

On the definitions of the $\gamma^*N \rightarrow N^*$ helicity amplitudes

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We present definitions and formulas that may be useful for a consistent computation of helicity amplitudes for the process $\gamma^*N \rightarrow N^*$ in theoretical approaches. Of particular importance is the correct determination of the common sign of the amplitudes and of the relative sign between the transverse ($A_{1/2}, A_{3/2}$) and longitudinal ($S_{1/2}$) amplitudes. This clarification is necessary to clear up confusions present in theoretical works. Using the definitions presented in this paper will enable a direct comparison with amplitudes extracted from experimental data.

I. INTRODUCTION

The excitation of nucleon resonances in electromagnetic interactions has long been known as an important source of information on the baryon structure and long- and short-range interaction in the domain of quark confinement. Constituent quark models have been developed that relate electromagnetic resonance transition form factors to fundamental quantities, such as the quark confining potential. While the picture of the N and N^* built from 3 quarks is recognized as a basic starting point in the description of these states, the inter-quark interaction does not exclude the possibility of additional degrees of freedom, namely, the presence of $3q - q\bar{q}$ components in the N and N^* . There is also a possibility of alternative structures, such as hybrid q^3G states and resonances dynamically generated in the meson-baryon interaction.

The Q^2 dependence of the $\gamma^*N \rightarrow N^*$ transition amplitudes is highly sensitive to different descriptions of the nucleon resonances. Rich information on these amplitudes, both transverse and longitudinal, has become available in a wide Q^2 range for the $N(1440)P_{11}$, $N(1520)D_{13}$, $N(1535)S_{11}$ resonances due to the recent high precision JLab-CLAS measurements of the $\bar{e}p \rightarrow ep\pi^0, en\pi^+$ reactions [1, 2, 3, 4, 5, 6, 7, 8]. More results are expected in π and 2π electroproduction.

While comparing the predictions obtained in different approaches with the $\gamma^*N \rightarrow N^*$ amplitudes extracted from experimental data, we found that there are two points which introduce confusion in theoretical predictions and in most cases do not allow to compare properly the results of different approaches with each other and with experimental data:

(i) It is known that experimental results on the $\gamma^*N \rightarrow N^*$ helicity amplitudes $A_{1/2}$, $A_{3/2}$, $S_{1/2}$, extracted from the contribution of the diagram of Fig. 1 to $\gamma^*N \rightarrow N\pi$, contain the sign of the πNN^* vertex. However, by the definition accepted many years ago, this fact is not reflected explicitly in the amplitudes extracted from experiment. In many cases this is not taken into account in theoretical calculations and causes difficulties in dealing with the common sign of the predicted amplitudes.

(ii) The definition of the amplitudes $A_{1/2}$, $A_{3/2}$, $S_{1/2}$ through the hadron electromagnetic current which is

commonly used in theoretical approaches contains points which can introduce mistakes in the relative sign of longitudinal amplitude $S_{1/2}$ relative to the transverse ones $A_{1/2}$, $A_{3/2}$. This is the second source of difficulties in dealing with theoretical predictions.

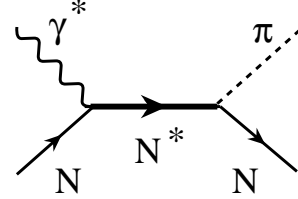


FIG. 1: The diagram corresponding to the N^* contribution to $\gamma^*N \rightarrow N\pi$.

Our goal in this paper is to present different definitions of the $\gamma^*p \rightarrow N^{*+}$ helicity amplitudes: through the $\gamma^*p \rightarrow N\pi$ multipole amplitudes, through the hadron electromagnetic current, in terms of the $\gamma^*p \rightarrow N^{*+}$ form factors, and in quark model, which are consistent with each other. To exclude possible sources of mistakes, we give explicitly the definitions of all quantities which enter the formulas.

We pay special attention to the way, how the sign of the πNN^* vertex should be taken into account. It is known that this sign comes from the relative contributions of the diagrams which present the resonance (Fig. 1) and Born terms (Fig. 2) contributions to $\gamma^*N \rightarrow N\pi$. We will give formulas which relate the $\gamma^*p \rightarrow N^{*+}$ helicity amplitudes extracted from experiment to those which are calculated through the hadron electromagnetic current and are multiplied by the sign of the ratio of the coupling constants in the vertices πNN and πNN^* .

In Sec. II, we present the definitions of the $\gamma^*p \rightarrow N^{*+}$ helicity amplitudes through the $\gamma^*p \rightarrow N\pi$ multipole amplitudes, which are used in the extraction of $A_{1/2}$, $A_{3/2}$, $S_{1/2}$ from the $\gamma^*p \rightarrow N\pi$ data. These definitions are well known and coincide, for example, with the definitions presented in Refs. [9, 10, 11, 12].

In Sec. III, we present the definition of the $\gamma^*p \rightarrow N^{*+}$ helicity amplitudes through the hadron electromagnetic current. To distinguish between these amplitudes and those extracted from experiment, we denote the amplitudes defined through the hadron electromagnetic cur-

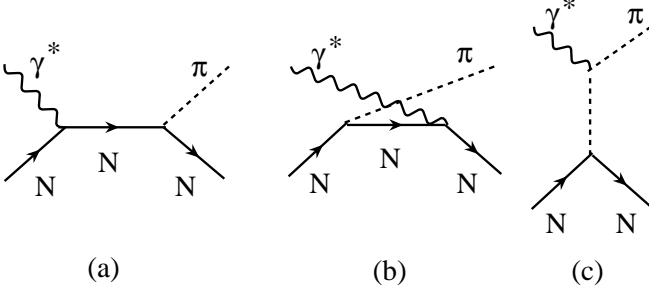


FIG. 2: The diagrams corresponding to the Born terms contributions to $\gamma^* N \rightarrow N\pi$.

rent by $\mathcal{A}_{1/2}$, $\mathcal{A}_{3/2}$, $\mathcal{S}_{1/2}$:

$$A_{\frac{1}{2}, \frac{3}{2}} = \zeta \mathcal{A}_{\frac{1}{2}, \frac{3}{2}}, \quad S_{\frac{1}{2}} = \zeta \mathcal{S}_{\frac{1}{2}}. \quad (1)$$

Here ζ is the sign which reflects the presence of the πNN^* vertex in Fig. 1.

In Sec. IV, using the definitions of Sec. III, we present the expressions for the amplitudes $\mathcal{A}_{1/2}$, $\mathcal{A}_{3/2}$, $\mathcal{S}_{1/2}$ in terms of the $\gamma^* N \rightarrow N^*$ form factors.

In Sec. V, the relation between ζ and the sign of the ratio of the πNN , πNN^* coupling constants is found from the covariant calculations of the Figs. 1,2 contributions to $\gamma^* N \rightarrow N\pi$.

