Quark Models of Excited Baryons

How are excited N and Δ states constructed in the constituent quark model?

What does this construction imply for the N and Δ spectrum?

How is this modified when we add strange quarks?

Is there a better way to construct the basis if we want to extend it?

Can we deal with momentum and coordinate-space operators in H?

Sample spectrum

Constructing Excited Baryon States

Think first about baryons made of light (u & d) quarks

Quarks have color, flavor, spin and spatial degrees of freedom

If strong interactions of u and d indistinguishable (isospin) then we need total exchange antisymmetry in the wave f'n:

$$\Psi = C_A \sum \psi \, \chi \, \phi$$

$$C_A = \frac{1}{\sqrt{6}}(rgb + gbr + brg - rbg - grb - bgr)$$

$$\psi(\vec{r}_1, \vec{r}_2, \vec{r}_3)$$

$$\chi(s_1, s_2, s_3),\ s_j = \uparrow, \downarrow$$

$$\phi(i_1, i_2, i_3),\ i_j = u,\ d$$

Sum performed to provide total symmetry & good J

Combining representations of S_3 exchange group

$$S_3 = \{\mathbf{1}, (12), (13), (23), (123), (132)\}$$

3x2x1 ways of arranging three objects

Representations are totally symmetric S, totally antisymmetric A, and a mixed symmetry pair $\{M^\rho, M^\lambda\}$ that transform into each other

Action of group elements, e.g.:

$$(12)M^\rho = -M^\rho, \; (12)M^\lambda = M^\lambda$$

$$(13)M^\rho = M^\rho/2 - \sqrt{3}M^\lambda/2$$

$$(13)M^\lambda = -\sqrt{3}M^\rho/2 - M^\lambda/2$$

Combining representations of S_3 exchange group

Combining two representations: $S_a \otimes S_b = S_{ab}$

$$S_a \otimes A_b = A_{ab}$$

$$A_a \otimes A_b = S_{ab}$$

$$\frac{1}{\sqrt{2}}(M_a^\rho \otimes M_b^\rho + M_a^\lambda \otimes M_b^\lambda) = S_{ab}$$

$$\frac{1}{\sqrt{2}}(M_a^\rho \otimes M_b^\lambda - M_a^\lambda \otimes M_b^\rho) = A_{ab}$$

$$\frac{1}{\sqrt{2}}(M_a^\rho \otimes M_b^\lambda + M_a^\lambda \otimes M_b^\rho) = M_{ab}^\rho$$

$$\frac{1}{\sqrt{2}}(M_a^\rho \otimes M_b^\rho - M_a^\lambda \otimes M_b^\lambda) = M_{ab}^\lambda$$

Ground states

Quark-spin wave functions: spin 1/2 have mixed symmetry

$$\chi^{\rho} = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \qquad \chi^{\lambda} = -\frac{1}{\sqrt{6}}(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)$$

spin 3/2 are symmetric $\chi^S = \uparrow\uparrow\uparrow$

Flavor wave functions: N flavor, e.g. proton

$$\phi^{\rho} = \frac{1}{\sqrt{2}}(udu - duu) \qquad \phi^{\lambda} = \frac{1}{\sqrt{6}}(udu + duu - 2uud)$$

Δ flavor, e.g. $\phi^S_{\Delta^+} = \frac{1}{\sqrt{3}}(uud + udu + duu)$

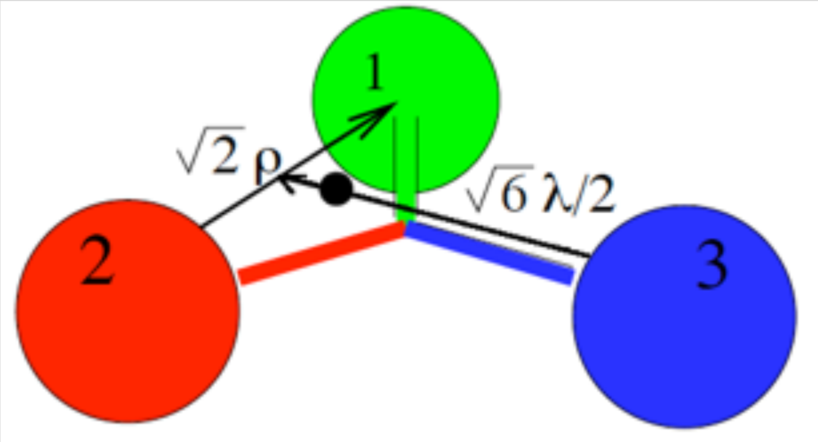
For ground state nucleon and Δ, symmetric sums

