Ways of introducing coupled channels

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• **Meson Spectroscopy with the Crystal–Barrel Detector**
  1. Data sets with different initial states but the same final state particles
  2. Data sets with particles belonging to the same isospin multiplet
  3. Data sets with different final states

• **Baryon Spectroscopy with CB–ELSA and CLAS**
  – Photoproduction of two–pion final states

• **Comparing ways of introducing coupled channels**
  – CMB model, K–matrix, Breit–Wigner models
1) Different Initial States $\rightarrow$ Same Final State Particles

Example: $\bar{p}p \rightarrow \pi^0 \pi^0 \eta$

- Probability for S– and P–wave annihilation depends on target pressure
  $\Rightarrow$ Dalitz plot changes with pressure
1) Different Initial States → Same Final State Particles

In single–channel analysis of LH$_2$ data set, P–wave annihilation was neglected (Crystal Barrel Collaboration, C. Amsler et al., Phys. Lett. **B333** (1994) 277)

Why? Inclusion leads to dramatic increase of number of parameters in the fit ⇒ often leads to unphysical solutions

Solution: Adding gaseous hydrogen data yields additional information ⇒ Inclusion of P–wave annihilation possible

- Amplitudes for specific initial state remain the same for LH$_2$ → GH$_2$
  ⇒ Relative fraction of the different initial states changes
- Relative ratios for S– and P–wave annihilation taken from cascade models

Coupled–channel analysis supports evidence for the exotic state $\pi_1(1400)$ observed before in $\bar{p}d → \pi^- \pi^0 \eta \, p_{\text{spectator}}$
Observation of an exotic $J^{PC} = 1^{-+}$ state

Most prominent candidate for an exotic hybrid state: $\pi_1(1400)$

52567 events in the Dalitz plot

$$PWA \left\{ \begin{array}{l} ^3S_1 \rho^-(770), \rho^-(1450), a_2(1320) \\ ^1P_1 a_0(980), a_0(1450) \end{array} \right. $$

$\Rightarrow$ confirmation of exotic state in analysis of $\overline{p}p \rightarrow \pi^0 \left( \pi^0 \eta \right)$

(PhD thesis Mario Herz, Bonn 1997)


*Exotic $\eta \pi$ state in $\overline{p}d$ annihilation at rest into $\pi^- \pi^0 \eta p_{\text{spectator}}$*

in agreement with D.R. Thompson et al. (E852 collaboration)
2) Isospin Coupling Analyses: $\bar{p}N$ Annihilation into $3\pi$

Additional constraints by combining data sets with particles belonging to the same isomultiplet
- isospin weights
- different interference effects

Similar to what Curtis pointed out for $\gamma p \rightarrow p 4\pi$
- However, GlueX much more complicated!
2) **Isospin Coupling Analyses: $\bar{p}N$ Annihilation into $3\pi$**

Three of these Dalitz plots have been part of a coupled–channel analysis (still in preliminary state):

- **Restriction of $\bar{p}n$ to $I = 1$ initial states**
  \[ \Rightarrow 1S_0, 3P_1, \text{ and } 3P_2 \]

- In $\bar{p}p \to \pi^+\pi^-\pi^0$, $3S_1$ and $1P_1$ also possible

- **Amplitudes for annihilation can be coupled by the following isospin relations**

\[
\begin{align*}
A_{(\pi^+\pi^-)-S-wave} &= -\sqrt{2} \cdot A_{(\pi^0\pi^0)-S-wave} \\
A_{(\pi^+\pi^-)-P-wave} &= +1 \cdot A_{(\pi^-\pi^0)-P-wave} \\
A_{(\pi^+\pi^-)-D-wave} &= -\sqrt{2} \cdot A_{(\pi^0\pi^0)-D-wave}
\end{align*}
\]

- **Overall scaling factor in order to account for different contributions from initial S– and P–wave annihilation in LH$_2$ and LD$_2$**

  \[ \Rightarrow \text{ Data are reasonably well described!} \]

