Some Scattering Stuff

Volker Credé

Florida State University
Tallahassee, FL

September 24, 2013

Outline

Scattering Processes: a + b $\rightarrow$ c + d

Elastic Scattering (target remains in ground state):
(particles before and after the scattering process are identical)

$$a + b \rightarrow a' + b'$$

Inelastic Scattering (target will be excited):

$$\begin{aligned} a + b \rightarrow a' + b^* & \\ b^* & \rightarrow c + d \end{aligned}$$

Rutherford Scattering

Scattering process of a particle with energy E and charge ze off an atomic nucleus with charge Ze is described by:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} = \frac{(zZe^2)^2}{(4\pi\epsilon_0)^2 \cdot (4E_{\text{kin}})^2 \sin^4\frac{\theta}{2}}$$

If the nucleus is sufficiently heavy and the energy of the projectile is not too big, the recoil can be ignored and the energy E and the magnitude of the momentum $\boldsymbol{p}$ are the same before and after the scattering.

Form Factor

Scattering off an extended charge distribution with finite volume V:

$$\frac{d\sigma}{d\Omega} = \frac{V^2 E'^2}{(2\pi)^2 (\hbar c)^4} \left| \langle \psi_f \,|\, \mathcal{H}_{\text{int}} \,|\, \psi_i \rangle \right|^2 .$$

If Ze small, (ψ_i, ψ_f) plane waves: $\psi_i = \frac{1}{\sqrt{V}} \, \mathrm{e}^{i\boldsymbol{px}/\hbar} \quad \psi_f = \frac{1}{\sqrt{V}} \, \mathrm{e}^{i\boldsymbol{p'x}/\hbar}$.

If $\rho(\boldsymbol{x}) = Ze\, f(\boldsymbol{x})$ charge distribution, $\boldsymbol{q} = \boldsymbol{p} - \boldsymbol{p'}$ momentum transfer:

$$\langle \psi_f \,|\, \mathcal{H}_{\text{int}} \,|\, \psi_i \rangle = \frac{e\,\hbar^2}{\epsilon_0 \cdot V\, |\boldsymbol{q}|^2} \int \rho(\boldsymbol{x})\, \mathrm{e}^{i\boldsymbol{qx}/\hbar}\, \mathrm{d}^3x$$

$$= \frac{Z \cdot 4\pi\alpha\hbar^3 c}{|\boldsymbol{q}|^2 \cdot V} F(\boldsymbol{q}) \quad \text{and} \quad F(\boldsymbol{q}) = \int \mathrm{e}^{i\boldsymbol{qx}/\hbar} f(\boldsymbol{x})\, \mathrm{d}^3x$$

$F(\boldsymbol{q})$ is the form factor of the charge distribution (Fourier transform).

Mott Scattering

Scattering of an electron off a target considering the electron spin:

$$\left(\frac{d\sigma}{d\Omega}\right)^*_{\mathrm{Mott}} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Rutherford}} \cdot \left(1 - \beta^2 \sin^2\frac{\theta}{2}\right) \quad \text{and } \beta = \frac{v}{c}$$

$$\approx \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Rutherford}} \cdot \cos^2\frac{\theta}{2} \quad \text{for } \beta \to 1$$

The recoil of the target is still not considered.

Form factor measurement:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{exp.}} = \left(\frac{d\sigma}{d\Omega}\right)^*_{\mathrm{Mott}} \cdot |F(\boldsymbol{q}^2)|^2 .$$

Outline

Form Factors of the Nucleon

Size of the nucleon is about 0.8 fm and a few hundred MeV are needed to study it. Thus, incoming energies are similar to the mass of the nucleon and the target recoil cannot be ignored:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \left(\frac{d\sigma}{d\Omega}\right)^{*}_{\text{Mott}} \cdot \frac{E'}{E} .$$

Energy loss of the electron due to the recoil cannot be ignored anymore (4-vector momentum transfer: $Q^2 = -q^2$). In addition o the interaction between the charge of the electron and the charge of the nucleus, interaction between electron current and magnetic moment of the nucleon needed.