$$\frac{1}{\sqrt{2}}\left(\chi^{\rho}\phi^{\rho} + \chi^{\lambda}\phi^{\lambda}\right) \qquad \chi^S\phi^S$$

Ground-state wave functions

Spatial wave functions: separate CM motion by using Jacobi coordinates

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \quad \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$$



For ground state nucleon and Δ(1232),
example symmetric spatial wave f'n (exact for HO potential) with $L^P=0^+$

$$\psi^S = \frac{\alpha^3}{\pi^{\frac{3}{2}}} exp\left\{-\alpha^2\left(\rho^2 + \lambda^2\right)\right\}$$

Recall ρρ + λλ ~ S

$$|N^2S_S\tfrac{1}{2}^+\rangle = C_A\psi^S_{00}\frac{1}{\sqrt{2}}(\phi^\rho_N\chi^\rho_{\frac{1}{2}} + \phi^\lambda_N\chi^\lambda_{\frac{1}{2}})$$

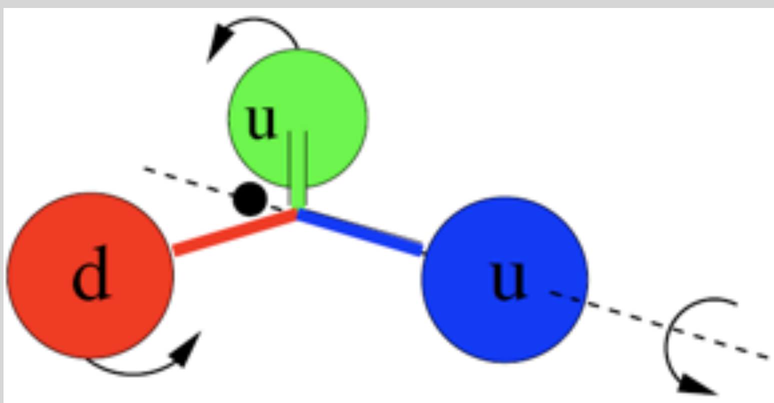
$$|\Delta^4S_S\tfrac{3}{2}^+\rangle = C_A\phi^S_\Delta\psi^S\chi^S_{\frac{3}{2}},$$

N and Δ, in [56,0^+]

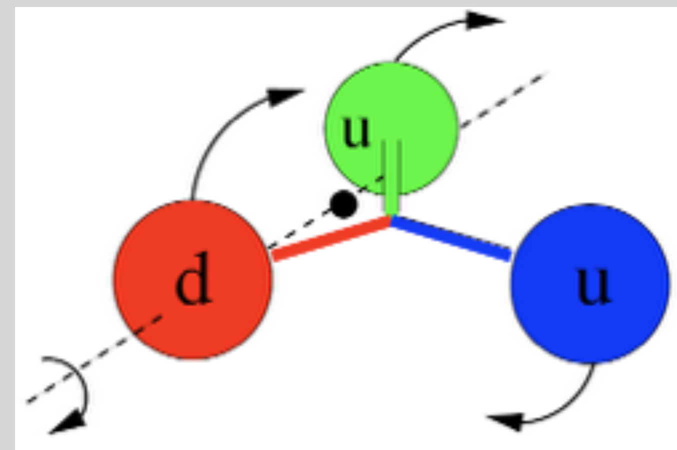
|Flavor L $_{\text{symmetry}}$ S J^P>

Orbital excited-state wave functions

Simplest (lowest energy) excited states have one unit of orbital angular momentum in either ρ or λ oscillator



