College Physics B - PHY2054C

Electric Flux and Potential

08/29/2022

My Office Hours:
Wednesday 1:00 - 3:00 PM

212 Keen Building

Reading Assignment & Recitation Problems

Read Sections 19.1 - 19.4

Problems for the recitations:

2.4, 2.6, 2.7 & 2.8

Further recommended problems:

19.16, 19.17, 19.24, 19.34

Announcement: ExpertTA problems on electric flux moved to this week's assignment.

Review: Coulomb's Law

Charles-Augustin de Coulomb
(14 June 1736 - 23 August 1806)

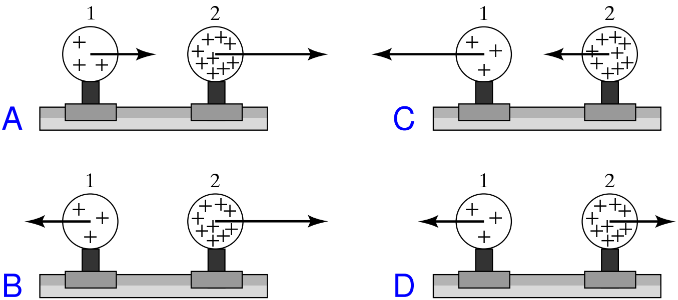
$$F = k\frac{q_1 q_2}{r^2}$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

ϵ_0 is called permittivity of free space

Review Question 1

Two uniformly charged spheres are firmly fastened to and electrically insulated from frictionless pucks on an air table. The charge on sphere 2 is three times the charge on sphere 1. Which force diagram correctly shows the magnitude and direction of the electrostatic forces?



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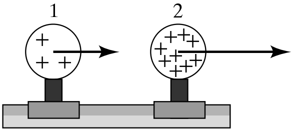

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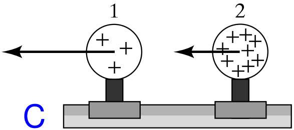

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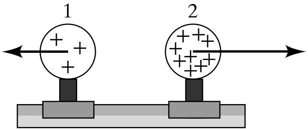
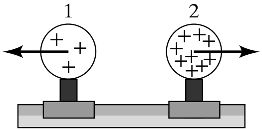


D



Review Question 2

A plastic comb can gain a net charge by combing hair. A charged comb attracts bits of paper because

A paper always has a net charge similar to that of a comb.

B paper always has a net charge opposite to that of a comb.

C the charge on the comb polarizes the charges in paper.

D paper is an insulator and attracts opposite charges from the surrounding air.

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Question 3

Have you looked over Chapter 19 in the textbook?

A Yes

B No

C I do not remember.

D Why would I do that?

Outline

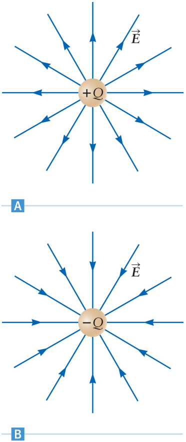
Review: Electric Field

An electric field gives another explanation of electric forces.

The presence of a charge produces an electric field:

- A positive charge produces field lines that radiate outward.
- For a negative charge, the field lines are directed inward, toward the charge.
- The electric field is a vector and denoted by $\vec{E}$.
- The density of the field lines is proportional to the magnitude of E.

  ➜ The field lines are most dense in the vicinity of charges.

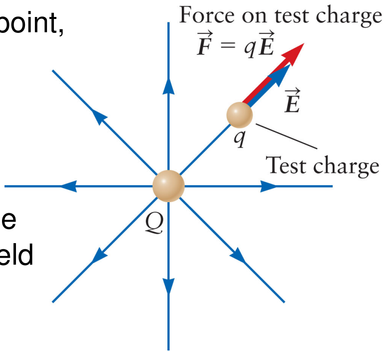


Review: Electric Field

The Electric Field is defined as the force exerted on a tiny positive test charge at that point divided by the magnitude of the test charge.

Consider a point in space where the electric field is E.

- If a charge q is placed at the point, the force is given by $F = qE$.
- The charge q is called a test charge.
- By measuring the force on the test charge, the magnitude and direction of the electric field can be inferred.