  (Parameters of the higher–mass vectors determined: $\rho(770)$, $\rho(1450)$, $\rho(1700)$)
2) Isospin Coupling Analyses: $\bar{p}N$ Annihilation into $3\pi$

Graphs showing distributions of $m(\pi\pi^0)$ for different isospin combinations: $\pi^- 2\pi^0$, $\pi^- 2\pi^0$, $\pi^+ \pi^- \pi^0$, $\pi^+ 2\pi^-$, $\pi^+ 2\pi^-$. Each graph illustrates the mass distribution in units of MeV for different isospin states.
3) Different Final States

\( 3\pi^0 \)

\( \pi^0 \pi^0 \eta \)

\( \pi^0 \eta \eta \)

\( m^2(\pi^0 \pi^0) \times 10^3 \)

\( m^2(\pi^0 \eta) \times 10^3 \)

\( m^2(\pi^0 \eta) \times 10^3 \)
3) Different Final States

Analysis by Crystal Barrel including $\bar{p}p \rightarrow 3\pi^0, 2\pi^0\eta, \pi^0\eta$

$+$ CERN–Munich scattering data


- Investigation of resonances in different decay modes
  $\Rightarrow$ Production of resonance with certain spectator does not depend on decay
- Masses, widths, and production strengths of resonances must be the same in all different channels
- Measurement of resonance coupling to one channel automatically determines corresponding inelasticity in the other channels
- $0^{++}$-wave described by using $3 \times 3$ $K$–matrix including couplings to the $\pi^0\pi^0$, $\eta\eta$, and $K\bar{K}$ channel
  $\Rightarrow$ Latter parameterizes inelasticity into all other open channels because no corresponding data was included in the analysis
  $\Rightarrow$ Allowed determination of couplings of $f_0(1500)$ to $\eta\eta$ relative to $\pi^0\pi^0$
The technique of Partial Wave Analysis

Measured intensity (incoherent sum over all possible $\bar{p}N$ initial states):

$$I = \sum_{JPC(\bar{p}N)} |A_{JPC}|^2$$

$$A_{JPC} = \sum_a \left( \sum_i \text{combinations} \right) \text{CG}_i \cdot H_{JPC,L,l}(\Theta) \cdot B_L(p) \cdot \hat{F}_l(q)$$

Angular dependence in terms of helicity amplitudes

$$H_{\lambda_1\lambda_2,M}(\theta, \phi) = D^J_{\lambda M}(\theta, \phi) \sum_{l s} \alpha_{l s} \langle J \lambda | l s 0 \lambda \rangle \langle s \lambda | s_1 s_2 \lambda_1, -\lambda_2 \rangle$$

In simplest case, $\hat{F}$ given as Breit–Wigner function

$$\Rightarrow \text{Parametrization of } \hat{F} \text{ in } K\text–\text{matrix formalism}$$

$$\hat{F} = (I - i\hat{K}\rho)^{-1} \hat{P}$$

Production and decay
Summary: Meson Spectroscopy

Procedure worked very well for Crystal Barrel data
⇒ Isobar model: series of successive two–body decays

Effects from
- direct three–body production
- t–channel exchange processes
- rescattering in the final state are neglected!
⇒ One has to be careful concerning the interpretations of broad poles found in the PWA!

What’s the applicability for the CB model?
- Radiative J/ψ decays and D decays, e.g. D→ Kππ (CLEO data) (✓)
- GlueX → would have to be extended! Much more complicated!
- What about baryons?
3–body final states are key for the discovery of missing states!