$l_\rho=1$



$l_\lambda=1$

$$\psi^{\rho}_{1m} = \left\{\rho_-, \sqrt{2}\rho_0, -\rho_+\right\}\psi^S$$

$$\psi^{\lambda}_{1m} = \left\{\lambda_-, \sqrt{2}\lambda_0, -\lambda_+\right\}\psi^S$$

$L^P = 1^- \otimes \{S = \frac{1}{2} \text{ or } S = \frac{3}{2}\}$ gives $J^P = 1/2^-, 3/2^-, 5/2^-$

mixed symmetry spatial wave f'ns

mixed symmetry (N) or symmetric (Δ) flavor

mixed symmetry (S=1/2) or symmetric (S=3/2) spin

Orbital excited-state wave functions

Quark-spin 3/2 Nucleon states

$$|N^4P_M(\tfrac{1}{2}^-,\tfrac{3}{2}^-,\tfrac{5}{2}^-)\rangle \;=\; C_A\chi^S_{\frac{3}{2}}\frac{1}{\sqrt{2}}(\phi^\rho_N\psi^\rho_{1M}+\phi^\lambda_N\psi^\lambda_{1M})$$

Quark-spin 1/2 Nucleon states

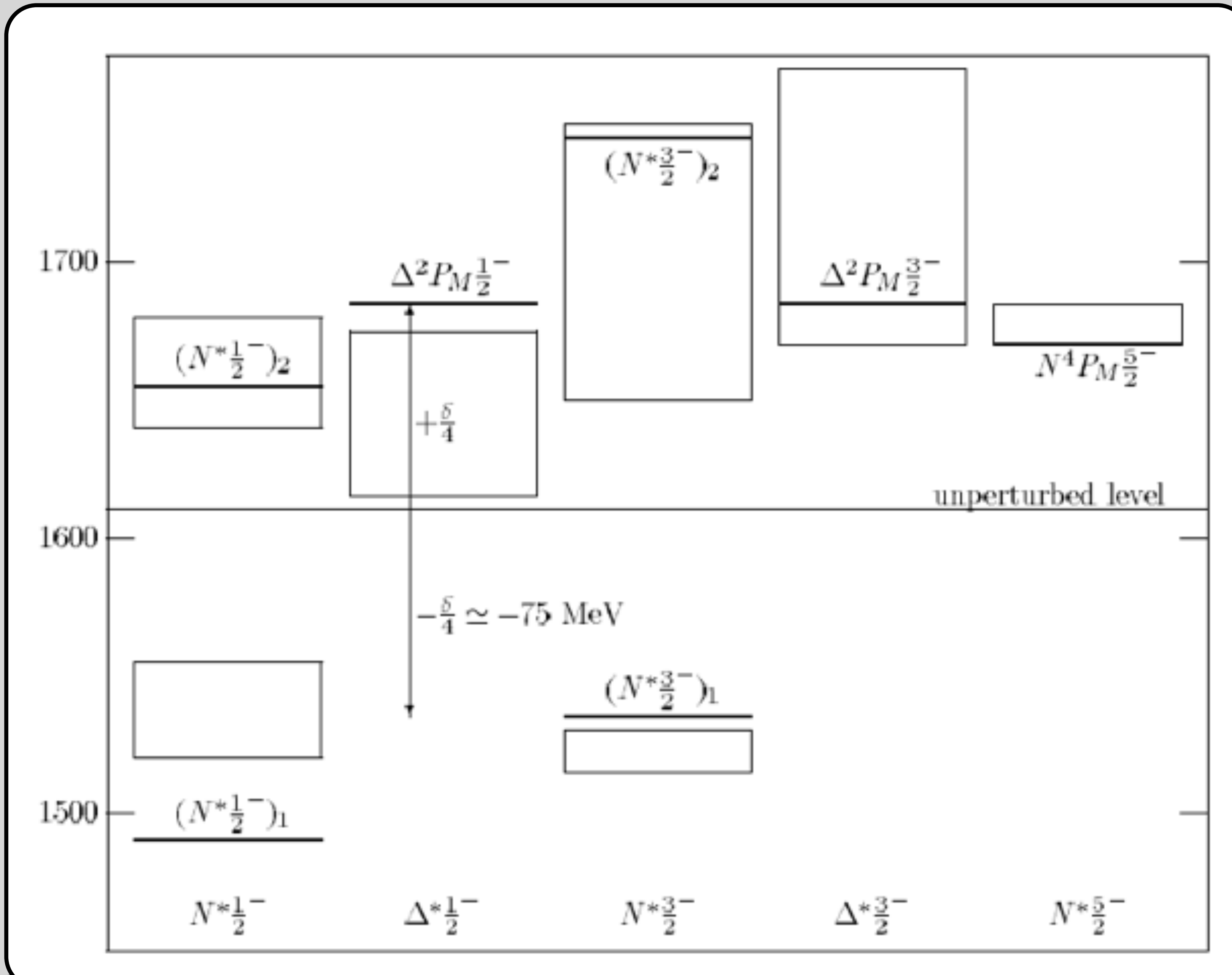
$$|N^2P_M(\tfrac{1}{2}^-,\tfrac{3}{2}^-)\rangle \;=\; C_A\frac{1}{2}\left\{\phi^\rho_N[\psi^\rho_{1M}\chi^\lambda_{\frac{1}{2}}+\psi^\lambda_{1M}\chi^\rho_{\frac{1}{2}}]+\phi^\lambda_N[\psi^\rho_{1M}\chi^\rho_{\frac{1}{2}}-\psi^\lambda_{1M}\chi^\lambda_{\frac{1}{2}}]\right\}$$