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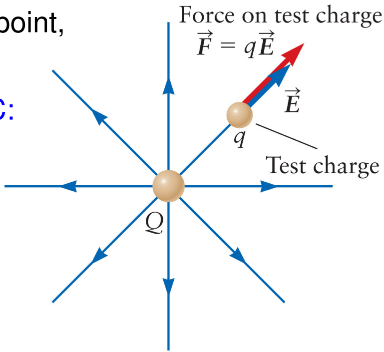
Consider a point in space where the electric field is E.

- If a charge q is placed at the point, the force is given by $F = q\,E$.
- Unit of the electric field is N/C:

$$F = k\,\frac{Q\,q}{r^2} = q\,E$$

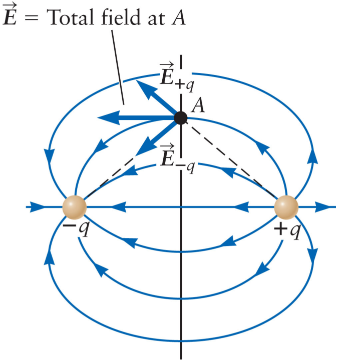
$$E = k\,\frac{Q}{r^2}$$



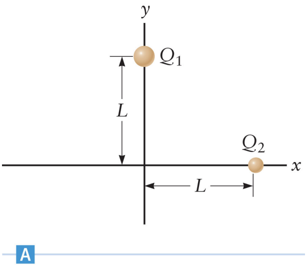
Rules for Drawing Field Lines

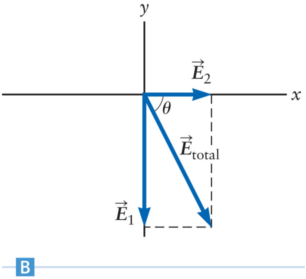
The lines must begin on a positive charge and terminate on a negative charge (or at infinity).

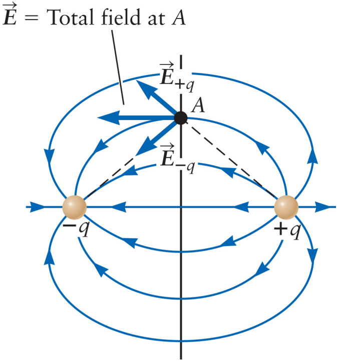
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No lines can cross.



Electric Fields are Vectors







Outline

Electric Flux

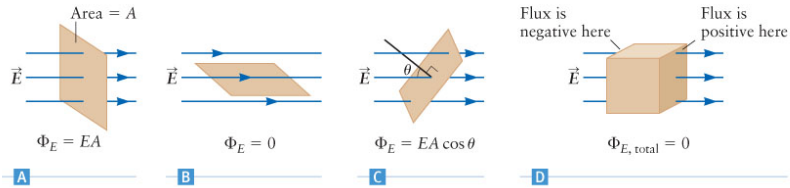
Gauss' Law can be used to find electric field of a complex charge distribution.

- Easier than treating it as a collection of point charges and using superposition.

To use Gauss' Law, a new quantity called the Electric Flux is needed.

- The electric flux is equal to the number of electric field lines that pass through a particular surface multiplied by the area of the surface.
- The electric flux is denoted by $\Phi_E = E A$.

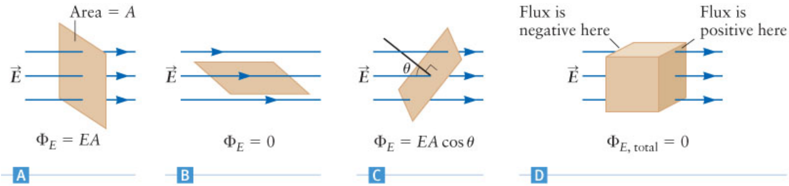
Electric Flux: Examples



A The electric field is perpendicular to the surface of A: $\rightarrow \Phi_E = E\,A$

B The electric field is parallel to the surface of A: $\rightarrow \Phi_E = E\,A = 0$

