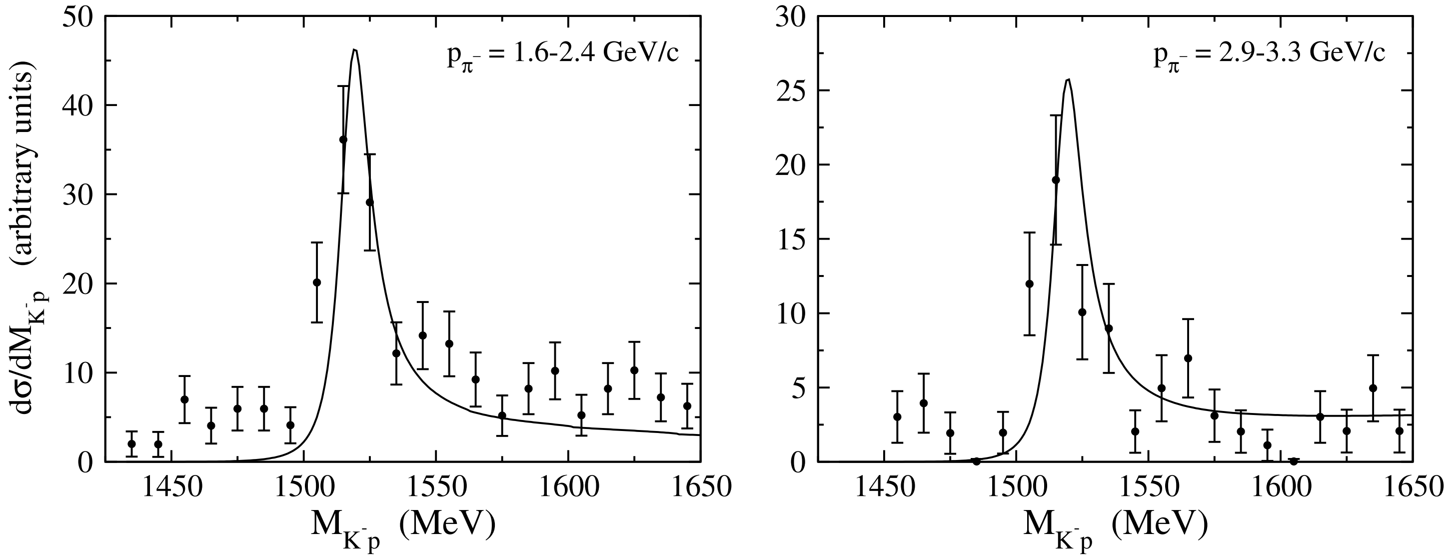
$\Lambda(1520)$



K^-p invariant mass distribution for the $\gamma p \to K^+K^-p$ reaction with photons in the range $E_\gamma = 2.8 - 4.8$ GeV.

Experimental data from D. P. Barber *et al.*, Z. Phys. C7 (1980) 17

$\Lambda(1520)$



K^-p invariant mass distribution for the $\pi^-p \to K^0K^-p$ reaction. Left: pions in the range $p_{\pi^-} = 1.6 - 2.4$ GeV/c; right: $p_{\pi^-} = 2.9 - 3.3$ GeV/c.

Experimental data from O. I. Dahl *et al.*, Phys. Rev. **163** (1967) 1377

Summary: $3/2^-$ resonances

- Systematic study of the interaction of the baryon decuplet with the meson octet taking the lowest order chiral Lagrangian (Weinberg Tomozawa term)
$\Rightarrow\ J^P = 3/2^-$ resonances
- Poles associated to established resonances:
$N^*(1520)$,$\Delta(1700)$,$\Lambda(1520)$,$\Sigma(1670)$,$\Sigma(1940)$,$\Xi(1820)$,$\Omega(2250)$
- Prediction of couplings of these resonances to the meson baryon decay channels $\Rightarrow$partial widths
- Prediction of extra resonances not yet observed: $\Xi(2160)$ with a width of 40 MeV, and some others too broad e.g. $\Delta \sim$ 1550 MeV.