Quark-spin 1/2 Δ states

$$|\Delta^2P_M(\tfrac{1}{2}^-,\tfrac{3}{2}^-)\rangle \;=\; C_A\phi^S_\Delta\frac{1}{\sqrt{2}}(\psi^\rho_{1M}\chi^\rho_{\frac{1}{2}}+\psi^\lambda_{1M}\chi^\lambda_{\frac{3}{2}})$$

Lowest orbital excitations

N. Isgur & G. Karl



Sample spectrum: boxes are masses, with uncertainties, from analyses of data (all states seen)

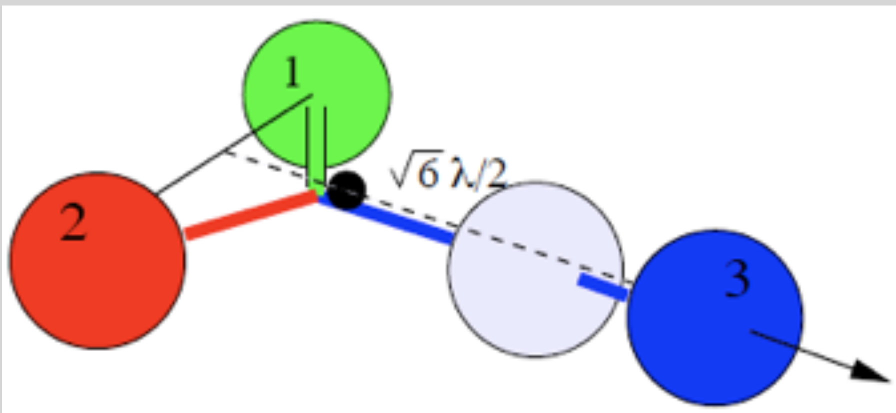
$\delta = M_\Delta - M_N$
~ 300 MeV

Degeneracy broken by tensor interaction

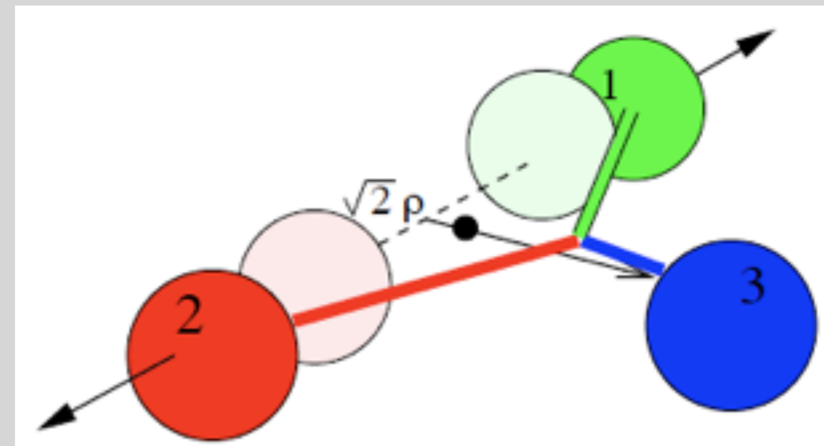
Also mixes S=1/2, S=3/2 wave f'ns, can explain N(1535) to Nη

Radial (doubly-excited) excitations