- Account for most of the cross section above $W \approx 1.7$ GeV
- 2–body final states largely explored $\Rightarrow$ No new states definitely found!
- Preliminary results: a) Polarization is key to unambiguous interpretation!

\[ \gamma p \rightarrow p \pi^0 \pi^0 \]
\[ \Rightarrow \]
\[ \gamma p \rightarrow p \pi^+ \pi^- \]
Advantages of $\pi^0\pi^0$ photoproduction:

- **No $\rho$ - amplitudes**
  - No diffractive $\rho$ - production
  - Reduces the number of possible $N^*$ decay amplitudes

- **No direct $\gamma p \rightarrow \Delta^{++}\pi^-$ production** (strong in $\gamma p \rightarrow \pi^+\pi^- p$)

Advantages of $\gamma p \rightarrow p \pi^+ \pi^-$:

- **Provides additional information on $N\rho$**

$\Rightarrow$ Idea: Combined analysis of both channels!
Rescattering, resonance mixing

Baryon decay (BW) with resonant final state, e.g. $N^* \rightarrow \rho p$

Meson decay (BW) with resonant final state, e.g. $D \rightarrow \rho \pi$

Looks too simple, what about these?

Also possible, in fact likely, called rescattering and coupled channels effects. Can happen many times!
**Definitions**

**Rescattering:**
When different states (same $Q$, $J\pi$) couple to each other with similar strength, any propagation must be a quantum-mechanical mixture of all states! (e.g. $\rho K \rightarrow K^*\pi$)

- Likely to be important for D decays

**Coupled Channel:**
If different final states are possible (same $Q$, $J\pi$), they must be treated on common basis (matrix) (e.g. $D \rightarrow K^*\pi^+ \rightarrow K^-\pi^0\pi^+$ versus $D \rightarrow K^*\pi^+ \rightarrow K^0_s\pi^-\pi^+$)

- Possibly important for D decays
How to analyse the data: Available Tools (models)

Two steps:

1. Partial wave decomposition (analysis):
   ⇒ Helicity Formalism, Covariant Formalism (operator formalism), etc.

2. Determination of resonance properties

<table>
<thead>
<tr>
<th>Model/Effect</th>
<th>Rescattering</th>
<th>Coupled Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breit–Wigner</td>
<td>Rudimentary</td>
<td>None</td>
</tr>
<tr>
<td>K–matrix</td>
<td>Good</td>
<td>Very good</td>
</tr>
<tr>
<td>CMB</td>
<td>Excellent</td>
<td>Excellent</td>
</tr>
</tbody>
</table>

Is this important?
⇒ Rescattering and coupled–channel effects known to be important for baryons!
**CMB model for baryons: scattering amplitude**

\[ T_{ab}^{CMB}(s) = \sum_{i,j} \sqrt{\rho_a(s)} f_a(s) \gamma_{ai} G_{ij}(s) \gamma_{jb} f_b \sqrt{\rho_b(s)} \]

\[ a = N\pi, \gamma N \]

\[ b = N\pi, N\eta, \Delta \pi, \pi N^*, N\rho, K\lambda, ... \]

\[ i, j = \text{res.} \]

- \( \gamma_{ai} \) is (real) coupling between \( a \) (e.g. \( N\pi \)) and \( i \) (e.g. \( S_{11}(1535) \))
- \( \rho_a \) is phase space for final state \( a \)
- \( f_a \) is a form factor (fixed, empirical)
- \( G_{ij} \) has all the action, full rescattering
- Every model has these elements in some form!

CMB model for baryons: *scattering amplitude*

\[ G_{ij} = G_{0}^{ij} + \sum_{k,l} G_{ik}^{0} \Sigma_{kl} G_{lj} \]

\[ = G_{0}^{ij} + \sum_{k,l} G_{ik}^{0} \Sigma_{kl} G_{lj}^{0} + \sum_{k,l,m,n} G_{ik}^{0} \Sigma_{kl} G_{lm}^{0} \Sigma_{mn} G_{nj}^{0} + ... \]

\[ G_{ij}^{0} = \frac{\epsilon_{i} \delta_{ij}}{s - s_{0,i}} \quad \text{Bare pole,} \]
\[ \epsilon_{i} = -1 \text{ for resonance} \]

\[ \Sigma_{kl} = \sum_{c} \gamma_{kc} \phi_{c} \gamma_{cl} \quad \text{Dyson equation generates width!!} \]

