Computational Physics Lab

Root-Finding Procedures

02/26/2009

Outline

Root-Finding Procedures

Chapter 14

1. Bisection Search
   (Section 14.1: 191 - 193)
2. False Position Method
3. Newton-Raphson Method
   (Section 14.2: 193 - 194)

Finding Zeros of Functions

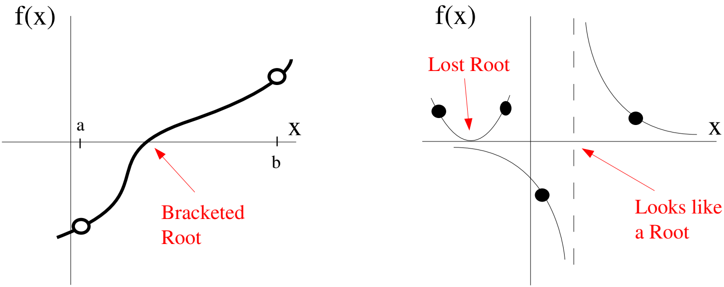
One of the most basic tasks:

Solving equations numerically

1. $F(x) = 0$ N-dimensional case
   1. Generic
      - N equations – N solutions
      - Distinct, Point-like, Separated
   2. Non-Generic
      - Degenerate (Continuous family of solutions)
      - Nonlinear (May have no real solution.)
2. $F(x) = 0$ 1-dimensional case
   - Possible to trap a root between bracketing values, and then hunt it down like a rabbit.

Bracketing & Bisection

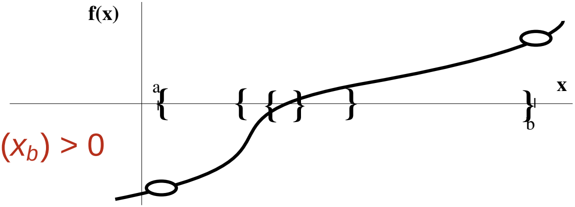
If in [a,b], f(x) is a continuous function and f(a) & f(b) have opposite signs then (at least) one root must exist.



Basic Approach

1. Calculate the midpoint between x_a and x_b.
2. Calculate the value of $f(x_{\mathrm{mid}})$.
3. If then
   1. $f(x_{\mathrm{mid}}) = 0$ stop
   2. $f(x_{\mathrm{mid}}) < 0$ replace x_a with x_{mid}
   3. $f(x_{\mathrm{mid}}) > 0$ replace x_b with x_{mid}
4. Repeat steps 1-3 as needed.

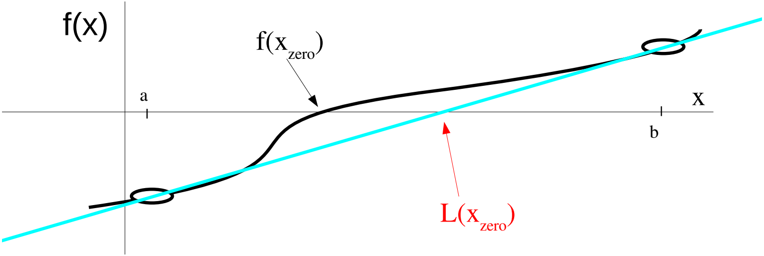
$f(x_a) < 0$ & $f(x_b) > 0$



False Position Method

Improve rate of convergence by using information about the values of the function.

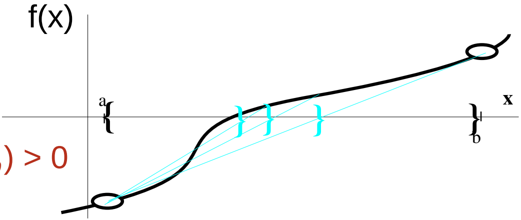
- Assume the function is linear between x_a and x_b
- Use the *linear zero* intersection $L(x_{zero}) = 0$ to estimate $f(x_{zero})$.