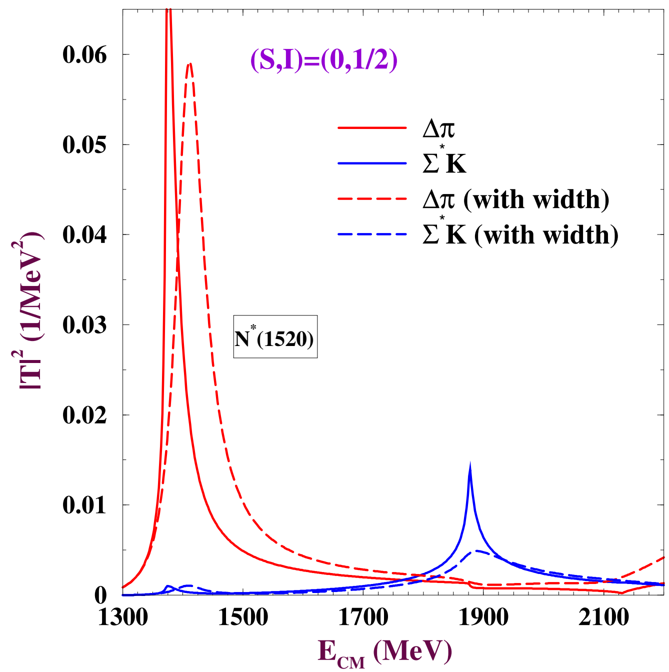
Summary: $\Lambda(1520)$

- Phenomenological introduction of the $d-wave$ channels $\bar{K}N$ and $\pi\Sigma$ into the coupled channel scheme
- Introduction of the Σ^* width in the $\pi\Sigma^*$ loop function
- We find that the $\Lambda(1520)$ couples strongly to the $\pi\Sigma^*$ channel though the branching ratios to the $\bar{K}N$ and $\pi\Sigma$ channels are much larger
- Prediction of amplitudes and couplings of the $\Lambda(1520)$ to all the channels
- We obtain good agreement with experimental data in the reactions $K^-p \to \Lambda\pi\pi$, $\gamma p \to K^+K^-p$ and $\pi^-p \to K^0K^-p$ at energies close to and above the $\Lambda(1520)$ region.

Extras

Results: $S = 0, I = 1/2\ \ (N^*)$

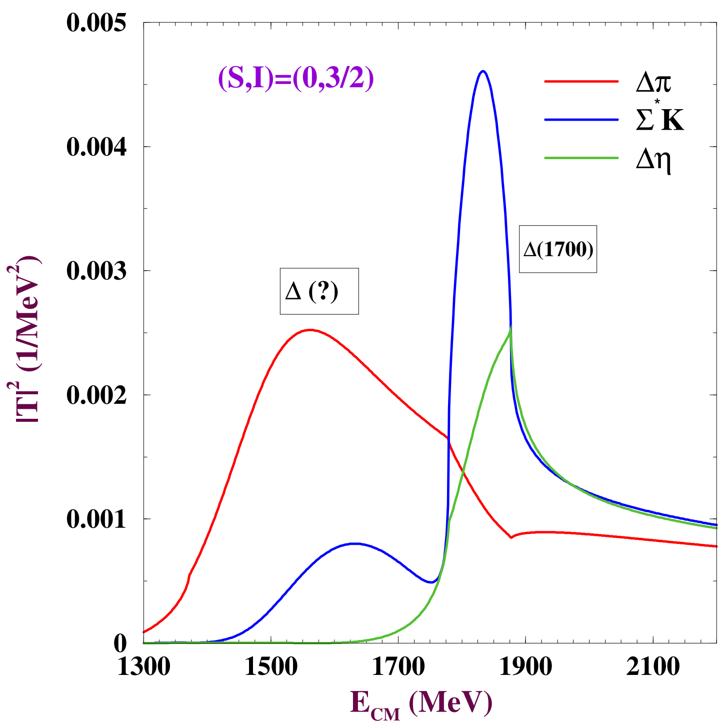
- States: $\Delta\pi$ and $\Sigma^* K$.



