Ξ^--Ξ^0 Mass Difference*

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The method of Dashen is applied to the calculation of the Ξ^--Ξ^0 mass difference. Assuming that the isoscalar electromagnetic form factors of the Ξ are dominated by the ω and φ poles and that the relevant coupling constants satisfy certain $SU(3)$ relations, approximate expressions for the required form factors are obtained by comparison with the Stanford data on nucleons. The mass difference calculated with a crude linear D function has the correct sign, but the magnitude is subject to large uncertainties, owing primarily to the fact that the $\pi\Xi\Xi$ coupling constant is very poorly known.

1. INTRODUCTION

About two years ago Dashen and Frautschi[1] developed a method of calculation of electromagnetic mass differences which gave excellent results for the neutron-proton mass difference.[2] More recently, certain aspects of their method have been criticized.[3] Without entering into the debate we wish to point out that a straightforward application of Dashen's method to the cascade particles under some additional assumptions described below gives a mass difference in qualitative agreement with experiment.

Following Dashen,[2] we treat the Ξ as a bound state in the $P_{1/2}$, $I=\frac{1}{2}$ channel in the $\pi\Xi$ system.[4] The quantities needed are then the $\pi\Xi\Xi$ coupling constant, the isoscalar electromagnetic form factors of the Ξ, and the D function for $\pi\Xi$ scattering in the $P_{1/2}$, $I=\frac{1}{2}$ state. These quantities are discussed in the next section, where $SU(3)$ is invoked to determine the relevant coupling constants. It turns out that under reasonable assumptions the Pauli form factor contributes considerably less to the mass difference than the Dirac form factor.

2. THE FORM FACTORS AND COUPLING CONSTANTS

The isoscalar electromagnetic form factor $F_{1S}(N)$ for the nucleons has been discussed by Dashen and Sharp.[5] Assuming ω, ϕ dominance and that the ω and the ϕ are superpositions of an unperturbed member Y of a unitary octet and a singlet B, it follows on diagonalizing the mass matrix that[5]

$$|\phi\rangle=\cos\theta|Y\rangle+\sin\theta|B\rangle,\qquad |\omega\rangle=\cos\theta|B\rangle-\sin\theta|Y\rangle, \tag{1}$$

with $\cos\theta=0.78$, $\sin\theta=0.62$, and that[5]

$$F_{1S}(N)=1-f_{YNN}/f_Y+\frac{\cos\theta f_{\phi NN}}{f_Y(1-t/m_\phi^2)}-\frac{\sin\theta f_{\omega NN}}{f_Y(1-t/m_\omega^2)}, \tag{2}$$

where

$$f_{\phi NN}=\cos\theta f_{YNN}+\sin\theta f_{BNN},\qquad f_{\omega NN}=\cos\theta f_{BNN}-\sin\theta f_{YNN}. \tag{3}$$

Comparing with the Stanford data,[6] one finds

$$f_{YNN}/f_Y=1.15,\quad \cos\theta f_{\phi NN}/f_Y=-1.04,\quad \sin\theta f_{\omega NN}/f_Y=-2.19, \tag{4}$$

so that Eqs. (3) give

$$f_{BNN}/f_Y=-3.58.$$

The isoscalar Dirac form factor for the Ξ is, in the same approximation,

$$f_{1S}(\Xi)=1-f_{Y\Xi\Xi}/f_Y+\frac{\cos\theta f_{\phi\Xi\Xi}}{f_Y(1-t/m_\phi^2)}-\frac{\sin\theta f_{\omega\Xi\Xi}}{f_Y(1-t/m_\omega^2)}, \tag{5}$$

with

$$f_{\phi\Xi\Xi}=\cos\theta f_{Y\Xi\Xi}+\sin\theta f_{B\Xi\Xi},\quad f_{\omega\Xi\Xi}=\cos\theta f_{B\Xi\Xi}-\sin\theta f_{Y\Xi\Xi}.$$