In Sec. VI, we present formulas for the calculation of the amplitudes $\mathcal{A}_{1/2}$, $\mathcal{A}_{3/2}$, $\mathcal{S}_{1/2}$ in nonrelativistic quark model. To avoid possible sources of mistakes, we give explicitly the definitions of all quantities which enter these formulas and present in explicit form definitions of wave functions for the N , $P_{11}(1440)$, and $S_{11}(1535)$. Further, in Sec. VI, we present formulas for the amplitudes $\mathcal{A}_{1/2}$, $\mathcal{S}_{1/2}$ for the transitions $\gamma^* p \rightarrow P_{11}(1440)$, $S_{11}(1535)$ in nonrelativistic quark model.

In Sec. VII, we present information on the signs of the πNN^* coupling constants available in quark model, and demonstrate the importance of using these signs for the correct presentation of quark model predictions. In other approaches, such as dynamically generated resonances, these signs should be found within these approaches. Only in this case, we can make proper comparison of the predictions obtained in different approaches with each other and with the results extracted from experiment.

II. DEFINITION OF THE $\gamma^* p \rightarrow N^{*+}$ HELICITY AMPLITUDES THROUGH THE $\gamma^* p \rightarrow N\pi$ MULTIPOLE AMPLITUDES

The $\gamma^* N \rightarrow N^*$ helicity amplitudes, extracted from experimental data on the reaction $eN \rightarrow eN\pi$, are usually presented through the $\gamma^* N \rightarrow N\pi$ multipole amplitudes. In the case of the $\gamma^* p \rightarrow N^{*+}$ amplitudes, the corresponding formulas are following.

For $l+$ multipole amplitudes:

$$A_{1/2} = -\frac{1}{2} [(l+2)\mathcal{E}_{l+} + l\mathcal{M}_{l+}], \quad (2)$$

$$A_{3/2} = \frac{[l(l+2)]^{1/2}}{2} (\mathcal{E}_{l+} - \mathcal{M}_{l+}), \quad (3)$$

$$S_{1/2} = -\frac{1}{\sqrt{2}} (l+1) \mathcal{S}_{l+}. \quad (4)$$

For $(l+1)-$ multipole amplitudes:

$$A_{1/2} = \frac{1}{2} [(l+2)\mathcal{M}_{(l+1)-} - l\mathcal{E}_{(l+1)-}], \quad (5)$$

$$A_{3/2} = -\frac{[l(l+2)]^{1/2}}{2} (\mathcal{E}_{(l+1)-} + \mathcal{M}_{(l+1)-}), \quad (6)$$

$$S_{1/2} = -\frac{1}{\sqrt{2}} (l+1) \mathcal{S}_{(l+1)-}. \quad (7)$$

Here $\mathcal{M}_{l\pm}(\mathcal{E}_{l\pm}, \mathcal{S}_{l\pm})$ are the amplitudes which are related to the resonance contributions to the multipole amplitudes at the resonance positions via:

$$ImM_{l\pm}(E_{l\pm}, S_{l\pm})(W=M) \equiv a\mathcal{M}_{l\pm}(\mathcal{E}_{l\pm}, \mathcal{S}_{l\pm}), \quad (8)$$

$$a \equiv C_I \left[\frac{1}{(2J+1)\pi} \frac{K}{q_r} \frac{m}{M} \frac{\beta_{\pi N}}{\Gamma} \right]^{1/2}, \quad (9)$$

$$C_{1/2} = -\sqrt{\frac{1}{3}}, \quad C_{3/2} = \sqrt{\frac{2}{3}} \text{ for } \gamma^* p \rightarrow \pi^0 p, \quad (10)$$

$$C_{1/2} = -\sqrt{\frac{2}{3}}, \quad C_{3/2} = -\sqrt{\frac{1}{3}} \text{ for } \gamma^* p \rightarrow \pi^+ n. \quad (11)$$

In Eq. (9), Γ , M , J and I are the total width, mass, spin and isospin of the resonance, $\beta_{\pi N}$ is its branching ratio to the πN channel, m is the nucleon mass, $K \equiv (M^2 - m^2)/2M$ and q_r are momenta of the real photon and pion at the resonance position in the c.m.s.

With this definition, the $N^* \rightarrow N\gamma$ width is

$$\Gamma(N^* \rightarrow N\gamma) = \frac{2K^2}{\pi(2J+1)} \frac{m}{M} (|A_{1/2}|^2 + |A_{3/2}|^2). \quad (12)$$

Below we give the relations between multipole amplitudes $M_{l\pm}(W, Q^2)$, $E_{l\pm}(W, Q^2)$, $S_{l\pm}(W, Q^2)$ and the $\gamma^* N \rightarrow \pi N$ differential cross section. It is convenient to present these relations in terms of the intermediate amplitudes $F_{1,2,\dots,6}(W, \cos\theta, Q^2)$:

$$F_1 = \sum \{ (lM_{l+} + E_{l+})P'_{l+1}(x) + [(l+1)M_{l-} + E_{l-}]P'_{l-1}(x) \}, \quad (13)$$

$$F_2 = \sum [(l+1)M_{l+} + lM_{l-}]P'_l(x), \quad (14)$$

$$F_3 = \sum [(E_{l+} - M_{l+})P''_{l+1}(x) + (E_{l-} + M_{l-})P''_{l-1}(x)], \quad (15)$$

$$F_4 = \sum (M_{l+} - E_{l+} - M_{l-} - E_{l-})P'_l(x), \quad (16)$$

$$F_5 = \sum [(l+1)S_{l+}P'_{l+1}(x) - lS_{l-}P'_{l-1}(x)], \quad (17)$$

$$F_6 = \sum [lS_{l-} - (l+1)S_{l+}]P'_l(x). \quad (18)$$

The amplitudes $F_{1,2,\dots,6}(W, \cos\theta, Q^2)$ are related to the helicity amplitudes and the cross section of the $\gamma^*N \rightarrow \pi N$ reaction in the following way:

$$H_1 = \frac{-1}{\sqrt{2}} \sin\theta (F_3 + F_4 \cos\theta), \quad (19)$$

$$H_2 = \frac{-1}{\sqrt{2}} \sin\theta (2F_1 - 2F_2 \cos\theta + F_4 \sin^2\theta), \quad (20)$$

$$H_3 = \frac{-1}{\sqrt{2}} F_4 \sin^2\theta, \quad (21)$$

$$H_4 = \frac{1}{\sqrt{2}} \sin\theta (2F_2 + F_3 + F_4 \cos\theta), \quad (22)$$

$$H_5 = \frac{Q}{|\mathbf{k}|} (F_5 + F_6 \cos\theta), \quad (23)$$

$$H_6 = \frac{Q}{|\mathbf{k}|} F_6 \sin\theta, \quad (24)$$

$$\begin{aligned} \frac{d\sigma}{d\Omega_\pi} = & \sigma_T + \epsilon\sigma_L + \epsilon\sigma_{TT} \cos 2\phi \\ & + \sqrt{2\epsilon(1+\epsilon)} \sigma_{LT} \cos\phi, \end{aligned} \quad (25)$$

$$\sigma_T = \frac{|\mathbf{q}|}{2K} (|H_1|^2 + |H_2|^2 + |H_3|^2 + |H_4|^2), \quad (26)$$

$$\sigma_L = \frac{|\mathbf{q}|}{K} (|H_5|^2 + |H_6|^2), \quad (27)$$

$$\sigma_{TT} = \frac{|\mathbf{q}|}{K} \text{Re}(H_3 H_2^* - H_4 H_1^*), \quad (28)$$

$$\sigma_{LT} = \frac{|\mathbf{q}|}{\sqrt{2}K} \text{Re}[(H_1 - H_4)H_5^* + (H_2 + H_3)H_6^*]. \quad (29)$$

In Eqs.(13-29), \mathbf{k} and \mathbf{q} are the momenta of the virtual photon and pion in the c.m.s. of the reaction $\gamma^*N \rightarrow \pi N$, θ and ϕ are the polar and azimuthal angles of the pion, ϵ is the polarization factor of the virtual photon, $x \equiv \cos\theta$.