- Fitting constants \( s_{0} \) and \( \gamma_{i,a} \)
  - \( \Rightarrow \) 1 bare mass for every resonance and one coupling constant for each open channel (e.g. \( N\pi \))
- For baryons, up to 9 constants per resonance (many)
- Unitarity for 2–body and quasi–2–body final states
- Analyticity through dispersion relations
Relationship of CMB to K–matrix model

\[ T_{ab}^{\text{CMB}}(s) = \sum_{i,j} \sqrt{\rho_a(s)} f_a(s) \gamma_{ai} G_{ij}(s) \gamma_{jb} f_b \sqrt{\rho_b(s)} \]

\[ K_{ab}^i(s) = \sqrt{\rho_a(s)} f_a(s) \gamma_{ai} G_{0ij}(s) \gamma_{jb} f_b \sqrt{\rho_b(s)} \]

\[ K_{ab} = \sum_{i \in \text{res}} K_{ab}^i K_{ab}^{\text{nonres}} \]

\[ T_{ab} = \frac{K_{ab}}{1 - i K_{ab}} \]

N.B. This is very similar to Chung, Klempt (Z. Phys. (1995))
Case study: $S_{11}$ states near $N\eta$ threshold

Notation: $L_{2I,2J}$, where

$L$ = orbital angular momentum as if $N\pi$ (e.g. S, P, D, etc.)
$I$ = isospin of $N^*$ resonance ($I = 1/2, 3/2$)
$J$ = total angular momentum of $N^*$ ($J = 1/2, 3/2, 5/2$, etc.)

For this reason, $S_{11}$ is an $L = 0$ state with $I = J = 1/2$, parity = −

- $S_{11}$ refers to $L = 0$ in the $N\pi$ system
- P–wave excitation of 1 quark in the NRQM

2 states, $S_{11}(1535)$ and $S_{11}(1650)$, and each couples mostly to $N\pi$ and $N\eta$!
What’s the nature of the $S_{11}(1535)$?

Why does it decay strongly into $N\eta$ and $S_{11}(1650)$ does not?

- Two states $S_{11}$ have appreciable mixing ($\approx 30^\circ$) 
  (N. Isgur and G. Karl, Phys. Lett. 72B (1977) 109.)
- Phenomenological fit to baryon decays ($\approx 30^\circ$)
- Coupled $\Sigma K-p\eta$ effect (Kaiser, Siegel and Weise) 
  $\Rightarrow$ No genuine 3-quark resonance required
- Amplitude analysis (G. Hoehler) 
  $\Rightarrow$ No pole is needed for $N(1535)S_{11}$
- Quark-diquark structure (Glozman and Riska)

$\Rightarrow$ Extraction of resonance properties important!
## Interpretation of $S_{11}(1535)$

<table>
<thead>
<tr>
<th></th>
<th>$\Gamma_{\text{full}}$ (MeV)</th>
<th>BR$_{N\pi}$</th>
<th>$A_{1/2}$</th>
<th>comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIT(1535)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VPI(96)</td>
<td>105</td>
<td>0.31</td>
<td>60 ± 15</td>
<td>$N\pi \rightarrow N\pi$</td>
</tr>
<tr>
<td>Drechsel(99)</td>
<td>80</td>
<td>0.40</td>
<td>67</td>
<td>$\gamma p \rightarrow p\pi$</td>
</tr>
<tr>
<td>Krusche(97)</td>
<td>212</td>
<td>0.45</td>
<td>120 ± 20</td>
<td>$\gamma p \rightarrow p\eta$</td>
</tr>
<tr>
<td>Pitt–ANL(00)</td>
<td>126</td>
<td>0.34</td>
<td>87 ± 3</td>
<td>All</td>
</tr>
<tr>
<td>PDG</td>
<td>100 – 200</td>
<td>0.35 – 0.55</td>
<td>90 ± 30</td>
<td>Averaging</td>
</tr>
</tbody>
</table>

1. If we use $N\pi$ or $N\eta$ data, we get different answers!!
2. If we use coupled-channel model, we get intermediate result.
We expect rescattering and resonance interference to matter!

