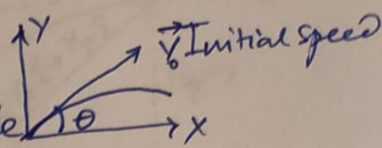
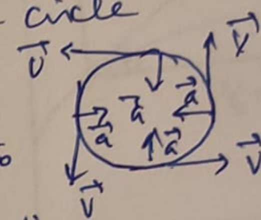
Knight Text Book Ch: 4

Projectile motion: 2D motion under the influence of gravity only. Projectile motion follows a parabolic trajectory characterized by Initial speed V_0 and launch Angle

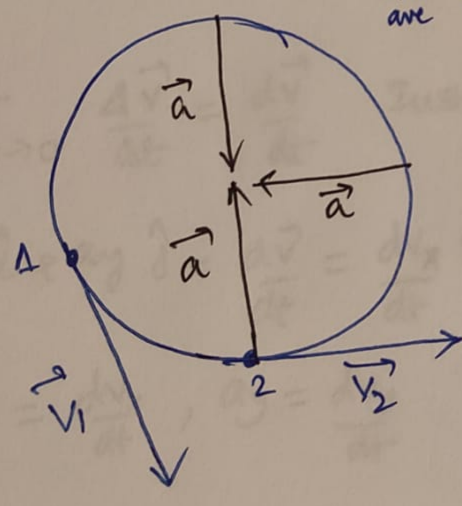


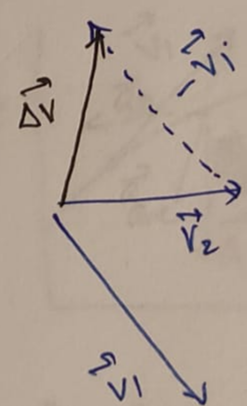
Circular motion: uniform circular motion with constant speed. Direction of velocity changes and acceleration points toward the center of the circle



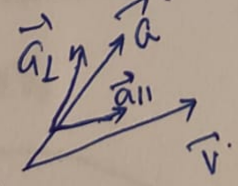
Angular position 'θ', Angular velocity 'ω' and Angular acceleration 'α' are Analogous to linear position x, velocity $\vec{V}$, and acceleration.

$$\vec{a}_{ave} = \frac{\Delta \vec{V}}{\Delta t}$$





$\vec{a}_{\parallel}$ is the acceleration component parallel to velocity causes the object to change speed

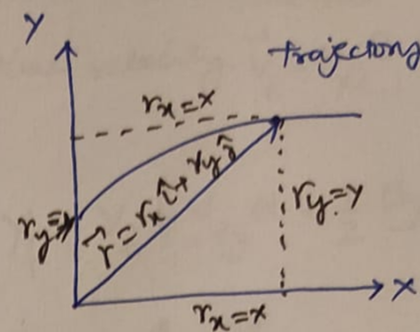


$a_{\perp}$ is ~~perpendicular~~ acceleration component ~~of a~~ perpendicular to velocity causes object to change direction.

$\vec{r} = r_x \hat{i} + r_y \hat{j}$

$\vec{r} = x\hat{i} + y\hat{j}$

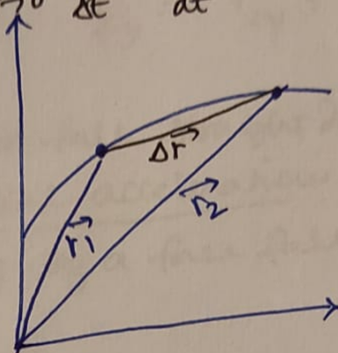
$$\vec{v}_{ave} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$$

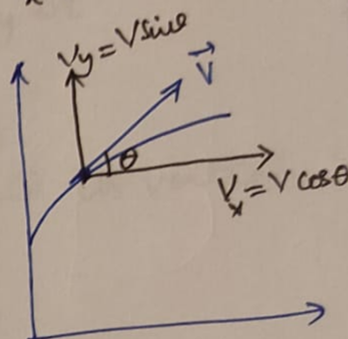


Instantaneous velocity $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$





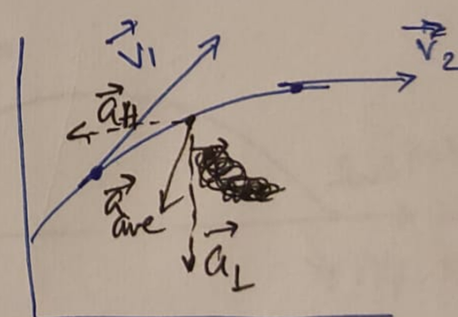
$V_x = v\cos\theta$; $V_y = V\sin\theta$