III. DEFINITION OF THE $\gamma^*N \rightarrow N^*$ HELICITY AMPLITUDES THROUGH THE HADRON ELECTROMAGNETIC CURRENT

The definition of the $\gamma N \rightarrow N^*$ helicity amplitudes through the hadron electromagnetic current, which is commonly used for the calculation of these amplitudes in theoretical approaches, is:

$$\mathcal{A}_{\frac{1}{2}} = \sqrt{\frac{2\pi\alpha}{K}} \frac{1}{e} \langle S_z^* = \frac{1}{2} | \epsilon_\mu^{(+)} J^\mu | S_z = -\frac{1}{2} \rangle, \quad (30)$$

$$\mathcal{A}_{\frac{3}{2}} = \sqrt{\frac{2\pi\alpha}{K}} \frac{1}{e} \langle S_z^* = \frac{3}{2} | \epsilon_\mu^{(+)} J^\mu | S_z = \frac{1}{2} \rangle, \quad (31)$$

$$\mathcal{S}_{\frac{1}{2}} = \sqrt{\frac{2\pi\alpha}{K}} \frac{1}{e} \langle S_z^* = \frac{1}{2} | \frac{|\mathbf{k}|}{Q} \epsilon_\mu^{(0)} J^\mu | S_z = \frac{1}{2} \rangle \quad (32)$$

In order to avoid the mistakes in using these formulas, below we give explicitly the definitions of all quantities which enter Eqs. (30-32).

Let us denote the 4-momenta of the virtual photon, nucleon and resonance in the vertex $\gamma N \rightarrow N^*$ through k, p, p^* , respectively:

$$p^* = p + k. \quad (33)$$

The z-axis is directed along the photon 3-momentum (\mathbf{k}) in the N^* rest frame, $Q \equiv \sqrt{-k^2}$, and S_z, S_z^* are the projections of the nucleon and resonance spins on the z-axis.

With the definition $a_\mu \equiv (a_0, -\mathbf{a})$, we have

$$\epsilon_\mu^{(0)} = \frac{1}{Q} (|\mathbf{k}|, 0, 0, -k_0), \quad (34)$$

$$\epsilon_\mu^{(+)} = (0, -\epsilon^{(+)}) , \quad \epsilon^{(+)} = -\frac{1}{\sqrt{2}} (1, i, 0), \quad (34')$$

and

$$\epsilon_\mu^{(+)} J^\mu = \frac{\mathbf{J}_x + i\mathbf{J}_y}{\sqrt{2}}, \quad (35)$$

$$\frac{|\mathbf{k}|}{Q} \epsilon_\mu^{(0)} J^\mu = J_0, \quad (36)$$

where in the last expression we have taken into account the gauge invariance condition: $\mathbf{J}_z = J_0 \frac{Q}{|\mathbf{k}|}$.

IV. THE $\gamma^*N \rightarrow N^*$ HELICITY AMPLITUDES IN TERMS OF THE $\gamma^*N \rightarrow N^*$ FORM FACTORS

In some theoretical approaches, it is convenient to use the definition of the $\gamma^*N \rightarrow N^*$ helicity amplitudes in terms of the $\gamma^*N \rightarrow N^*$ form factors. For the $J^P = \frac{1}{2}^\pm$ resonances, we will present this definition explicitly using the form factors introduced in Ref. [13].

For the $J^P = \frac{1}{2}^+$ resonances, the definition [13] is:

$$\langle N^* | J_\mu | N \rangle \equiv e \bar{u}(p^*) \tilde{J}_\mu u(p), \quad (37)$$

$$\begin{aligned} \tilde{J}_\mu = & [k^2 \gamma_\mu - (k\gamma) k_\mu] G_1(Q^2) \\ & - [(Pk) \gamma_\mu - (k\gamma) P_\mu] G_2(Q^2), \end{aligned} \quad (38)$$

For the $J^P = \frac{1}{2}^-$ resonances:

$$\langle N^* | J_\mu | N \rangle \equiv e \bar{u}(p^*) \tilde{J}_\mu \gamma_5 u(p), \quad (39)$$

$$\begin{aligned} \tilde{J}_\mu = & [k^2 \gamma_\mu - (k\gamma) k_\mu] G_1(Q^2) \\ & + [(Pk) \gamma_\mu - (k\gamma) P_\mu] G_2(Q^2), \end{aligned} \quad (40)$$

where $P \equiv \frac{1}{2}(p^* + p)$, and the γ matrices are defined in the following way:

$$\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \quad \gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (41)$$

$$\gamma_5 = - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (42)$$

$$\gamma^i = -\gamma_i = \vec{\gamma}_i, \quad \gamma_5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3. \quad (43)$$

The Dirac equation and the Dirac spinor are:

$$(\gamma_\mu p^\mu - m)u(p) = 0, \quad (44)$$

$$u_{s_z}(p) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} 1 \\ \frac{\sigma \mathbf{p}}{E+m} \end{pmatrix} \varphi_{s_z}, \quad (45)$$

E is the nucleon energy.

Using the definitions (30,32,37-40), we obtain the following relations between the $\gamma^*N \rightarrow N^*$ helicity amplitudes and form factors.

The $J^P = \frac{1}{2}^+$ resonances:

$$\mathcal{A}_{\frac{1}{2}} = [2Q^2 G_1(Q^2) - (M^2 - m^2)G_2(Q^2)] b, \quad (46)$$

$$\mathcal{S}_{\frac{1}{2}} = \tilde{S} b \frac{|\mathbf{k}|}{\sqrt{2}}, \quad (47)$$

$$\tilde{S} = 2(M+m)G_1(Q^2) + (M-m)G_2(Q^2),$$

$$b = e \sqrt{\frac{E-m}{8mK}}.$$

The $J^P = \frac{1}{2}^-$ resonances:

$$\mathcal{A}_{\frac{1}{2}} = [2Q^2 G_1(Q^2) - (M^2 - m^2)G_2(Q^2)] b, \quad (48)$$

$$\mathcal{S}_{\frac{1}{2}} = -\tilde{S} b \frac{|\mathbf{k}|}{\sqrt{2}}, \quad (49)$$