Electric Flux: Examples



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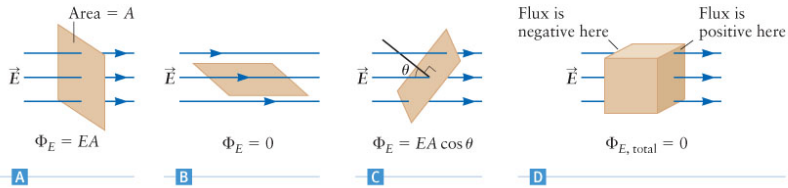
→ $\Phi_E = E\,A$

B The electric field is parallel to the surface of A:
→ $\Phi_E = E\,A = 0$

C The electric field forms an angle with the surface A:
→ $\Phi_E = E\,A\cos\Theta$

Electric Flux: Examples



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C The electric field forms an angle with the surface A:
➜ $\Phi_E = E\,A\cos\Theta$

D The flux is positive if the field is directed out of the region surrounded by the surface and negative if going into the region: $\Phi_E = 0$

Gauss' Law

Carl Friedrich Gauss
(30 April 1777 - 23 February 1855)

Gauss' Law says that the electric flux through any closed surface is proportional to the charge Q inside the surface:

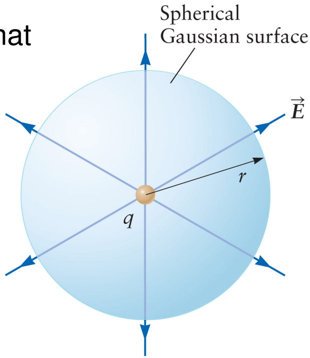
$$\Phi_E = \frac{Q}{\epsilon_0}$$

ϵ_0 is called permittivity of free space

Field of a Point Charge

Choose a Gaussian surface

- Choose a surface that will make the calculation easy.
- Surface should match the symmetry of the problem.
  ➜ For a point charge, the field lines have a spherical symmetry.
- The spherical symmetry means that the magnitude of the electric field depends only on the distance from the charge.
- The magnitude of the field is the same at all points on the surface (due to the symmetry).
- The field is perpendicular to the sphere at all points.



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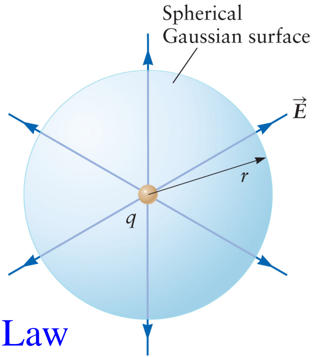


$$\begin{aligned}\Phi_E &= E\,A_{\text{sphere}} \\ &= E\,4\pi r^2 \\ &= q/\epsilon_0\end{aligned}$$

$$E = \frac{q}{4\pi\epsilon_0\,r^2} \quad \rightsquigarrow \quad \text{Coulomb's Law}$$

Question 4

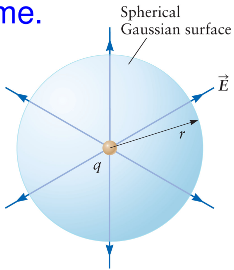
A spherical surface surrounds a point charge it its center. If the charge is doubled and if the radius of the surface is also doubled, what happens to the electric flux Φ_E out of the surface and the magnitude E of the electric field at the surface as a result of these doublings?

A Φ_E and E do not change.

B Φ_E increases and E remains the same.

C Φ_E increases and E decreases.

D Φ_E increases and E increases.



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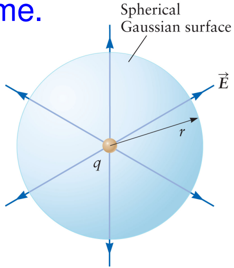
$$\Phi_E = \frac{Q}{\epsilon_0}$$

$$\Phi_E = E\,A_{\text{sphere}} = E\,4\pi r^2$$



Problem Solving Strategy

1. **Recognize the Principle**
   - Calculate the electric flux through a Gaussian surface.
   - The choice of the Gaussian surface is key!
2. **Sketch the Problem**
   - Draw the charge distribution.
   - Use symmetry of distribution to sketch the electric field.
   - The Gaussian surface should match the symmetry of the electric field.
3. **Identify the Relationships**
   - Your Gaussian surface should satisfy one or more of the following conditions:
     - The field has constant magnitude over all or much of the surface and makes a constant angle with the surface.
     - The field may be zero over a portion of the surface.
       ➜ The flux through that part of the surface is zero.
     - The field may be parallel to some part of the surface.
       ➜ The flux through that part of the surface is zero.