Assuming that the meson-baryon coupling constants satisfy $SU(3)$ relations,[7] we have

$$f_{B\Xi\Xi}/f_Y=f_{BNN}/f_Y=-3.58,\quad f_{Y\Xi\Xi}/f_Y=-(g/\sqrt{3}f_Y)(1+2\lambda),\quad g\equiv f_{\rho NN},\quad f_{YNN}/f_Y=(g/\sqrt{3}f_Y)(4\lambda-1)=1.15. \tag{6}$$

One more relation is needed to evaluate the two quantities λ and g/f_Y. For this purpose we make the drastic one-pole approximation for the isovector Dirac form

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[1] R. F. Dashen and S. C. Frautschi, Phys. Rev. **135**, B1190 (1964).
[2] R. F. Dashen, Phys. Rev. **135**, B1196 (1964).
[3] R. Sawyer (unpublished report); G. Shaw and D. Wong (unpublished report); and G. Barton (unpublished report). See also H. Fried and T. Truong, Phys. Rev. Letters **16**, 559 (1966).
[4] In a consistent $SU(3)$ calculation the $\bar{K}\Lambda$ and $\bar{K}\Sigma$ channels should also be taken into account. The thresholds of these channels, however, are some 160 and 230 MeV above the $\pi\Xi$ threshold (i.e., both are above the production threshold for $\pi\pi\Xi$). Since we are using $SU(3)$ here only as an aid in obtaining the electromagnetic form factors, we shall neglect the effects of these channels.
[5] R. F. Dashen and D. Sharp, Phys. Rev. **133**, B1585 (1964). Their constants a and b are equal to our $\cos\theta$ and $\sin\theta$, respectively.
[6] E. B. Hughes, T. A. Griffy, M. R. Yearian, and R. Hofstadter, Phys. Rev. **139**, B458 (1965).
[7] J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963). We follow his notations with trivial modifications.

factor of the nucleon[8]:

$$f_{1V}(N)=1-\frac{f_{\rho NN}}{f_\rho}+\frac{f_{\rho NN}}{f_\rho(1-t/m_\rho^2)}. \quad (7)$$

The coupling constant f_ρ is equal to $2f_Y/\sqrt{3}$,[5,8] so that $f_{\rho NN}/f_\rho \equiv g/f_\rho = g\sqrt{3}/2f_Y$. Comparison of (7) with experiment then provides the necessary fourth relation which together with (6) suffice to determine all the coefficients in (5). The result is

$$F_{1S}(\Xi)=2.12-\frac{2.42}{1-t/m_\phi^2}+\frac{1.30}{1-t/m_\omega^2}. \quad (5')$$

Since the Pauli form factor $F_{2S}(t)$ is usually believed to vanish for large t, we write in analogy with (2) but without the constant term

$$F_{2S}(N)\approx\frac{\cos\theta\, g_{\phi NN}}{g_Y(1-t/m_\phi^2)}-\frac{\sin\theta\, g_{\omega NN}}{g_Y(1-t/m_\omega^2)}, \quad (8)$$

and a similar equation with N replaced by Ξ. Using again the data[9] of Ref. 6, we obtain under assumptions similar to those for F_{1S} (i.e., all the relations above with the f's replaced by the g's)

$$F_{2S}(\Xi)\approx\frac{-2.29}{1-t/m_\phi^2}+\frac{0.35}{1-t/m_\omega^2}. \quad (9)$$

Our method of determining the residues of the poles in (5′) and (9) is clearly subject to large uncertainties. The individual residues may be uncertain by about 30%, although the combined error may be considerably less (see Ref. 12 below).

For the $\pi\Xi$ coupling constant we note that according to $SU(3)$, one has[7]

$$g_{\pi\Xi\Xi}^2/g_{\pi NN}^2=(1-2\alpha_p)^2. \quad (10)$$

A recent analysis of the $Y_1^*(1385)$ by Martin[10] gave the value $\alpha_p=0.75$. Other analyses of meson-baryon scattering give the estimate[10]

$$0.55\leq\alpha_p\leq0.75, \quad (11)$$

which corresponds to

$$0.01\leq g_{\pi\Xi\Xi}^2/g_{\pi NN}^2\leq0.25. \quad (12)$$

Since π-Ξ interactions appear to be quite strong [the $\Xi^*(1530)$ resonance occurs at ~70-MeV kinetic energy in the $\pi\Xi$ system], it is unlikely that $g_{\pi\Xi\Xi}^2$ can be down two orders of magnitude from $g_{\pi NN}^2$. The correct ratio is probably much closer to the upper value 0.25 than to the lower value 0.01.