Radial excitations have a radial node in either the ρ or λ oscillator, with $L^P=0^+$



$n_\rho=1$



$n_\lambda=1$

$$\sqrt{\tfrac{2}{3}}\left\{\tfrac{3}{2}-\alpha^2\lambda^2\right\}\psi^S$$

$$\sqrt{\tfrac{2}{3}}\left\{\tfrac{3}{2}-\alpha^2\rho^2\right\}\psi^S$$

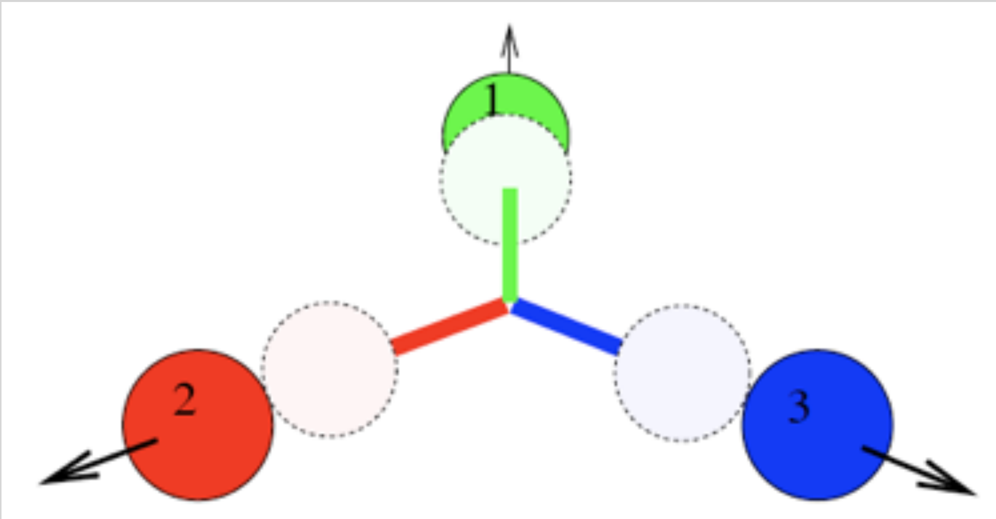
not states of definite symmetry

Additional true 3-body state with $l_\rho=1$ and $l_\lambda=1$ combined to $L^P=0^+$

$$\psi_{00}^{M^\rho}=-\frac{2}{\sqrt{3}}\frac{\alpha^5}{\pi^{\frac{3}{2}}}\boldsymbol{\rho}\cdot\boldsymbol{\lambda}e^{-\frac{\alpha^2}{2}(\rho^2+\lambda^2)}$$