Problem Solving Strategy

Solve

- Calculate the total electric flux through the entire Gaussian surface.
- Find the total electric charge inside the surface.
- Apply Gauss' Law to solve for the electric field.

Check

- Consider what the answer means.
- Check if the answer makes sense.

Outline

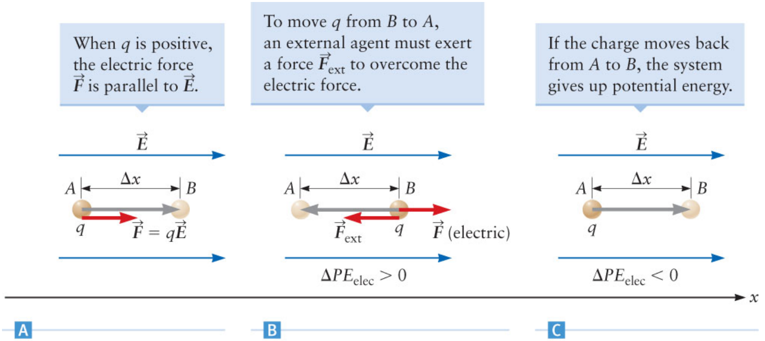
Electric Potential Energy

A point charge in an electric field experiences a force:

$$\vec{F} = q\,\vec{E}$$

Assume the charge moves a distance Δx: $W = F\,\Delta x$.

Electric force is conservative: Work done independent of path.

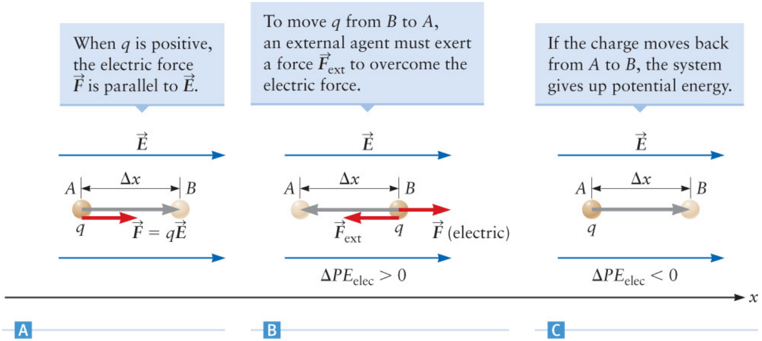


Electric Potential Energy

The change in electric potential energy is

$$\Delta PE_{\text{elec}} = -W = -F\,\Delta x = -q\,E\,\Delta x$$

The change in potential energy depends on the endpoints of the motion, but not on the path taken.



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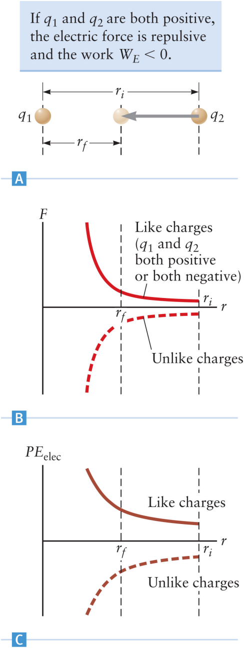
A positive amount of energy can be stored in a system that is composed of the charge and the electric field.

Stored energy can be taken out of the system:

- This energy may show up as an increase in the kinetic energy of the particle.