- Pole at $1372 - i\ 20$ MeV with strong coupling to $\Delta\pi$ channel.
- PDG $N^*(1520)$, $\Gamma = 120$ MeV
- The peak at 1877 MeV is a threshold effect.

Results: $S = 0, I = 3/2\ (\Delta)$

- States: $\Delta\pi$, $\Sigma^* K$ and $\Delta\eta$.
- Two poles in the complex plane



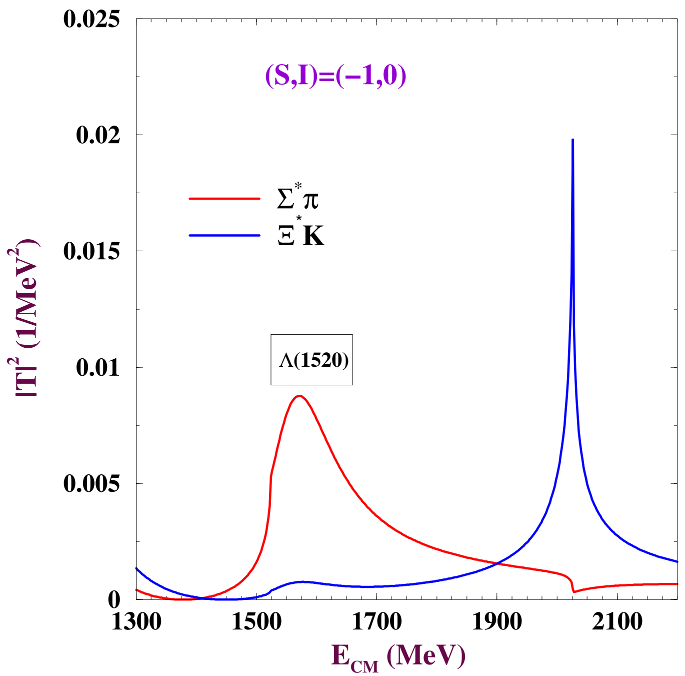
- Pole at $1478 - i\ 165$ MeV with strong coupling to $\Delta\pi$ channel.
- No counterpart in PDG. *Missing resonance ?*
- Pole at $1827 - i\ 108$ MeV with strong coupling to $\Sigma^* K$ channel.
- PDG $\Delta(1700)$, $\Gamma = 300$ MeV

Couplings of Δ to various channels

z_R	$1478 - i165$		$1827 - i108$	
	g_i	$\|g_i\|$	g_i	$\|g_i\|$
$\Delta\pi$	$2.0 - i1.9$	2.8	$0.5 + i0.8$	1.0
$\Sigma^* K$	$1.6 - i1.6$	2.3	$3.3 + i0.7$	3.4
$\Delta\eta$	$0.3 - i0.1$	0.3	$1.7 - i1.4$	2.2

Results: $S = -1, I = 0 \ (\Lambda)$

- States: $\Sigma^* \pi$ and $\Xi^* K$.



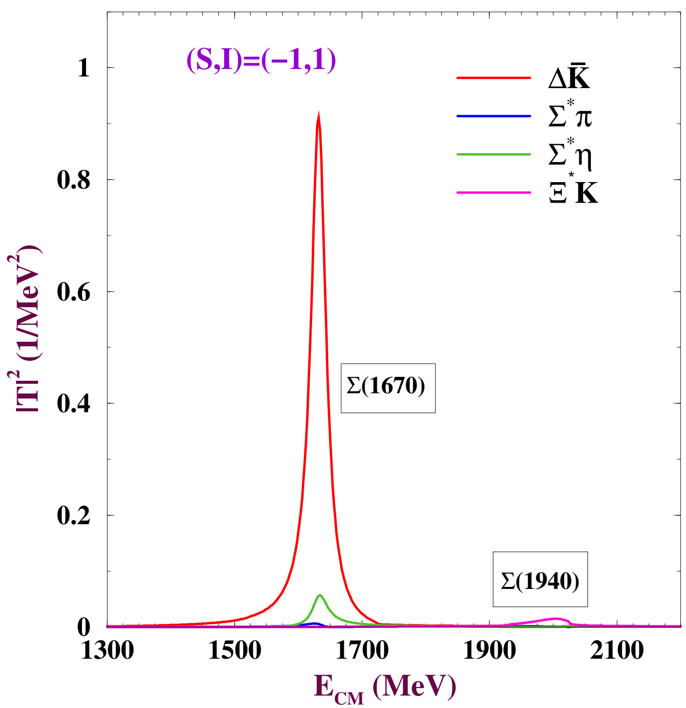
- Pole at $1550 - i\ 67$ MeV couples strongly to $\Sigma^* \pi$ channel.
- PDG $\Lambda(1520)$
- The peak around 2000 MeV is a threshold effect.

