Computational Physics Lab

Numerical Differentiation & Simple Differential Equations

03/17/2009

Outline

This Week's Project

Radioactive Decays

$$\frac{dN(t)}{dt} = \frac{-N(t)}{\tau}$$

$$Set \ \frac{dN(t)}{dt} = D^{+}_{\Delta t}(N(t))$$

and solve for the incremental equation of state

$$N(t+\Delta) = \_\_\_\_\_$$

Euler Method: Radioactive Decay

Differential equation describing radioactive decay

$$\frac{dN(t)}{dt} = -\frac{N(t)}{\tau}$$

Using the forward difference:

$$\frac{N(t+\Delta t) - N(t)}{\Delta t} + O(\Delta t) = -\frac{N(t)}{\tau}$$

$$N(t+\Delta t) = N(t) - \Delta t \frac{N(t)}{\tau} + O(\Delta t)^2$$

Dropping the error term

$$\boxed{N_{n+1} = N_n - \Delta t \frac{N_n}{\tau}}$$

Euler Method Procedure

Calculation of Decay (the incremental equation)

1. Specify the initial conditions: N_1 & τ.
2. Choose a time step Δt.
3. Use $N_{n+1} = N_n - \Delta t \frac{N_n}{\tau}$ to compute new N.
4. Go to step 3 until enough trajectory points have been computed.

Euler-Cromer Method

$$N_{n+1} = N_n - \Delta t \frac{N_n}{\tau}$$