- Since $S_{11}(1535) (\Gamma = 130\text{MeV})$ and $S_{11}(1650) (\Gamma = 200\text{MeV})$ overlap, we must consider quantum mechanical interference.
- Since $S_{11}(1535)$ decays roughly equally to $N\pi$ and $N\eta$, we must consider coupled-channel effects.

For this reason, design a little study that tests model dependence of
(a) Breit–Wigner versus (b) K–matrix versus (c) CMB model:
- Use identical data input
- Use models as close as possible to others

⇒ Work of Alvin Kiswandhi at FSU
- Error bars are stat., syst.
- Traditionally unitless
- Argand plot is $\text{Im } T(W)$ versus $\text{Re } T(W)$
- Sign of resonance: Peak in $\text{Im } T(W)$ Zero in $\text{Re } T(W)$
Two–resonance, two–channel CMB model

- Fit not perfect due to missing channels ⇒ e.g. $\Delta\pi$
- Non–res. important close to $N\pi$ threshold
- Errors of many parameters within errors of full model

--- total amp.

... resonant amp.

--- non–resonant
K–matrix model

- Fit very similar to CMB model
- At $W \approx 1.6$ GeV, difference in $\text{Im } T(N\pi \rightarrow N\pi)$
Breit–Wigner model
- Extra phase necessary to get good fit
- Fits with BW best and worse
- Difference in res. versus nonres. with respect to CMB and K–matrix!

--- total amp.
• • • resonant amp.
--- non–resonant
## Results

<table>
<thead>
<tr>
<th>Model</th>
<th>CMB</th>
<th>K</th>
<th>BW</th>
<th>PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>S^{11}_{11}(1535)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass [MeV]</td>
<td>1532 ± 2</td>
<td>1533 ± 1</td>
<td>1538 ± 2</td>
<td>1520 – 1555</td>
</tr>
<tr>
<td>Width [MeV]</td>
<td>124 ± 6</td>
<td>119 ± 3</td>
<td>130 ± 6</td>
<td>100 – 200</td>
</tr>
<tr>
<td>BR$_{N\pi}$ [%]</td>
<td>30 ± 2</td>
<td>33 ± 1</td>
<td>38 ± 1</td>
<td>35 – 55</td>
</tr>
<tr>
<td><strong>S^{11}_{11}(1650)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass [MeV]</td>
<td>1685 ± 2</td>
<td>1682 ± 2</td>
<td>1647 ± 2</td>
<td>1640 – 1680</td>
</tr>
<tr>
<td>Width [MeV]</td>
<td>168 ± 6</td>
<td>184 ± 5</td>
<td>109 ± 5</td>
<td>145 – 190</td>
</tr>
<tr>
<td>BR$_{N\pi}$ [%]</td>
<td>79 ± 2</td>
<td>75 ± 1</td>
<td>51 ± 1</td>
<td>55 – 90</td>
</tr>
<tr>
<td>$\chi^2/N$</td>
<td>3.8</td>
<td>3.7</td>
<td>5.0</td>
<td></td>
</tr>
</tbody>
</table>

Errors shown come from Minuit
Interpretation and Conclusion

- Results not far from PDG for CMB and K–matrix
  ⇒ Truncated model is ok
- Range of model results comparable to PDG error range
  ⇒ PDG averages over various model results, i.e. it includes both statistical and systematic errors
- BW results are not close to CMB and K–matrix
  ⇒ Lack of theoretical constraints is problem and requires ad–hoc parameters to fit the real data

- CMB and K–matrix results 1–2σ apart
  ⇒ Is this large or small?
- CMB model better constrained theoretically: Should it be preferred model?
  ⇒ Simplified dynamics of the K–matrix model has practical advantage
- How do we treat multi–particle final states?
Basic resonance shapes are identical

- $M = 1710$ MeV
- $\Gamma = 215$ MeV
An isolated resonance – $D_{15}(1675)$

Very inelastic, peak in Im T at 0.4 (unitary bound is 1.0)
Low smooth background, no thresholds nearby
Strong signal in $\pi N \rightarrow \pi \Delta$ also