Couplings of Λ various to channels

z_R	$1550 - i67$	
	g_i	$\lvert g_i \rvert$
$\Sigma^* \pi$	$2.0 - i1.5$	2.5
$\Xi^* K$	$0.9 - i0.8$	1.2

Results: $S = -1, I = 1\ (\Sigma)$

- States: $\Delta\bar{K}$, $\Sigma^*\pi$, $\Sigma^*\eta$ and Ξ^*K.
- We find three poles in the complex energy plane



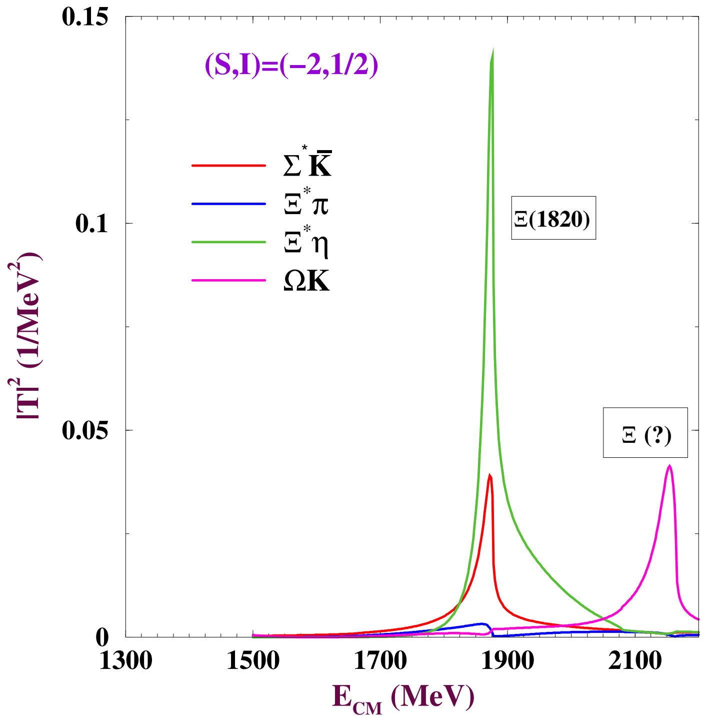
- Pole at $1632 - i\ 15$ MeV with strong coupling to $\Delta\bar{K}$ channel.
- PDG: $\Sigma(1670)$, $\Gamma = 60$ MeV
- Pole at $1687 - i\ 178$ MeV. Too broad !
- Pole at $2021 - i\ 45$ is associated to $\Sigma(1940)$ with $\Gamma = 220$ MeV in the PDG

Couplings of Σ to various channels

z_R	$1632 - i15$		$1687 - i178$		$2021 - i45$	
	g_i	$\lvert g_i \rvert$	g_i	$\lvert g_i \rvert$	g_i	$\lvert g_i \rvert$
$\Delta\overline{K}$	$3.7 - i0.03$	3.7	$0.4 - i1.7$	1.8	$0.4 - i0.5$	0.6
$\Sigma^*\pi$	$1.1 + i0.4$	1.1	$2.2 - i2.0$	3.0	$0.3 + i0.8$	0.8
$\Sigma^*\eta$	$1.8 - i0.3$	1.9	$1.9 + i0.6$	1.9	$1.0 - i0.7$	1.2
Ξ^*K	$0.3 + i0.5$	0.6	$2.7 - i1.4$	3.0	$2.5 + i1.0$	2.7