$$\tilde{S} = 2(M-m)G_1(Q^2) + (M+m)G_2(Q^2),$$

$$b = e \sqrt{\frac{E+m}{8mK}}.$$

For the resonances with $J \geq \frac{3}{2}$, the relations between the $\gamma^*N \rightarrow N^*$ helicity amplitudes and form factors can be found using the results of Ref. [13]. In Ref. [13], the amplitudes h_1, h_2, h_3 are introduced, which are proportional, respectively, to $\mathcal{S}_{1/2}, \mathcal{A}_{3/2}, \mathcal{A}_{1/2}$; the relations between h_1, h_2, h_3 and the $\gamma^*N \rightarrow N^*$ form factors are presented too. These relations are quite cumbersome. For this reason, here we give only the relations between $\mathcal{A}_{1/2}, \mathcal{A}_{3/2}, \mathcal{S}_{1/2}$ and h_3, h_2, h_1 , which for the resonances with $J^P = \frac{3}{2}^\pm, \frac{5}{2}^\mp, \dots$ have the form:

$$\mathcal{A}_{1/2} = h_3 X, \quad (50)$$

$$\mathcal{A}_{3/2} = \pm \sqrt{3} \frac{h_2}{l} X, \quad (51)$$

$$\mathcal{S}_{1/2} = h_1 \frac{|\mathbf{k}|}{\sqrt{2}M} X, \quad (52)$$

$$X \equiv \sqrt{\pi\alpha \frac{(M \mp m)^2 + Q^2}{24MmK}},$$

where $l = J - \frac{1}{2}$.

V. THE RELATION BETWEEN THE $\gamma^*N \rightarrow N^*$ AMPLITUDES EXTRACTED FROM $\gamma^*N \rightarrow N\pi$ AND DEFINED THROUGH HADRON ELECTROMAGNETIC CURRENT

In this Section, our goal is to find the explicit relation between the $\gamma^*N \rightarrow N^*$ form factors contributions to the

$\gamma^*N \rightarrow N\pi$ multipole amplitudes and the $\gamma^*N \rightarrow N^*$ vertex. This will allow us to find the connection between ζ in Eqs. (1) and the sign of the ratio of the $\pi NN, \pi NN^*$ coupling constants. This will allow us also to check the consistency of the relative sign between longitudinal ($S_{1/2}$) and transverse ($A_{1/2}, A_{3/2}$) amplitudes in the definitions through multipole amplitudes (2-7) and hadron electromagnetic current (30-32). With this aim, we will present in detail the results of the covariant calculations of the Figs. 1,2 contributions to $\gamma^*N \rightarrow N\pi$ for the resonances with $J^P = \frac{1}{2}^\pm$. For the resonances with $J \geq \frac{3}{2}$, the relation between ζ and the sign of the ratio of the $\pi NN, \pi NN^*$ coupling constants can be found from the results of Ref. [13]. We will present it in the end of this Section along with that for the $J^P = \frac{1}{2}^\pm$ resonances.

We will use the definitions (37-40) for the $\gamma^*N \rightarrow N^*$ vertices and will define the πNN^* coupling constants according to Ref. [13] in the following form.

The $J^P = \frac{1}{2}^-$ resonances:

$$\langle N | J_\pi(0) | N^{*+} \rangle = C_I g^* \bar{u}(p') u(p^*), \quad (53)$$

$$\Gamma(N^{*+} \rightarrow N\pi) = \frac{g^{*2}}{4\pi} \frac{E' + m}{M} q_\pi. \quad (54)$$

The $J^P = \frac{1}{2}^+$ resonances:

$$\langle N | J_\pi(0) | N^{*+} \rangle = -C_I g^* \bar{u}(p') \gamma_5 u(p^*), \quad (55)$$

$$\Gamma(N^{*+} \rightarrow N\pi) = \frac{g^{*2}}{4\pi} \frac{E' - m}{M} q_\pi. \quad (56)$$

In Eqs. (54,56), E' and p' are the energy and 4-momentum of the final nucleon in the reaction $\gamma^*N \rightarrow N\pi$ in the c.m.s., q_π is the pion 3-momentum in this system.

The πNN coupling constant is defined according to

$$\langle N^+ | J_\pi(0) | N^+ \rangle = g \bar{u}(p') \gamma_5 u(f), \quad (57)$$

where f is the 4-momentum of the intermediate nucleon in the diagram of Fig. 2(a). For the clarity, we will take only that part of the nucleon electromagnetic current which is related to the $F_1(Q^2)$ Pauli form factor:

$$\langle N^+, f | J_\mu | N^+, p \rangle = F_1^p(Q^2) \bar{u}(f) \gamma_\mu u(p). \quad (58)$$

Now let us write the matrix elements for the contributions of the Fig. 1, 2(a) diagrams to $\gamma^*p \rightarrow \pi^0 p$:

$$\text{Fig. 2a: } M = g F_1^p(Q^2) \bar{u}(p') \gamma_5 \frac{m + \hat{f}}{m^2 - f^2} \hat{e} u(p), \quad (59)$$

$$\text{Fig. 1: } M = \pm C_I e g^* G_1(Q^2) \bar{u}(p') \gamma_5 \frac{\pm M + \hat{f}}{M^2 - f^2}$$

$$[k^2(\gamma\epsilon) - (\gamma k)(k\epsilon)]u(p), \quad J^P = \frac{1}{2}^\pm, \quad (60)$$

$$f = p + k = p' + q, \quad Q^2 = -k^2, \quad (61)$$

where $\hat{a} \equiv (a\gamma)$, and again for the simplicity, we have taken only the part of the $\gamma^*N \rightarrow N^*$ vertex related to the $G_1(Q^2)$ form factor.

The general form of the $\gamma^*N \rightarrow N\pi$ matrix element according to the definition of Ref. [14] is:

$$\begin{aligned} M &= \bar{u}(p')\gamma_5 J\epsilon u(p), \\ J\epsilon &= \frac{B_1(Q^2)}{2}[(\gamma\epsilon)(\gamma k) - (\gamma k)(\gamma\epsilon)] + \\ &+ 2B_2(Q^2) \left[(\mathcal{P}\epsilon) - (\mathcal{P}k)\frac{k\epsilon}{k^2} \right] + \\ &+ 2B_3(Q^2) \left[(q\epsilon) - (qk)\frac{k\epsilon}{k^2} \right] - \\ &- B_5(Q^2) \left[(\gamma\epsilon) - (\gamma k)\frac{k\epsilon}{k^2} \right] + \\ &+ B_6(Q^2)(\gamma k) \left[(\mathcal{P}\epsilon) - (\mathcal{P}k)\frac{k\epsilon}{k^2} \right] + \\ &+ B_8(Q^2)(\gamma k) \left[(q\epsilon) - (qk)\frac{k\epsilon}{k^2} \right], \end{aligned} \quad (62)$$

where $\mathcal{P} \equiv \frac{1}{2}(p + p')$.