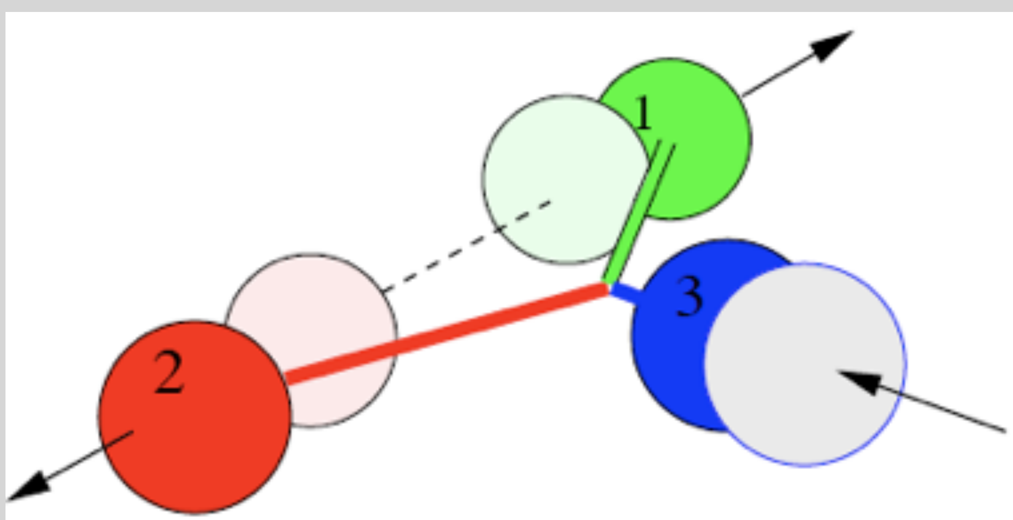
has M^ρ symmetry

States of definite symmetry



breathing (lowest frequency) mode

$$\psi_{00}^{S'} = \sqrt{\frac{2}{3}}\frac{\alpha^5}{\pi^{\frac{3}{2}}}\frac{1}{\sqrt{2}}(\rho^2 - \frac{3}{2\alpha^2} + \lambda^2 - \frac{3}{2\alpha^2})e^{-\frac{\alpha^2}{2}(\rho^2+\lambda^2)}$$



antisymmetric (higher frequency) mode

$$\psi_{00}^{M^\lambda} = \sqrt{\frac{2}{3}}\frac{\alpha^5}{\pi^{\frac{3}{2}}}\frac{1}{\sqrt{2}}(\rho^2 - \lambda^2)e^{-\frac{\alpha^2}{2}(\rho^2+\lambda^2)}$$

paired with

$$\psi_{00}^{M^\rho} = -\frac{2}{\sqrt{3}}\frac{\alpha^5}{\pi^{\frac{3}{2}}}\boldsymbol{\rho}\cdot\boldsymbol{\lambda}e^{-\frac{\alpha^2}{2}(\rho^2+\lambda^2)}$$

L=0 positive-parity excited-state wave functions

Radial recurrences of N ($J^P=1/2^+$) and Δ ($J^P=3/2^+$) ground states

$$[56',0^+]: \quad |N^2S_{S'}\tfrac{1}{2}^+\rangle = C_A\psi^{S'}\frac{1}{\sqrt{2}}(\phi_N^\rho\chi_{\frac{1}{2}}^\rho + \phi_N^\lambda\chi_{\frac{1}{2}}^\lambda)$$

$$|\Delta^4S_{S'}\tfrac{3}{2}^+\rangle = C_A\phi_\Delta^S\psi^{S'}\chi_{\frac{3}{2}}^S$$

Other $L^P=0^+$ states ($J^P=1/2^+$, $3/2^+$)

$$[70,0^+]: \quad |N^4S_M\tfrac{3}{2}^+\rangle = C_A\chi_{\frac{3}{2}}^S\frac{1}{\sqrt{2}}(\phi_N^\rho\psi_{00}^\rho + \phi_N^\lambda\psi_{00}^\lambda)$$

$$|\Delta^2S_M\tfrac{1}{2}^+\rangle = C_A\phi_\Delta^S(\psi_{00}^\rho\chi_{\frac{1}{2}}^\rho + \psi_{00}^\lambda\chi_{\frac{1}{2}}^\lambda)$$

$$|N^2S_M\tfrac{1}{2}^+\rangle = C_A\frac{1}{2}\left\{\phi_N^\rho[\psi_{00}^\rho\chi_{\frac{1}{2}}^\lambda + \psi_{00}^\lambda\chi_{\frac{1}{2}}^\rho] + \phi_N^\lambda[\psi_{00}^\rho\chi_{\frac{1}{2}}^\rho - \psi_{00}^\lambda\chi_{\frac{1}{2}}^\lambda]\right\}$$