Results: $S = -2, I = 1/2\ \ (\Xi)$

- States: $\Sigma^* \bar{K}$, $\Xi^* \pi$, $\Xi^* \eta$ and ΩK.
- We find four poles in the complex energy plane



- The pole at $1877 - i\ \ 15$ MeV is associated with the $\Xi(1820)$ which has $\Gamma = 24^{+15}_{-10}$ MeV
- width (on real axis) appears reduced due to Flatt$\acute{e}$ effect
- Poles at $1832 - i\ 182$ and $1920 - i\ 137$ MeV are too broad to show up

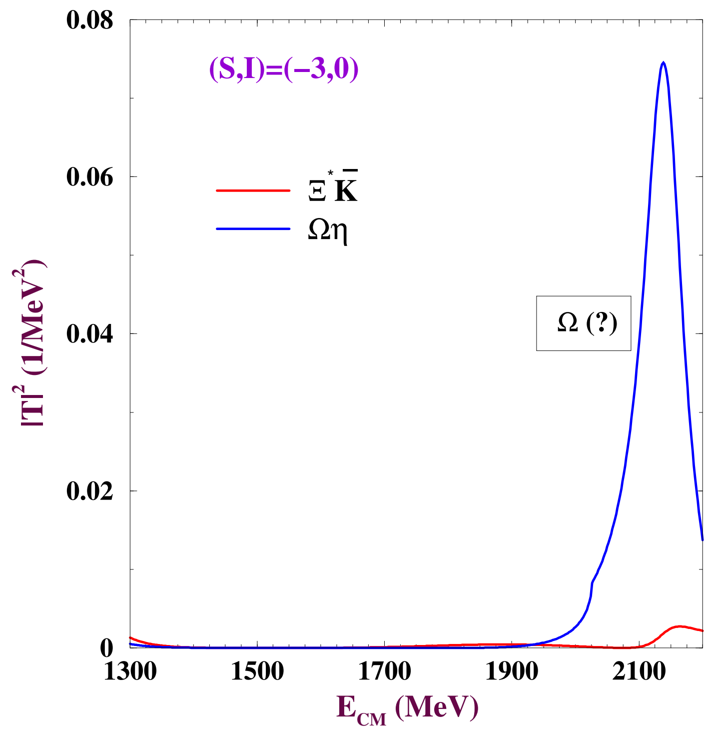
- Pole at $2162 - i\ 19$ MeV couples strongly to ΩK $\rightarrow$ quasibound state !

Couplings of Ξ to various channels

z_R	$1863 - i14 (x = 0.9)$		$1832 - i182$		$1920 - i137$		$2162 - i19$	
	g_i	$\|g_i\|$	g_i	$\|g_i\|$	g_i	$\|g_i\|$	g_i	$\|g_i\|$
$\Sigma^* \overline{K}$	$1.9 + i0.7$	2.0	$1.8 - i1.1$	2.1	$1.1 + i0.1$	1.1	$0.3 - i0.4$	0.5
$\Xi^* \pi$	$0.5 + i0.9$	1.1	$2.3 - i1.8$	2.9	$1.1 - i1.7$	2.0	$0.2 + i0.7$	0.7
$\Xi^* \eta$	$2.5 + i0.2$	2.6	$1.4 + i1.3$	1.9	$3.5 + i1.7$	3.8	$0.4 - i0.3$	0.5
ΩK	$0.1 - i0.7$	0.7	$2.3 - i0.9$	2.4	$1.6 - i0.4$	1.7	$2.1 + i0.9$	2.3

Results: $S = -3$, $I = 0$ (Ω)

- States: $\Xi^* \bar{K}$ and $\Omega\eta$.