3. THE MASS DIFFERENCE

The calculation of the Ξ^--Ξ^0 mass difference is essentially the same as that of the neutron-proton difference,[2] apart from the fact that the isoscalar part of the Pauli form factor here is not quite negligible as it is for the nucleons. In the absence of reliable information on the D function for π-Ξ scattering in the $I=\frac{1}{2}$, $P_{1/2}$ channel we make the crude linear approximation $D(W)=W-m$, where m denotes the average mass of the Ξ's. The relevant integrals can then be evaluated without difficulty. Keeping in mind the possibility of better determinations of the electromagnetic form factors in the future, we rewrite (5′) and (9) in more general form:

$$F_{1S}(\Xi)=a+\frac{b}{1-t/m_\omega^2}+\frac{c}{1-t/m_\phi^2},\quad a+b+c=1; \quad (5'')$$

$$F_{2S}(\Xi)=\frac{\beta}{1-t/m_\omega^2}+\frac{\gamma}{1-t/m_\phi^2}. \quad (9')$$

Defining δA to be $A(I_3=-\frac{1}{2})-A(I_3=+\frac{1}{2})$, and similarly for the other dynamical quantities, we find that the analog of the first term within the brackets of Eq. (4) of Ref. 2 has the value[11]

$$I=\frac{1}{2\pi i}\int_{\supset L}\frac{\delta A(W')D^2(W')dW'}{W'-m}=\frac{-2\alpha}{3m}\bigg[a+b+c+a\ln(m_\rho/2\mu)+b\frac{m_\omega^2\ln(m_\rho/2\mu)-m_\rho^2\ln(m_\omega/2\mu)}{m_\omega^2-m_\rho^2}$$
$$+c\frac{m_\phi^2\ln(m_\rho/2\mu)-m_\rho^2\ln(m_\phi/2\mu)}{m_\phi^2-m_\rho^2}+\left(\frac{m_\rho}{m}\right)^2\left(\beta\frac{m_\omega^2}{m_\omega^2-m_\rho^2}\ln(m_\omega/m_\rho)+\gamma\frac{m_\phi^2\ln(m_\phi/m_\rho)}{m_\phi^2-m_\rho^2}\right)\bigg], \quad (13)$$

with $D(W)=W-m$ and if the infrared divergence is handled in the same way as Refs. 1 and 2. Owing to the factor $(m_\rho/m)^2\approx0.34$, the contribution due to F_{2S} is considerably suppressed. Inserting the values of a, b, c, and β, γ, we have

$$I\approx-1.16\alpha/m. \quad (14)$$

Taking into account kinematical singularities and Ξ exchange in the same approximation as Ref. 2, we find that the mass difference is given by

$$\delta m=m(\Xi^-)-m(\Xi^0)=\frac{0.337\alpha\mu(\mu/m)}{f_\Xi^2}=\frac{0.035}{f_\Xi^2}\text{ MeV}, \quad (15)$$

[8] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 965 (1961). Their γ_ρ and $\gamma_{\rho NN}$ are equal to half of our f_ρ and $f_{\rho NN}$, respectively. (See also Ref. 5.)

[9] The result of Ref. 6 for F_{2S} is given in terms of two poles and a small constant term which we neglect.

[10] B. R. Martin, Phys. Rev. **138**, B1136 (1965). This paper contains further references to other estimates of α_p.