Magnetic moment of spin-1/2 particle: $\mu = g \cdot \frac{e}{2M} \cdot \frac{\hbar}{2}$.

($g = 2$ for Dirac particles without inner structure)

Rosenbluth Formula

Scattering of an electron off a nucleon:

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \cdot \left[\frac{G_{\mathrm{E}}^2(Q^2) + \tau G_{\mathrm{M}}^2(Q^2)}{1+\tau} + 2\tau G_{\mathrm{M}}^2(Q^2)\tan^2\frac{\theta}{2}\right],$$

where $G_{\mathrm{E}}(Q^2)$ and $G_{\mathrm{M}}(Q^2)$ are the electric and magnetic form factors that depend on Q^2. For the case $Q^2 \to 0$:

$$G_{\mathrm{E}}^{\mathrm{p}}(Q^2=0) = 1 \qquad G_{\mathrm{E}}^{\mathrm{n}}(Q^2=0) = 0$$

$$G_{\mathrm{M}}^{\mathrm{p}}(Q^2=0) = 2.79 \qquad G_{\mathrm{M}}^{\mathrm{n}}(Q^2=0) = -1.91$$

Magnetic moments of the nucleons ($\mu_{\mathrm{N}} = \frac{e\hbar}{2M_{\mathrm{p}}}$):

$$\mu_{\mathrm{p}} = \frac{g_{\mathrm{p}}}{2}\mu_{\mathrm{N}} = +2.79\cdot\mu_{\mathrm{N}} \quad \text{and} \quad \mu_{\mathrm{n}} = \frac{g_{\mathrm{n}}}{2}\mu_{\mathrm{N}} = -1.91\cdot\mu_{\mathrm{N}}$$

Outline

Structure Functions

In deep inelastic scattering, the dynamics of the process is described by form factors. However, they are usually called structure functions.

Elastic scattering: $2M\nu - Q^2 = 0$.
(ν is energy transfer, Q^2 is 4-momentum transfer, $W = M$)

Inelastic scattering: $2M\nu - Q^2 > 0$.

➜ Structure functions and cross sections are then functions of two independent parameters: (E', θ) or (Q^2, ν).

Instead of the Rosenbluth formula:

$$\frac{d^2\sigma}{d\Omega\, dE'} = \left(\frac{d\sigma}{d\Omega}\right)^{*}_{\text{Mott}} \left[W_2(Q^2, \nu) + 2W_1(Q^2, \nu) \tan^2\frac{\theta}{2} \right].$$

Structure Functions

Bjorken scale variable is a measure for the inelasticity of the process:

$$x := \frac{Q^2}{2P\,q} = \frac{Q^2}{2M\,\nu}\,,$$

where $x = 1$ for elastic scattering: $W = M$ and $2M\nu - Q^2 = 0$.
For inelastic processes: $0 < x < 1$.

Instead of the two structure functions $W_1\,(Q^2,\,\nu)$ and $W_2\,(Q^2,\,\nu)$, the following two dimensionless structure functions are used:

$$F_1\,(x,\,Q^2) = M\,c^2\,W_1\,(Q^2,\,\nu)\,,$$

$$F_2\,(x,\,Q^2) = \nu\,W_2\,(Q^2,\,\nu)\,.$$

For fixed values of x, they depend little on Q^2.

Structure Functions

If the structure functions do not depend on Q^2, the scattering occurs off a point charge. Since the nucleon is an extended object, it has substructure:

- The structure function F_1 stems from the magnetic interaction. It vanishes in the scattering off a spin-0 particle.
- For Dirac particles with spin-1/2: $2x\,F_1(x) = F_2(x)$.

This *Callan-Gross Relation* is fulfilled very well for the scattering off the nucleon. ➜ The point-like constituents of the nucleon have spin 1/2.