Doubly excited-state wave functions with L=1,2

Can also form spatial wave f'ns with $L^P=1^+$ and 2^+, from

$$l_\rho = 0 \otimes l_\lambda = 2,\ l_\rho = 2 \otimes l_\lambda = 0, \text{ and } l_\rho = 1 \otimes l_\lambda = 1$$

$$\psi_{11}^{A} = -\frac{\alpha^5}{\pi^{\frac{3}{2}}}(\rho_+\lambda_0 - \rho_0\lambda_+)e^{-\frac{\alpha^2}{2}(\rho^2+\lambda^2)}$$

Antisymmetric $L^P=1^+$

$$\begin{aligned}
\psi_{22}^{S} &= \frac{1}{\sqrt{2}}\frac{\alpha^5}{\pi^{\frac{3}{2}}}(\rho_+^2 + \lambda_+^2)e^{-\frac{\alpha^2}{2}(\rho^2+\lambda^2)} \\
\psi_{22}^{M\lambda} &= \frac{1}{\sqrt{2}}\frac{\alpha^5}{\pi^{\frac{3}{2}}}(\rho_+^2 - \lambda_+^2)e^{-\frac{\alpha^2}{2}(\rho^2+\lambda^2)} \\
\psi_{22}^{M\rho} &= \frac{\alpha^5}{\pi^{\frac{3}{2}}}\rho_+\lambda_+ e^{-\frac{\alpha^2}{2}(\rho^2+\lambda^2)},
\end{aligned}$$

Symmetric and mixed symmetry $L^P=2^+$

L=1,2 positive-parity excited-state wave functions

$L^P=2^+$ states with S=1/2, 3/2 All J values up to 7/2

$$[56,2^+]: \quad |\Delta^4 D_S(\tfrac{1}{2}^+,\tfrac{3}{2}^+,\tfrac{5}{2}^+,\tfrac{7}{2}^+)\rangle = C_A\phi^S_\Delta\psi^S_{2M}\chi^S_{\frac{3}{2}},$$

$$|N^2 D_S(\tfrac{3}{2}^+,\tfrac{5}{2}^+)\rangle = C_A\psi^S_{2M}\frac{1}{\sqrt{2}}(\phi^\rho_N\chi^\rho_{\frac{1}{2}}+\phi^\lambda_N\chi^\lambda_{\frac{1}{2}})$$

$$[70,2^+]: \quad |N^4 D_M(\tfrac{1}{2}^+,\tfrac{3}{2}^+,\tfrac{5}{2}^+,\tfrac{7}{2}^+)\rangle = C_A\chi^S_{\frac{3}{2}}\frac{1}{\sqrt{2}}(\phi^\rho_N\psi^\rho_{2M}+\phi^\lambda_N\psi^\lambda_{2M})$$

$$|\Delta^2 D_M(\tfrac{3}{2}^+,\tfrac{5}{2}^+)\rangle = C_A\phi^S_\Delta(\psi^\rho_{2M}\chi^\rho_{\frac{1}{2}}+\psi^\lambda_{2M}\chi^\lambda_{\frac{1}{2}})$$

$$|N^2 D_M(\tfrac{3}{2}^+,\tfrac{5}{2}^+)\rangle = C_A\frac{1}{2}\left\{\phi^\rho_N[\psi^\rho_{2M}\chi^\lambda_{\frac{1}{2}}+\psi^\lambda_{2M}\chi^\rho_{\frac{1}{2}}]+\phi^\lambda_N[\psi^\rho_{2M}\chi^\rho_{\frac{1}{2}}-\psi^\lambda_{2M}\chi^\lambda_{\frac{1}{2}}]\right\}$$