[11] Our result for the Dirac part differs from that of Ref. 2 by a factor of $\frac{4}{3}$ owing to the fact that we did not make approximations such as $W+m\approx2m$, etc., as did Dashen in certain parts of the integral.

where f_Ξ is the renormalized $\pi\Xi\Xi$ coupling constant:

$$\frac{f_\Xi^2}{f_N^2}=\left(\frac{m_N}{m}\right)^2\frac{g_{\pi\Xi\Xi}^2}{g_{\pi NN}^2}\approx\tfrac{1}{2}(1-2\alpha_p)^2\,,\quad (f_N^2=0.08)\,;\qquad(16)$$

or

$$\delta m=\frac{0.88}{(1-2\alpha_p)^2}=3.5\text{ MeV}\quad\text{for}\quad\alpha_p=0.75\quad\text{(Martin's value)}\,,\qquad(17)$$

as compared to the experimental value $\delta m=(6.5\approx1)$ MeV. The expression (17) becomes catastrophically large (88 MeV) if $\alpha_p=0.55$, which, as was noted in Sec. 2, corresponds to the highly unlikely case $g_{\pi\Xi\Xi}^2/g_{\pi NN}^2=0.01$. For $0.55\leq\alpha_p\leq0.75$, Eq. (17) gives 3.5 MeV $\leq\delta m\leq88$ MeV, which is consistent with experiment. In view of the crudity of the D function used and the drastic assumptions made in extracting the form factors, an uncertainty on the order of a factor of 2 in the coefficient of (15) is probably not an overestimate. With the choice $D(W)\approx W-m$, however, the sign of the mass difference appears quite stable[12] within the framework of the Dashen-Frautschi method.

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[12] To estimate the uncertainity in the mass difference due to variations in the numbers a, b, c, etc., we obtained values of the latter using the older Cornell data [K. Berkelman, in *Proceedings of the 1963 International Conference on Nucleon Structure* (Stanford University Press, Stanford, California, 1964), p. 45]. Some of these values differ considerably from those given in (5′) and (9). Upon evaluating the mass difference, however, there is much cancellation, with the result that δm suffers only minor changes.

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Conservation Laws and Symmetries. II

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The reciprocal relationship between conservation laws and symmetries is established for those theories wherein the equations of motion are derivable from a variational principle. It is shown, for a general variational problem with arbitrary number of independent and dependent variables, that to every divergenceless vector there corresponds another which differs from it, in general, by terms that vanish when the Euler-Lagrange equations are satisfied and which has the structure obtained by applying Noether's theorem to some symmetry transformation. Thus existence of a continuity equation implies some invariance property of the variational problem (converse of Noether's theorem). The Lagrangian is invariant, in general, up to a divergence. Derivatives of dependent variables of any arbitrary finite order are allowed to appear in the Lagrangian; it is assumed, however, that it does not contain independent variables explicitly. A systematic procedure is formulated to deduce the invariance property associated with a given conservation law and is illustrated by some examples.

I. INTRODUCTION

IN a previous paper[1] an attempt was made to prove, in the Lagrangian formalism of local field theory, that every conservation law has associated with it some symmetry property of the (coupled) field system. An analogous proof can, of course, also be worked out[2] in particle mechanics. The proofs given in Ref. 1 and in a previous work by Horn[3] have rather severe limitations. Apart from assuming the existence of the space integrals of the time components of the conserved currents, they involve very restrictive assumptions about the structure of the conserved quantities and of the Lagrangian. In this Paper we shall give a more general proof whose scope has been outlined in the abstract.

Instead of specializing to specific dynamical systems, we shall speak of a general variational problem leaving the nature of the variables unspecified. Indeed, Noether's theorem[4–6] itself deals with variational problems in general, without having any physics associated with it. Like every other theorem in mathematics, it becomes a statement of a physical law only when the variables are identified with the dynamical variables of some physical system.

In Sec. II we collect some useful formulas from the calculus of variation. The next section contains the above-mentioned proof of the converse of Noether's

[1] Tulsi Dass, Phys. Rev. **145**, 1011 (1966).
[2] Tulsi Dass (unpublished).
[3] D. Horn, Ann. Phys. (N. Y.) 32, 444 (1965).

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[5] E. L. Hill, Rev. Mod. Phys. **23**, 253 (1951).
[6] A. Trautman, in *Brandeis Summer Institute in Theoretical Physics, 1964* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1965), Vol. I.