$|\vec{V}| = \sqrt{V_x^2 + V_y^2}$; $\tan\theta = \frac{V_y}{V_x}$

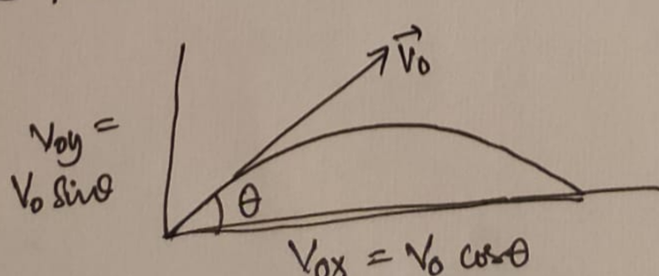
$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t} = \frac{d\vec{V}}{dt}$$ Instantaneous acceleration

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}$$



- Projectile motion is an extension of free fall motion.



$V_{ox} = V_0\cos\theta$

$V_{oy} = V_0\sin\theta$.

Particle is moving with a constant acceleration from initial Position $\vec{r}_i = x_i \hat{i} + y_i \hat{j}$ with initial velocity $\vec{V}_i = V_{xi}\hat{i} + V_{yi}\hat{j}$

final Position and velocities are.

$$x_f = x_i + V_{ix}\Delta t + \frac{1}{2} a_x (\Delta t)^2 \; ; \; Y_f = Y_i + V_{iy}\Delta t + \frac{1}{2} a_y (\Delta t)^2$$

$$V_{fx} = V_{ix} + a_x \Delta t \; ; \; V_{fy} = V_{iy} + a_y \Delta t.$$

object released from rest fall straight down

* Projectile has no Horizontal acceleration and its vertical acceleration is simply of a free fall.

$$a_x = 0 \quad a_y = -g$$

(assume started at 0)

- $x_f = x_0 + V_{0x}\Delta t$ (since $a_x = 0$)

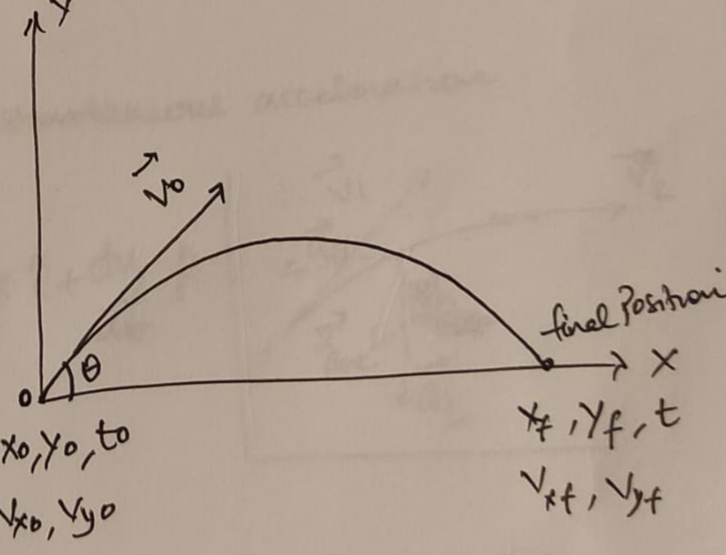
$x_f = V_{0x}\Delta t = V_0 \cos\theta \, \Delta t$

$Y_f = Y_0 + V_{0y}\Delta t - \frac{1}{2} g (\Delta t)^2$ ($Y_0 = 0$)

$= V_0 \sin\theta \, \Delta t - \frac{1}{2} g (\Delta t)^2$

final $Y_f = 0 \Rightarrow V_0 \sin\theta = \frac{1}{2} g \Delta t$

$\Delta t = 0$ or $\boxed{\frac{2V_0 \sin\theta}{g} = \Delta t}$



$$\boxed{x_f = V_0 \cos\theta \cdot \frac{2V_0 \sin\theta}{g} = \frac{2V_0^2 \sin\theta \cos\theta}{g} = \frac{V_0^2 \sin(2\theta)}{g}.}$$