Multipole amplitudes E_{0+} , S_{0+} , M_{1-} , S_{1-} are related to the invariant amplitudes $B_i(Q^2)$ by [15]:

$$E_{0+} = \tilde{E}_{0+} \frac{\sqrt{(E+m)(E'+m)}}{8\pi W}, \quad (63)$$

$$M_{1-} = \tilde{M}_{1-} \frac{\sqrt{(E-m)(E'-m)}}{8\pi W}, \quad (64)$$

$$S_{0+} = \tilde{S}_{0+} \frac{\sqrt{(E-m)(E'+m)}}{8\pi W Q^2}, \quad (65)$$

$$S_{1-} = \tilde{S}_{1-} \frac{\sqrt{(E+m)(E'-m)}}{8\pi W Q^2}, \quad (66)$$

$$\begin{aligned} \tilde{E}_{0+} &= (W-m)B_1 - B_5, \\ \tilde{M}_{1-} &= -(W+m)B_1 - B_5, \\ \tilde{S}_{0+} &= X(E+m) - \beta Y, \\ \tilde{S}_{1-} &= X'(E-m) + \beta Y', \end{aligned}$$

$$\begin{aligned} \beta &= \frac{W-E}{2}(t-m_\pi^2+Q^2) - Q^2(W-E'), \\ X &= Q^2 B_1 + (W-m)B_5 + \\ &2W(E-m) \left(B_2 - \frac{W+m}{2}B_6 \right), \\ Y &= 2B_3 - B_2 + (W+m) \left(\frac{B_6}{2} - B_8 \right), \\ X' &= -Q^2 B_1 + (W+m)B_5 - \\ &2W(E+m) \left(B_2 + \frac{W-m}{2}B_6 \right), \\ Y' &= 2B_3 - B_2 - (W-m) \left(\frac{B_6}{2} - B_8 \right). \end{aligned}$$

Now from Eqs. (59,62) it is easy to find the contribution of the Fig. 2(a) diagram to $\gamma^*p \rightarrow \pi^0 p$:

$$B_1(Q^2) = \frac{gF_1^p(Q^2)}{s-m^2}, \quad (67)$$

$$B_2(Q^2) = 2B_3(Q^2) = -B_1(Q^2), \quad (68)$$

$$B_5(Q^2) = B_6(Q^2) = B_8(Q^2) = 0. \quad (69)$$

These relations coincide with the commonly used expressions for the Born term which corresponds to the s -channel nucleon exchange.

For the contribution of the Fig. 1 diagram to $\gamma^*p \rightarrow \pi^0 p$ we have:

$$B_1(Q^2) = A, \quad (70)$$

$$B_2(Q^2) = 2B_3(Q^2) = -B_1(Q^2), \quad (71)$$

$$B_6(Q^2) = B_8(Q^2) = 0, \quad (72)$$

$$B_5(Q^2) = (-m \pm M)A, \quad J^P = \frac{1}{2}^\pm, \quad (73)$$

$$A \equiv C_I Q^2 \frac{eg^* G_1(Q^2)}{M^2 - s - iM\Gamma}. \quad (74)$$

From the relations (2-9,63-66,70-74), we find for the $J^P = \frac{1}{2}^+$ resonances:

$$A_{\frac{1}{2}} = 2Q^2 G_1(Q^2)b', \quad (75)$$

$$S_{\frac{1}{2}} = 2(M+m)G_1(Q^2)b' \frac{|\mathbf{k}|}{\sqrt{2}}, \quad (76)$$

$$b' = -e \frac{C_I g^*}{a} \frac{\sqrt{(E-m)(E'+m)}}{8\pi M\Gamma}.$$

For the $J^P = \frac{1}{2}^-$ resonances:

$$A_{\frac{1}{2}} = 2Q^2 G_1(Q^2)b', \quad (77)$$

$$S_{\frac{1}{2}} = -2(M-m)G_1(Q^2)b' \frac{|\mathbf{k}|}{\sqrt{2}}, \quad (78)$$

$$b' = -e \frac{C_I g^*}{a} \frac{\sqrt{(E+m)(E'+m)}}{8\pi M\Gamma}.$$

Now from the comparison with Eqs. (1,46-49) and using Eqs. (54,56), we find:

$$\zeta = -\text{sign}(g^*/g). \quad (79)$$

Using the results of Ref. [14], it can be shown that the same relation between ζ and $\text{sign}(g^*/g)$ is correct also for the resonances with $J \geq \frac{3}{2}$, if the πNN^* vertices are defined in the following way: for the resonances with $J^P = \frac{3}{2}^+, \frac{5}{2}^-, \dots$

$$\langle N | J_\pi(0) | N^{*+} \rangle = C_I g^* \bar{u}(p') p'_{\nu_1} \dots p'_{\nu_l} u^{\nu_1 \dots \nu_l}(p^*), \quad (80)$$

and for the resonances with $J^P = \frac{3}{2}^-, \frac{5}{2}^+, \dots$

$$\langle N | J_\pi(0) | N^{*+} \rangle = -C_I g^* \bar{u}(p') p'_{\nu_1} \dots p'_{\nu_l} \gamma_5 u^{\nu_1 \dots \nu_l}(p^*), \quad (81)$$

where $l = J - \frac{1}{2}$ and $u^{\nu_1 \dots \nu_l}(p^*)$ is the generalized Rarita-Schwinger spinor.

VI. $\gamma^* p \rightarrow N^*$ HELICITY AMPLITUDES IN NONRELATIVISTIC QUARK MODEL

In this Section we will present explicit formulas for the calculation of the $\gamma^* p \rightarrow N^*$ helicity amplitudes in non-relativistic quark model. We will present also in explicit form wave functions for the N , $P_{11}(1440)$, $S_{11}(1535)$, and final formulas for the $\gamma^* p \rightarrow P_{11}(1440)$, $S_{11}(1535)$ helicity amplitudes.

In nonrelativistic quark model, the matrix elements which enter Eqs. (30-32) can be written in the form:

$$\langle N^{*+}, S_z^* | J_{t,l} | N^+, S_z \rangle = \quad (82)$$

$$\begin{aligned} & \sum_i \int d\mathbf{q}_\lambda d\mathbf{q}_\rho \langle S_z^i | J_{t,l}^i | S_z^i \rangle \Phi_N(\mathbf{q}_\lambda, \mathbf{q}_\rho) \Phi_{N^*}(\mathbf{q}'_\lambda, \mathbf{q}'_\rho) \\ & \equiv 3 \int d\mathbf{q}_\lambda d\mathbf{q}_\rho \langle S_z^3 | J_{t,l}^3 | S_z^3 \rangle \Phi_N(\mathbf{q}_\lambda, \mathbf{q}_\rho) \Phi_{N^*}(\mathbf{q}'_\lambda, \mathbf{q}'_\rho), \end{aligned}$$

where $\Phi_N(\mathbf{q}_\lambda, \mathbf{q}_\rho)$, $\Phi_{N^*}(\mathbf{q}'_\lambda, \mathbf{q}'_\rho)$ are the radial parts of the N , N^* wave functions, and according to Eqs. (30-32,35,36), we have made notations: $J_t \equiv \frac{\mathbf{J}_x + i\mathbf{J}_y}{\sqrt{2}}$, $J_l \equiv J_0$. In the last part of Eq. (82), it is supposed that photon interacts with the 3-rd quark. In the calculations, it is convenient to define the 3-rd quark momentum in the N and N^* in the form:

$$\mathbf{q}_3 = \tilde{\mathbf{Q}} - \frac{\mathbf{k}}{2}, \quad (83)$$

$$\mathbf{q}'_3 = \tilde{\mathbf{Q}} + \frac{\mathbf{k}}{2}, \quad (84)$$

where \mathbf{k} is the photon 3-momentum directed along the z-axis in the N^* rest frame, and $\mathbf{q}'_3 = \mathbf{q}_3 + \mathbf{k}$. The momenta, which have appropriate symmetries under $\mathbf{q}_1, \mathbf{q}_2$ exchanges, are:

$$\mathbf{q}_\rho = \frac{\mathbf{q}_1 - \mathbf{q}_2}{\sqrt{2}}, \quad \mathbf{q}'_\rho = \mathbf{q}_\rho, \quad (85)$$

$$\mathbf{q}_\lambda = \tilde{\mathbf{Q}}_\lambda + \frac{\mathbf{k}}{\sqrt{6}}, \quad \mathbf{q}'_\lambda = \tilde{\mathbf{Q}}_\lambda - \frac{\mathbf{k}}{\sqrt{6}}, \quad (86)$$

$$\tilde{\mathbf{Q}}_\lambda \equiv \frac{\mathbf{q}_1 + \mathbf{q}_2 - 2\tilde{\mathbf{Q}}}{\sqrt{6}}. \quad (87)$$

The matrix elements of the quark electromagnetic current have the following form:

$$\begin{aligned} \langle S_z^3 | J_t^3 | S_z^3 \rangle &= e_3 \bar{u}(q'_3) \frac{\vec{\gamma}_x + i\vec{\gamma}_y}{\sqrt{2}} u(q_3) \\ &= e_3 \frac{\sqrt{2}}{m_q} \phi_z^3 \begin{pmatrix} \tilde{Q}_x + i\tilde{Q}_y & \frac{|\mathbf{k}|}{2} \\ 0 & \tilde{Q}_x + i\tilde{Q}_y \end{pmatrix} \phi_z^3, \end{aligned} \quad (88)$$

$$\langle S_z^3 | J_l^3 | S_z^3 \rangle = e_3 \bar{u}(q'_3) \gamma_0 u(q_3) = e_3 \phi_z^3 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \phi_z^3, \quad (89)$$

where m_q is the quark mass, and ϕ_z, ϕ'_z are the quark spin wave functions in the initial and final states.

The radial part of the nucleon wave function with harmonic oscillator potential is

$$\Phi_N(\mathbf{q}_\lambda, \mathbf{q}_\rho) = \frac{1}{\pi^{3/2} \beta^3} \exp\left(-\frac{\mathbf{q}_\rho^2 + \mathbf{q}_\lambda^2}{2\beta^2}\right), \quad (90)$$

where β is the harmonic-oscillator parameter.

In the classification over group $SU(6) \times O(3)$, the proton is the member of the octet from the multiplet $[56, 0^+]$. The spin part of the proton wave function is:

$$|p, S_z \rangle = \frac{1}{\sqrt{2}} \left(|p >_\rho | \frac{1}{2}, S_z >_\rho + |p >_\lambda | \frac{1}{2}, S_z >_\lambda \right), \quad (91)$$

where $|p >_{\rho, \lambda}$ are ρ and λ -type flavor wave functions for the octet:

$$|p >_\rho = \frac{1}{\sqrt{2}} (udu - duu), \quad (92)$$

$$|p >_\lambda = \frac{1}{\sqrt{6}} (2uud - udu - duu), \quad (93)$$

and $|\frac{1}{2}, S_z >_{\rho, \lambda}$ are ρ and λ -type spin $\frac{1}{2}$ wave functions:

$$|\frac{1}{2} \frac{1}{2} >_\rho = \frac{1}{\sqrt{2}} (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow), \quad (94)$$

$$|\frac{1}{2} \frac{1}{2} >_\lambda = \frac{1}{\sqrt{6}} (2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow), \quad (95)$$

$$|\frac{1}{2} - \frac{1}{2} >_\rho = \frac{1}{\sqrt{2}} (\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow), \quad (96)$$

$$|\frac{1}{2} - \frac{1}{2} >_\lambda = -\frac{1}{\sqrt{6}} (2\downarrow\downarrow\uparrow - \downarrow\uparrow\downarrow - \uparrow\downarrow\downarrow). \quad (97)$$

In the explicit form this gives:

$$\begin{aligned} |p, \frac{1}{2} \rangle = & \frac{1}{\sqrt{18}} (2u\uparrow u\uparrow d\downarrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow \\ & - u\uparrow d\uparrow u\downarrow + 2u\uparrow d\downarrow u\uparrow - u\downarrow d\uparrow u\uparrow \\ & - d\uparrow u\uparrow u\downarrow - d\uparrow u\downarrow u\uparrow + 2d\downarrow u\uparrow u\uparrow), \end{aligned} \quad (98)$$

$$\begin{aligned} |p, -\frac{1}{2} \rangle = & -\frac{1}{\sqrt{18}} (2u\downarrow u\downarrow d\uparrow - u\downarrow u\uparrow d\downarrow - u\uparrow u\downarrow d\downarrow \\ & - u\downarrow d\downarrow u\uparrow + 2u\downarrow d\uparrow u\downarrow - u\uparrow d\downarrow u\downarrow \\ & - d\downarrow u\downarrow u\uparrow - d\downarrow u\uparrow u\downarrow + 2d\uparrow u\downarrow u\downarrow). \end{aligned} \quad (99)$$

A. $\gamma^* p \rightarrow P_{11}(1440)$

In the classification over group $SU(6) \times O(3)$, we will consider the resonance $P_{11}(1440)$ as the member of the octet from the multiplet $[56, 0^+]_r$. The spin part of the $P_{11}(1440)$ wave function in this case is the same as for the nucleon (91,98,99), and the radial part is:

$$\begin{aligned} \Phi_{N^*}(\mathbf{q}'_\lambda, \mathbf{q}'_\rho) = & \quad (100) \\ & \frac{1}{\sqrt{3}\pi^{3/2}\beta^3} \left(3 - \frac{\mathbf{q}_\rho'^2 + \mathbf{q}_\lambda'^2}{\beta^2} \right) \exp\left(-\frac{\mathbf{q}_\rho'^2 + \mathbf{q}_\lambda'^2}{2\beta^2}\right). \end{aligned}$$

The $\gamma^* p \rightarrow P_{11}(1440)$ helicity amplitudes obtained using the definitions (30,32) and the results presented in this

Section are:

$$\mathcal{A}_{\frac{1}{2}} = -\sqrt{\frac{2\pi\alpha}{K}} \frac{|\mathbf{k}|^3}{6\sqrt{6}m_q\beta^2} e^{-\frac{\mathbf{k}^2}{6\beta^2}}, \quad (101)$$

$$\mathcal{S}_{\frac{1}{2}} = -\sqrt{\frac{2\pi\alpha}{K}} \frac{\mathbf{k}^2}{6\sqrt{3}\beta^2} e^{-\frac{\mathbf{k}^2}{6\beta^2}}. \quad (102)$$