Basic Approach

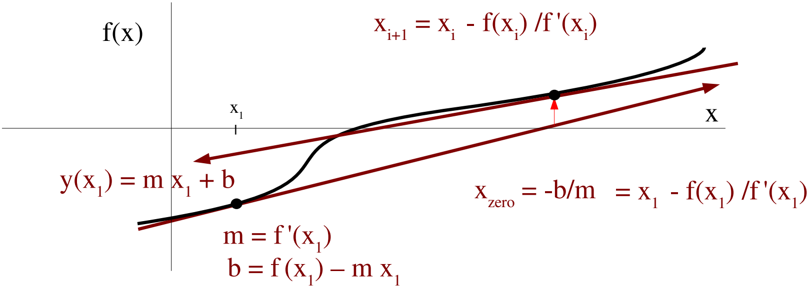
1. Calculate the slope $m = (f(x_b) - f(x_a))/(x_b - x_a)$
2. Calculate the intercept $b = f(x_a) - m\,x_a$
3. Determine linear $x_{zero} = -b/m$
    1. $f(x_{zero}) = 0$ stop
    2. $f(x_{zero}) < 0$ replace x_a with x_{zero}
    3. $f(x_{zero}) > 0$ replace x_b with x_{zero}
4. Repeat steps 1-3 as needed.

$f(x_a) < 0$ & $f(x_b) > 0$



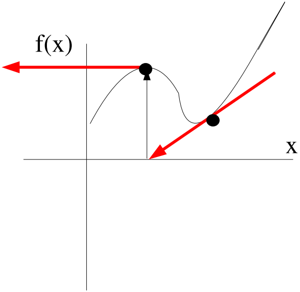
Newton-Raphson Method

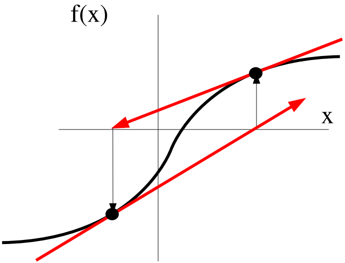
- Most Commonly Used Root-Finding Routine
  - Uses only one starting point
  - Calculates $f(x_{start})$ & $f'(x_{start})$
  - Uses the tangent line's zero crossing $L_T(x_{zero})=0$ to estimate $f(x_{zero})=0$



Newton-Raphson Method

Drawbacks of the method





Newton-Raphson Method

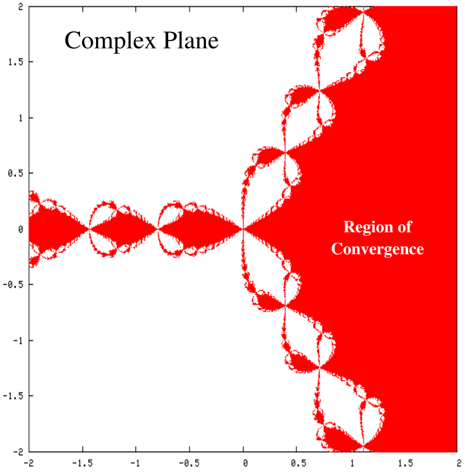
Also works for
Complex Functions

$f(z) = 0$

$f(z) = z^3 - 1 = 0$

Roots:

$z=1,\ z=e^{\pm 2\pi i/3}$



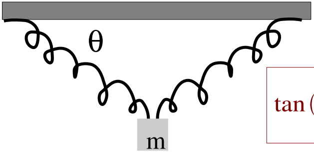
Newton-Raphson Method

$$z_{j+1} = z_j - (z_j^3 - 1)/(3z_j^2)$$

Look for Convergence

This Week's Project

Finding the Zero of a function



$$\tan(\theta) - \sin(\theta) - \frac{mg}{2kL_o} = 0$$

1. Example given for Bisection Method
2. Solve for False Position Method
3. Solve for Newton-Raphson Method