Om

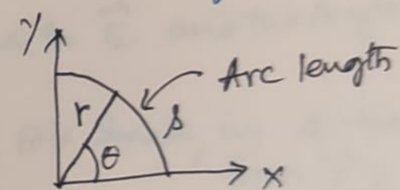
Uniform Circular motion: Particle moving at constant speed around a circle of radius 'r'. $\vec{v}$ is always tangential to the circle. Speed is constant. So length of $\vec{v}$ is always same length.

Time taken to go around circle once (one revolution) is called period of motion. Period is represented by 'T'.

$$V = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{T} \quad ; \quad \text{frequency } f = \frac{1}{\text{Period}} = \frac{\#\text{ Cycles}}{\text{time}}$$

Angular position: The Angle 'θ' is the angular position of the particle

$$\theta\ (\text{radians}) = s/r = \frac{\text{Arc length}}{\text{radius}}$$



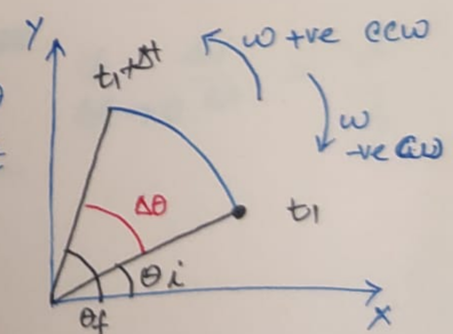
$$\theta_{\text{full circle}} = \frac{2\pi r}{r} = 2\pi \ ;$$

$$1\text{rad} = 1\text{rad} \times \frac{360°}{2\pi} = 57.3° \quad ; \quad \text{Arclength } (s) = r\theta$$

Angular velocity:

$$\text{Average angular velocity} \equiv \frac{\Delta\theta}{\Delta t}$$

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \quad ; \quad \theta_f = \theta_i + \omega t$$



$$|\omega| = \frac{2\pi \text{ rad}}{T} \quad \text{or} \quad T = \frac{2\pi}{|\omega|} \text{ rad}$$

ccw: Counter clockwise
cw: clockwise.

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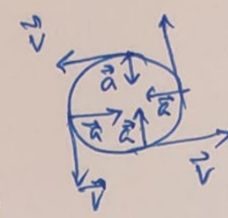
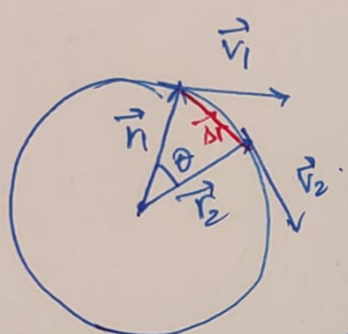
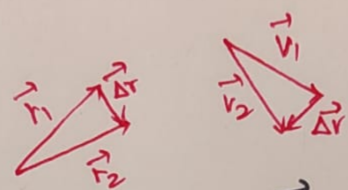
velocity and acceleration in uniform circular motion



$$\theta = \frac{s}{r} \Rightarrow \theta = \frac{1}{r} \cdot s$$

$$\frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt} = \frac{1}{r} v \Rightarrow v_t = r\frac{d\theta}{dt} = r\omega$$





$$\vec{\Delta r} = \vec{r_2} - \vec{r_1} \qquad \vec{\Delta v} = \vec{v_2} - \vec{v_1}$$

$\vec{v_1}$ is perpendicular to $\vec{r_1}$; $\vec{v_2}$ is perpendicular to $\vec{r_2}$ and the angle 'θ' are similar $\quad \frac{\Delta v}{v} = \frac{\Delta r}{r}$; suppose θ is small, in Δt time from $\vec{r_1}$ to $\vec{r_2}$

$$\frac{v \cdot \Delta t}{r} \simeq \frac{\Delta v}{v} \Rightarrow \frac{v^2}{r} \simeq \frac{\Delta v}{\Delta t} = \bar{a}_{ave} \qquad \Delta r \simeq v\Delta t$$

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

$$\boxed{a = \frac{v^2}{r}}$$

Uniform circular motion