B. $\gamma^*p \rightarrow S_{11}(1535)$

In the classification over group $SU(6) \times O(3)$, we will consider the resonance $S_{11}(1535)$ as the ${}^28_{\frac{1}{2}}$ member of the multiplet $[70, 1^-]$. The spin part of the $S_{11}(1535)$ wave function in this case is:

$$|N^{*+}, \frac{1}{2}\rangle = -\frac{1}{2\sqrt{3}} (|p >_{\lambda} X_{\lambda} + |p >_{\rho} X_{\rho}), \quad (103)$$

$$X_{\lambda} \equiv -|\frac{1}{2}\frac{1}{2}\rangle_{\lambda} |\lambda, 0\rangle + \sqrt{2} |\frac{1}{2} - \frac{1}{2}\rangle_{\lambda} |\lambda, +1\rangle + |\frac{1}{2}\frac{1}{2}\rangle_{\rho} |\rho, 0\rangle - \sqrt{2} |\frac{1}{2} - \frac{1}{2}\rangle_{\rho} |\rho, +1\rangle, \quad (104)$$

$$X_{\rho} \equiv |\frac{1}{2}\frac{1}{2}\rangle_{\rho} |\lambda, 0\rangle - \sqrt{2} |\frac{1}{2} - \frac{1}{2}\rangle_{\rho} |\lambda, +1\rangle + |\frac{1}{2}\frac{1}{2}\rangle_{\lambda} |\rho, 0\rangle - \sqrt{2} |\frac{1}{2} - \frac{1}{2}\rangle_{\lambda} |\rho, +1\rangle, \quad (105)$$

where

$$|\lambda, 0\rangle \equiv \mathbf{q}'_{\lambda,z}, \quad |\lambda, +1\rangle \equiv -\frac{\mathbf{q}'_{\lambda,x} + i\mathbf{q}'_{\lambda,y}}{\sqrt{2}}, \quad (106)$$

$$|\rho, 0\rangle \equiv \mathbf{q}'_{\rho,z}, \quad |\rho, +1\rangle \equiv -\frac{\mathbf{q}'_{\rho,x} + i\mathbf{q}'_{\rho,y}}{\sqrt{2}}. \quad (107)$$

The radial part of the $S_{11}(1535)$ wave function is:

$$\Phi_{N^*}(\mathbf{q}'_{\lambda}, \mathbf{q}'_{\rho}) = \frac{\sqrt{2}}{\pi^{3/2}\beta^4} \exp\left(-\frac{\mathbf{q}'_{\rho}{}^2 + \mathbf{q}'_{\lambda}{}^2}{2\beta^2}\right). \quad (108)$$

The $\gamma^*p \rightarrow S_{11}(1535)$ helicity amplitudes obtained according to the definitions (30,32) and the results of this Section are:

$$\mathcal{A}_{1/2} = \sqrt{\frac{8\pi\alpha}{K}} \frac{\beta}{6m_q} \left(1 + \frac{\mathbf{k}^2}{2\beta^2}\right) \exp\left(-\frac{\mathbf{k}^2}{6\beta^2}\right), \quad (109)$$

$$\mathcal{S}_{1/2} = \sqrt{\frac{4\pi\alpha}{K}} \frac{|\mathbf{k}|}{6\beta} \exp\left(-\frac{\mathbf{k}^2}{6\beta^2}\right). \quad (110)$$

VII. INFORMATION ON THE SIGNS OF THE πNN^* COUPLING CONSTANTS AVAILABLE IN QUARK MODEL

In the framework of quark model, the signs of the πNN^* coupling constants were found in Ref. [16] for the resonances of the multiplet $[70, 1^-]$ using an approach based on PCAC. There are also results on the sign of the $\pi NP_{11}(1440)$ coupling constant obtained in Refs. [11, 17], respectively in 3P_0 model and using PCAC; they

coincide with each other. Quark model predictions for the $\gamma^*p \rightarrow N^*$ helicity amplitudes are presented in Refs. [11, 16, 17] taking into account the πNN^* signs obtained in these papers.

However, traditionally, quark model predictions for the $\gamma^*p \rightarrow N^*$ helicity amplitudes are presented with the common sign fixed by taking the sign of $A_{1/2}$ at $Q^2 = 0$ equal to that of the amplitude extracted from experimental data. Sometimes, such definition of the sign can bring to confusing and wrong results, as we will demonstrate below on the example of the $P_{11}(1440)$ resonance.

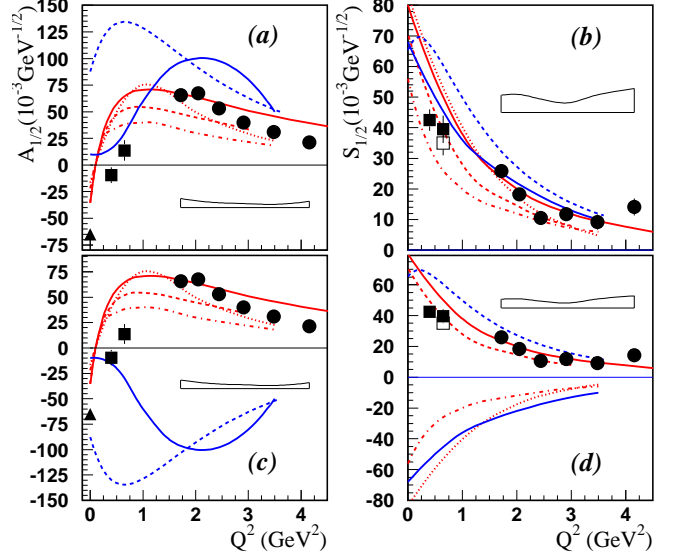


FIG. 3: Helicity amplitudes for the $\gamma^*p \rightarrow P_{11}(1440)$ transition. The full circles are the data extracted from the JLab-CLAS $\bar{e}p \rightarrow ep\pi^0, en\pi^+$ data [1, 2, 3, 4, 5, 6, 7, 8]. The bands present model uncertainties of the data. The full triangle at $Q^2 = 0$ is the RPP estimate [9]. The red curves correspond to the predictions of the light-front relativistic quark models: dashed - Ref. [11], solid - Ref. [17], dotted - Ref. [18], dashed-dotted - Ref. [19]. The blue curves are the results obtained via nonrelativistic calculations: solid - Ref. [20], dashed - Ref. [21]. The plots (c,d) present the predictions in that form as they are given in the papers. In plots (a,b), all results are presented with correct signs.