Two Point Charges

From Coulomb's Law:

$$F = \frac{k\, q_1 q_2}{r^2}$$

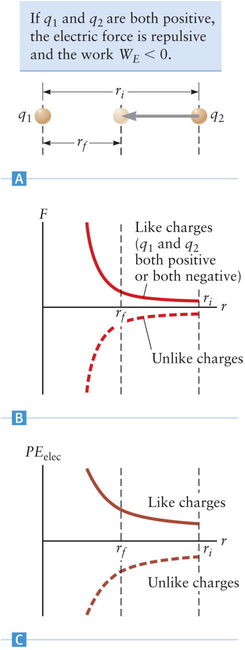
The electric potential energy is given by:

$$PE_{\text{elec}} = \frac{k\, q_1 q_2}{r} = \frac{q_1 q_2}{4\pi\epsilon_0\, r}$$

Note that PE_{elec} varies as $1/r$ while the force varies as $1/r^2$.

- PE_{elec} approaches zero when the two charges are very far apart.
- The electric force also approaches zero in this limit.

Two Point Charges



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The electric potential energy is given by:

$$PE_{\text{elec}} = \frac{k\, q_1 q_2}{r} = \frac{q_1 q_2}{4\pi\epsilon_0\, r}$$

The changes in the potential energy are important:

$$\Delta PE_{\text{elec}} = PE_{\text{elec, f}} - PE_{\text{elec, i}}$$

$$= \frac{k\, q_1 q_2}{r_f} - \frac{k\, q_1 q_2}{r_i}$$

Electric Potential: Voltage

Electric potential energy is a property of a system of charges or of a charge in an electric field, it is not a property of a single charge alone.

Electric potential energy can be treated in terms of a test charge, similar to the treatment of the electric field produced by a charge:

$$V = \frac{PE_{\text{elec}}}{q}$$

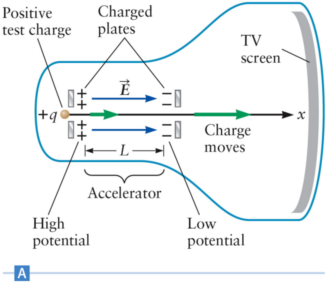
- Units are the Volt or [V]: 1 V = 1 J/C.
  (Named in honor of Alessandro Volta)
- The unit of the electric field can also be given in terms of the Volt: 1 N/m = 1 V/m.

Electric Potential: Voltage

The diagram shows the electric potential in a television picture tube: Two parallel plates are charged and form an "accelerator":

1. The electric force on the test charge is $F = q\,E$ and the charge moves a distance L.
2. The work done on the test charge by the electric field is $W = q\,E\,L$.
3. There is a potential difference between the plates of:

$$\Delta V = \frac{\Delta PE_{\text{elec}}}{q} = -E\,L$$



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3. There is a potential difference between the plates of:

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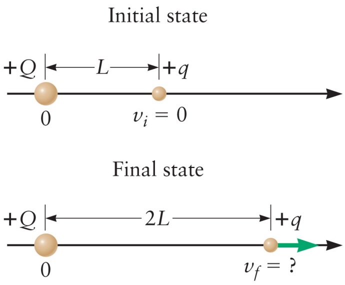
4. Conservation of Energy:

$$KE_i + PE_{\mathrm{elec},\,i} = KE_f + PE_{\mathrm{elec},\,f}$$



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3. There is a potential difference between the plates of:

$$\Delta V = \frac{\Delta PE_{\text{elec}}}{q} = -E\,L$$

4. Conservation of Energy: ($v_i = 0$ for an electron)

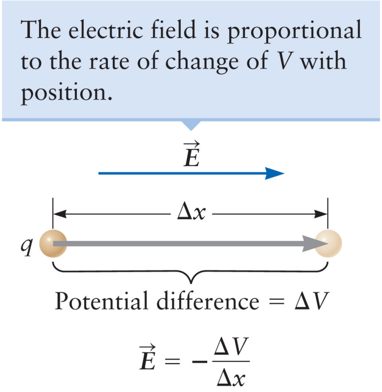
$$KE_f = -\Delta PE_{\text{elec}} = -q\,\Delta V = e\,\Delta V = 1/2\,m\,v_f^2$$

$$v_f = \sqrt{\frac{2e\,\Delta V}{m}}$$

Electric Field and Potential

The electric field may vary with position.