- We find a pole at $2141 - i\,38$ MeV
- Could be associated to the $\Omega(2250)$(***), $\Gamma = 55 \pm 18$ MeV in PDG

Couplings of Ω to various channels

z_R	$2141 - i38$	
	g_i	$\lvert g_i \rvert$
$\Xi^* \overline{K}$	$1.1 - i0.8$	1.4
$\Omega\eta$	$3.3 + i0.4$	3.4

Formulation

Example: $S=-1$, $Q=0$

	$\Delta^0\overline{K}^0$	Δ^+K^-	$\Sigma^{*-}\pi^+$	$\Sigma^{*0}\pi^0$	$\Sigma^{*0}\eta$	$\Sigma^{*+}\pi^-$	$\Xi^{*-}K^+$	$\Xi^{*0}K^0$
$\Delta^0\overline{K}^0$	2	2	-1	1	$-\sqrt{3}$	0	0	0
Δ^+K^-		2	0	-1	$-\sqrt{3}$	-1	0	0
$\Sigma^{*-}\pi^+$			2	2	0	0	2	0
$\Sigma^{*0}\pi^0$				0	0	-2	1	-1
$\Sigma^{*0}\eta$					0	0	$\sqrt{3}$	$\sqrt{3}$
$\Sigma^{*+}\pi^-$						2	0	2
$\Xi^{*-}K^+$							2	-1
$\Xi^{*0}K^0$								2

$$|\Sigma^*\pi\ I=0\rangle = \sqrt{\tfrac{1}{3}}\,|\Sigma^{*+}\pi^-\rangle - \sqrt{\tfrac{1}{3}}\,|\Sigma^{*0}\pi^0\rangle - \sqrt{\tfrac{1}{3}}\,|\Sigma^{*-}\pi^+\rangle;$$

$$|\Xi^*K\ I=0\rangle = \sqrt{\tfrac{1}{2}}\,|\Xi^{*0}K^0\rangle - \sqrt{\tfrac{1}{2}}\,|\Xi^{*-}K^+\rangle$$

to finally get for $I=0$

	$\Sigma^*\pi$	Ξ^*K
$\Sigma^*\pi$	4	$\sqrt{6}$
Ξ^*K		3

N/D Method

Unitarity states that, above threshold,

$$[Imt^{-1}(s)]_{ij} = -\frac{q_i \; M_i}{4\pi \; \sqrt{s}}\delta_{ij} = ImG(s)$$

Using a subtracted dispersion relation

$$t^{-1}(s) = -G(s) + V^{-1}(s)$$

where $G(S)$ contains an arbitrary subtraction constant and V^{-1} accounts for contact terms which remain at tree level when $G = 0$.

The above equation can be cast as

$$t = [1 - VG]^{-1} \; = \; V + VGt$$

$\Lambda(1520)$

The actual amplitudes are given by

$$
\begin{aligned}
t_{\pi\Sigma^*\to\pi\Sigma^*} &= T_{\pi\Sigma^*\to\pi\Sigma^*} \\
t_{K\Xi^*\to K\Xi^*} &= T_{K\Xi^*\to K\Xi^*} \\
t_{\bar{K}N\to\pi\Sigma^*} &= T_{\bar{K}N\to\pi\Sigma^*}\,\mathcal{C}(\tfrac{1}{2}\;2\;\tfrac{3}{2};m,M-m)Y_{2,m-M}(\hat{k})(-1)^{M-m}\sqrt{4\pi} \\
t_{\pi\Sigma\to\pi\Sigma^*} &= T_{\pi\Sigma\to\pi\Sigma^*}\,\mathcal{C}(\tfrac{1}{2}\;2\;\tfrac{3}{2};m,M-m)Y_{2,m-M}(\hat{k})(-1)^{M-m}\sqrt{4\pi} \\
t_{\bar{K}N\to\bar{K}N} &= T_{\bar{K}N\to\bar{K}N}\sum_{M}\mathcal{C}(\tfrac{1}{2}\;2\;\tfrac{3}{2};m,M-m)Y_{2,m-M}(\hat{k})\;\cdot \\
&\cdot\quad \mathcal{C}(\tfrac{1}{2}\;2\;\tfrac{3}{2};m',M-m')Y^{*}_{2,m'-M}(\hat{k}')(-1)^{m'-m}\,4\pi\;.
\end{aligned}
$$