In Fig. 3, we present the results for the $\gamma^*p \rightarrow P_{11}(1440)$ helicity amplitudes extracted from the JLab-CLAS data on the $\bar{e}p \rightarrow ep\pi^0, en\pi^+$ reactions [1, 2, 3, 4, 5, 6, 7, 8] in comparison with the quark model predictions obtained in the light-front dynamics [11, 17, 18, 19] and via nonrelativistic calculations [20, 21]. For clear understanding of the results presented in Fig. 3, it is important to note that the $A_{1/2}$ amplitude found with the same N and $P_{11}(1440)$ wave functions via nonrelativistic calculations and in the light-front relativistic approaches have at $Q^2 = 0$ opposite signs. For the first time this was mentioned in Ref. [11]. However, in the presentation of the results in traditional way (see plots (c,d)), the amplitudes $A_{1/2}$ at $Q^2 = 0$ from Refs. [11, 17, 18, 19] and [20, 21] appear with the same sign. As a result, for

higher Q^2 , we have strong disagreement between the corresponding predictions, and the quark model predictions [20, 21] strongly disagree with the data extracted from experiment.

In plots (a,b), we present all results with the $\pi NP_{11}(1440)$ sign found in Refs. [11, 17]. We have also corrected the relative signs between the $A_{1/2}$ and $S_{1/2}$ amplitudes from Refs. [18, 19, 21]. It can be seen that for $Q^2 > 0.4 \text{ GeV}^2$, this results in good agreement with experiment for the signs of amplitudes from all approaches [11, 17, 18, 19, 20, 21]; we have also better agreement of the results obtained in different approaches with each other.

For the resonances from the multiplet $[70, 1^-]$: $N(1520)D_{13}$, $N(1535)S_{11}$, $\Delta(1620)S_{31}$, $N(1650)S_{11}$, $N(1700)D_{13}$, and $\Delta(1700)D_{33}$, the signs found in traditional way and in Ref. [16] coincide with each other for all resonances, except $N(1700)D_{13}$. With the πNN^* coupling constants found in Ref. [16], nonrelativistic quark model prediction for the $A_{1/2}(\gamma^*p \rightarrow N^*)$ amplitude at $Q^2 = 0$ is positive for the resonances $N(1535)S_{11}$, $\Delta(1620)S_{31}$, $N(1650)S_{11}$, $N(1700)D_{13}$, and $\Delta(1700)D_{33}$, and negative for $N(1520)D_{13}$.

VIII. SUMMARY

By performing covariant calculations of the resonance (Fig. 1) and Born terms (Fig. 2) contributions to $\gamma^*p \rightarrow \pi N$, we have found the relations between the following definitions of the $\gamma^*p \rightarrow N^{*+}$ helicity amplitudes $A_{1/2}, A_{3/2}, S_{1/2}$:

(i) the definition through the $\gamma^*p \rightarrow \pi N$ multipole amplitudes which is commonly used for the extraction of $A_{1/2}, A_{3/2}, S_{1/2}$ from experimental data on $\gamma^*p \rightarrow \pi N$;

(ii) the definitions through the hadron electromagnetic current and the $\gamma^*p \rightarrow N^*$ form factors which are used in theoretical calculations.

These relations include the relative sign between the

coupling constants g^* and g which we have defined explicitly for the vertices πNN^* and πNN . For the proper comparison of theoretical predictions with the results extracted from experimental data, it is important to have both quantities, the $\gamma^*p \rightarrow N^{*+}$ amplitudes and the sign of the ratio (g^*/g) , calculated within the same theoretical approach. This is demonstrated on the example of the $\gamma^*p \rightarrow P_{11}(1440)$ transition.

The performed calculations allowed us also to check and present the formulas for the $\gamma^*p \rightarrow N^{*+}$ helicity amplitudes through the hadron electromagnetic current and the $\gamma^*p \rightarrow N^*$ form factors which give the correct relative sign between the longitudinal $S_{1/2}$ and transverse $A_{1/2}, A_{3/2}$ amplitudes, consistent with that for the amplitudes extracted from experiment. To avoid sources of mistakes in the calculation of this sign, we give definitions of all quantities which enter the formulas.

For completeness, we have presented formulas for the calculations of the $\gamma^*N \rightarrow N^*$ helicity amplitudes in nonrelativistic quark model along with the $N, P_{11}(1440), S_{11}(1535)$ wave functions and final formulas for the $\gamma^*p \rightarrow P_{11}(1440), S_{11}(1535)$ helicity amplitudes.

Taking into account the available information on the signs of the ratios (g^*/g) in the framework of quark model, we have presented, in the case of nonrelativistic quark model, the signs of the $A_{1/2}$ amplitudes for the resonances from the multiplet $[70, 1^-]$. This may be useful for the correct presentations of the signs of the helicity amplitudes $A_{1/2}, A_{3/2}, S_{1/2}$ in quark model.

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- [1] K. Joo et al., CLAS Collaboration, Phys. Rev. Lett. **88**, 122001 (2002).
 - [2] K. Joo et al., CLAS Collaboration, Phys. Rev. C **68**, 032201 (2003).
 - [3] K. Joo et al., CLAS Collaboration, Phys. Rev. C **70**, 042201 (2004).
 - [4] H. Egiyan et al., CLAS Collaboration, Phys. Rev. C **73**, 025204 (2006).
 - [5] K. Park et al., CLAS Collaboration, Phys. Rev. C **77**, 015208 (2008).
 - [6] I. G. Aznauryan, V. D. Burkert, H. Egiyan, et al., Phys. Rev. C **71**, 015201 (2005).
 - [7] I. G. Aznauryan, V. D. Burkert, et al., Phys. Rev. C **72**, 045201 (2005).
 - [8] I. G. Aznauryan et al., CLAS Collaboration, nucl-exp/0804.0447.
 - [9] W.-M. Yao et al. [Particle Data Group], Journal of Physics G **33**, 1 (2006).
 - [10] R. A. Arndt, R. L. Workman, Zh. Li, and L. D. Roper, Phys. Rev. C **42**, 1864 (1990).
 - [11] S. Capstick and B. D. Keister, Phys. Rev. D **51**, 3598 (1995).
 - [12] L. Tiator, D. Drechsel, S. Kamalov, and S. N. Yang, PiN Newslett. **16**, 41 (2002).
 - [13] R. C. E. Devenish, T. S. Eizenschitz and J. G. Körner, Phys. Rev. D **14**, 3063 (1976).
 - [14] R. C. E. Devenish and D. H. Lyth, Phys. Rev. D **5**, 47 (1972).
 - [15] I. G. Aznauryan, Phys. Rev. C **67**, 015209 (2003).
 - [16] I. G. Aznauryan and A. S. Bagdasaryan, Sov. J. Nucl. Phys. **41**, 158 (1985).
 - [17] I. G. Aznauryan, Phys. Rev. C **76**, 025212 (2007).

- [18] H. J. Weber, Phys. Rev. C **41**, 2783 (1990).
- [19] E. Pace, G. Salme, and S. Simula, Few Body Syst. Suppl. **10**, 407 (1999).
- [20] M. Warns, H. Schröder, W. Pfeil, and H. Rolnik, Z. Phys. C **46**, 627 (1990).
- [21] M. Aiello, M. M. Giannini, and E. Santopinto, J. Phys. G **24**, 753 (1998).