The magnitude and direction of the electric field are related to how the electric potential changes with position:



$$\Delta V = -E\,\Delta x$$

$$E = -\Delta V/\Delta x$$

It is convenient to define a unit of energy called the *electron-volt (eV)*. One electron volt is defined as the amount of energy gained or lost when an electron travels through a potential difference of 1 V:

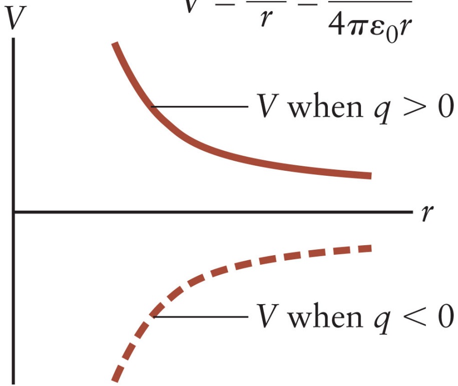
$$1\ \mathrm{eV} = 1.60 \times 10^{-19}\ \mathrm{J}$$

Outline

Point Charge

The electric potential at a distance r away from a single point charge q is given by:

$$V = \frac{kq}{r} = \frac{q}{4\pi\varepsilon_0 r}$$



Only changes in potential (and potential energy) are important.

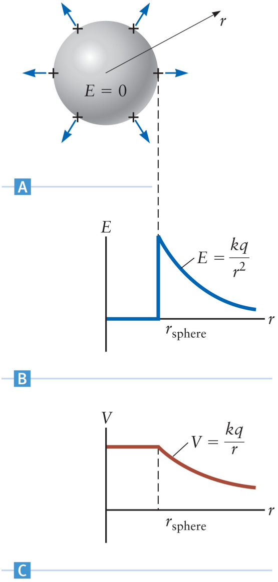
Reference point (usually):

$$V = 0 \text{ at } r = \infty$$

Sometimes:

Earth may be $V = 0$.

Electric Field Near a Metal



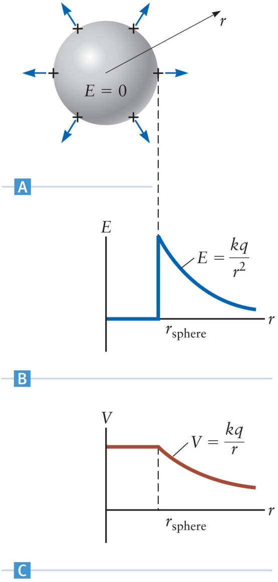
The Fig. A shows a solid metal sphere carrying an excess positive charge, q:

- The excess charge resides on the surface.
- The field inside the metal is zero.
- The field outside any spherical ball of charge is given by:

$$E = \frac{kq}{r^2} = \frac{q}{4\pi\epsilon_0 r^2},$$

where r is the distance from the center of the ball.

Electric Field Near a Metal



The Fig. A shows a solid metal sphere carrying an excess positive charge, q:

- The excess charge resides on the surface.
- The field inside the metal is zero.
- Since the electric field outside the sphere is the same as that of a point charge, the potential is also the same:

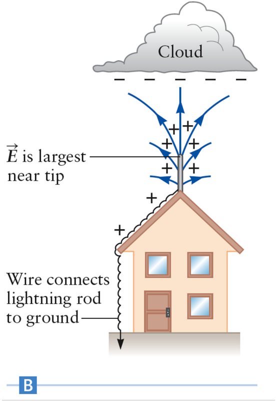
$$V = \frac{kq}{r} \quad \text{for } r > r_{\text{sphere}}$$

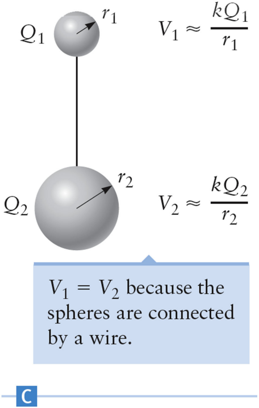
- The potential is constant inside the metal: $V = k\,q/r_{\text{sphere}}$.

Lightning Rod

Field lines are perpendicular to the surface of the metal rod.

The field lines are largest near the sharp tip of the rod and smaller near the flat side.





