Computational Physics Lab

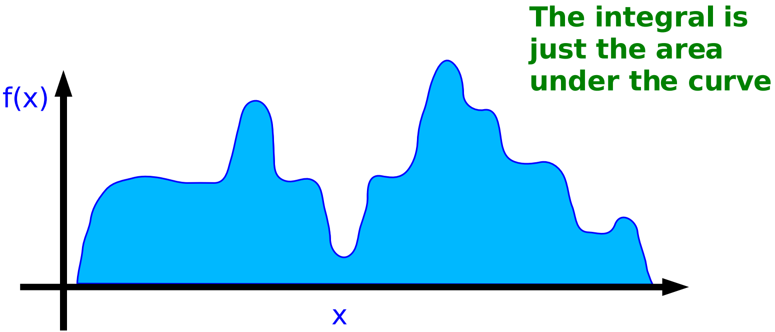
Numerical Integration

04/14/2009

Outline

Numerical Integration

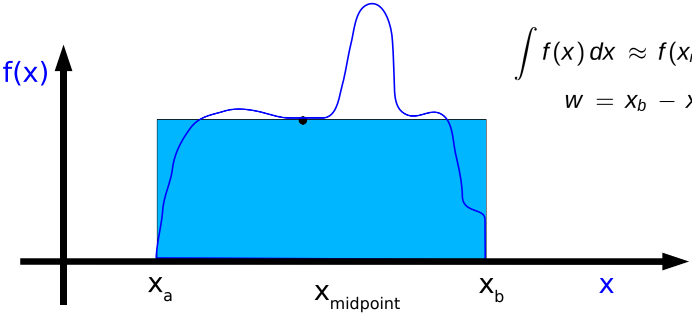
For a given function f(x), the solution can exist in an exact analytical form but frequently an analytical solution does not exist and it is therefore necessary to solve the integral numerically.



Rectangle Midpoint Rule

Newton-Cotes Method of Order Zero

- Approximate f(x) as a constant: $f(x) \approx f(x = x_{\text{midpoint}})$
- $\text{area} = f(x_{\text{mid}}) \cdot w$



$$\int f(x)\,dx \approx f(x_m) \cdot w$$

$$w = x_b - x_a$$

Composite Midpoint Rule

1. Break up interval into small pieces
2. Approximate interval area via rectangle

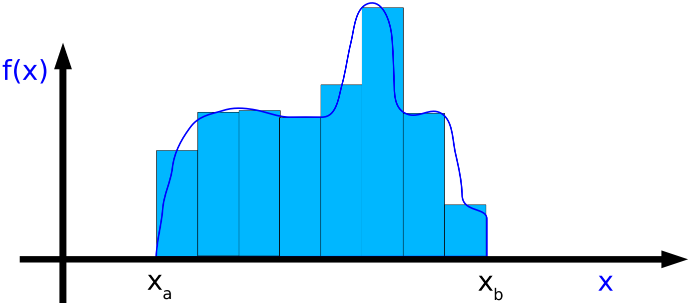
$$area = f(x_{\text{mid}}) \cdot w$$

3. Add up all of the areas

Composite Midpoint Method

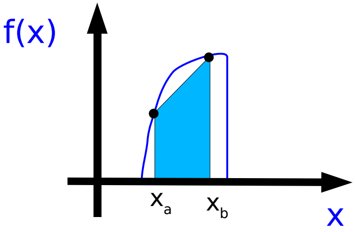
$$\int_{x_a}^{x_b} f(x)\,dx \approx w \sum_{i=0}^{n-1} f(x_a + w\,(i + 1/2))$$

$$w = (x_b - x_a)/n$$



Trapezoidal Rule

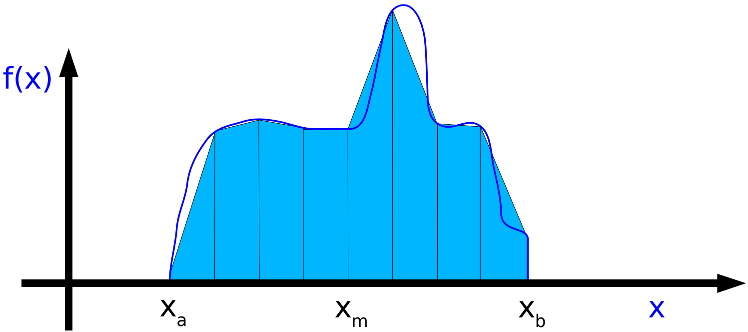
- Linear f(x) approximation
- Uses both start & end points
- area $= w \cdot (f(x_1) + f(x_2))/2$



Composite Trapezoidal Method

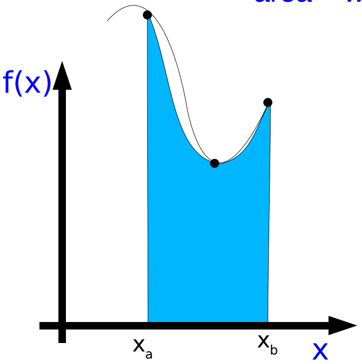
$$\int_{x_a}^{x_b} f(x)\,dx \approx \frac{w}{2}\sum_{i=0}^{n-1} f(x_a + w\,i) + f(x_a + w\,(i+1))$$

$$w = (x_b - x_a)/n$$



3-Point Simpson Method

- Quadratic f(x) approximation
- Uses start, mid, and end points
- area = $w \cdot (f(x_1) + 4f(x_m) + f(x_2))/6$



Composite 3-Point Simpson Method

$$\int_{x_a}^{x_b} f(x)\,dx \approx \frac{w}{6}\sum_{i=0}^{n-1} f(x_a+w\,i)+4f(x_a+w\,(i+\frac{1}{2}))+f(x_a+w\,(i+1))$$

$$w=(x_b-x_a)/n$$