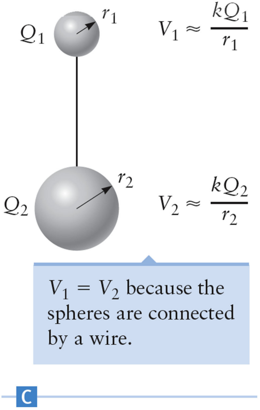
Lightning Rod

A lightning rod can be modeled as two metal spheres, which are connected by a metal wire.

The smaller sphere represents the tip and the larger sphere represents the flatter body:

$$V_1 = \frac{k\,Q_1}{r_1} = V_2 = \frac{k\,Q_2}{r_2}$$

$$\frac{Q_1}{r_1} = \frac{Q_2}{r_2}$$



Because the spheres are connected by a wire, they are a single piece of metal. They must have the same potential.

Lightning Rod

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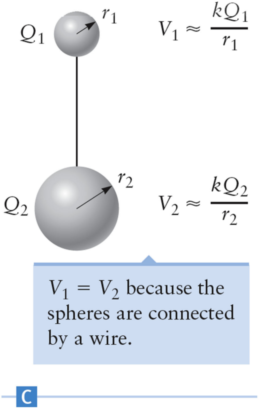
The smaller sphere represents the tip and the larger sphere represents the flatter body:

$$V_1 = \frac{k\,Q_1}{r_1} = V_2 = \frac{k\,Q_2}{r_2}$$

$$\frac{Q_1}{r_1} = \frac{Q_2}{r_2}$$

$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$



The field is large near the surface of the smaller sphere. This means the field is largest near the sharp edges of the lightning rod.

Outline

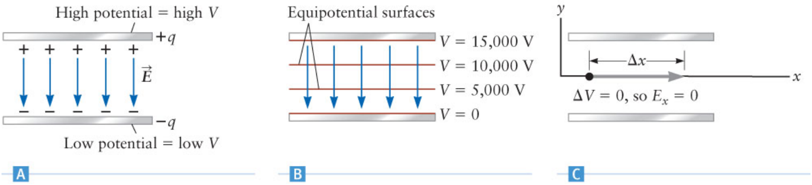
Equipotential Surfaces

A useful way to visualize electric fields is through plots of *equipotential surfaces:*

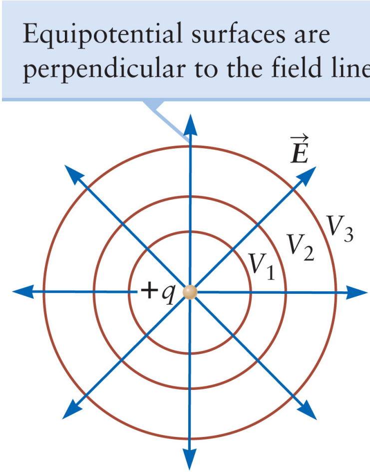
- Contours where the electric potential is constant.
- Equipotential lines are in two-dimensions.

The equipotential surfaces are always perpendicular to the direction of the electric field.

- For motion parallel to an equipotential surface, V is constant and $\Delta V = 0$.
- Electric field component parallel to the surface is zero.



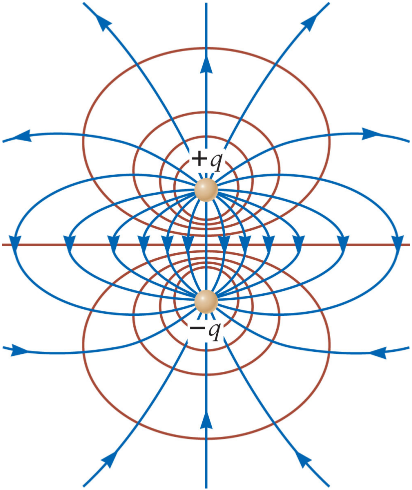
Example: Point Charge



The electric field lines emanate radially outward from the charge.

- The equipotential surfaces are perpendicular to the field.
- The equipotentials are a series of concentric spheres.
- Different spheres correspond to different values of V.

Example: Dipole



The dipole consists of charge $+q$ and $-q$.

- Field lines are plotted in blue.
- Equipotential lines are plotted in orange.