$L^P=1^+$ states with S=1/2 J =1/2, 3/2

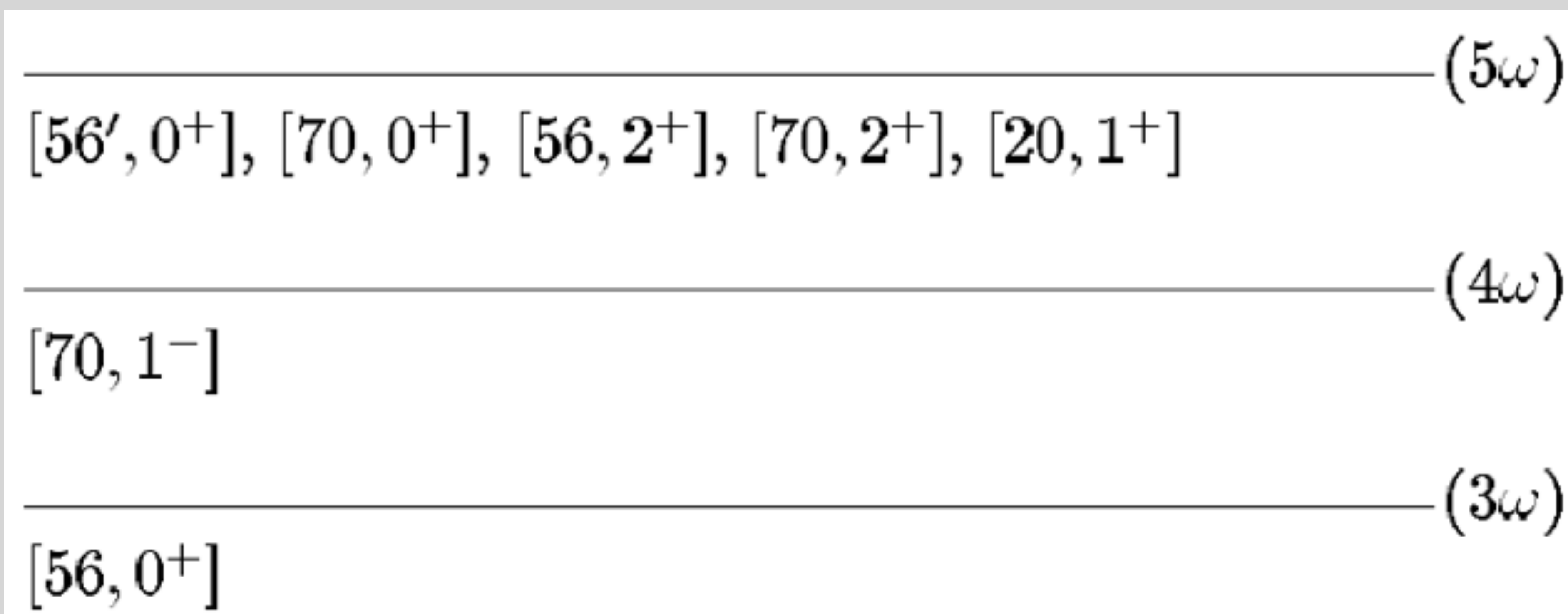
$$[20,1^+]: \quad |N^2 P_A(\tfrac{1}{2}^+,\tfrac{3}{2}^+)\rangle = C_A\psi^A_{1M}\frac{1}{\sqrt{2}}(\phi^\rho_N\chi^\lambda_{\frac{1}{2}}-\phi^\lambda_N\chi^\rho_{\frac{1}{2}})$$

Pattern of splitting of positive-parity excited states

Isgur & Karl: first order perturbation theory in anharmonicity $U = \Sigma_{i<j} U_{ij}$

(E.g. $U_{ij} = br_{ij} - 3Kr_{ij}^2/2$, $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$, but don't need to specify)

Starting point



Positive-parity excited-state spectrum

In [56,0+] ground states

$$\langle \phi^S_\Delta \psi^S_{00} \chi^S_{\frac{3}{2}} | \sum_{i<j} U_{ij} | \phi^S_\Delta \psi^S_{00} \chi^S_{\frac{3}{2}} \rangle = 3 \langle \psi^S_{00} | U(r_{12}) | \psi^S_{00} \rangle = 3 \frac{\alpha^3}{\pi^{\frac{3}{2}}} \int d^3\boldsymbol{\rho} U(\sqrt{2}\rho) e^{-\alpha^2\rho^2} =: a$$

In [70,1-] orbitally-excited states

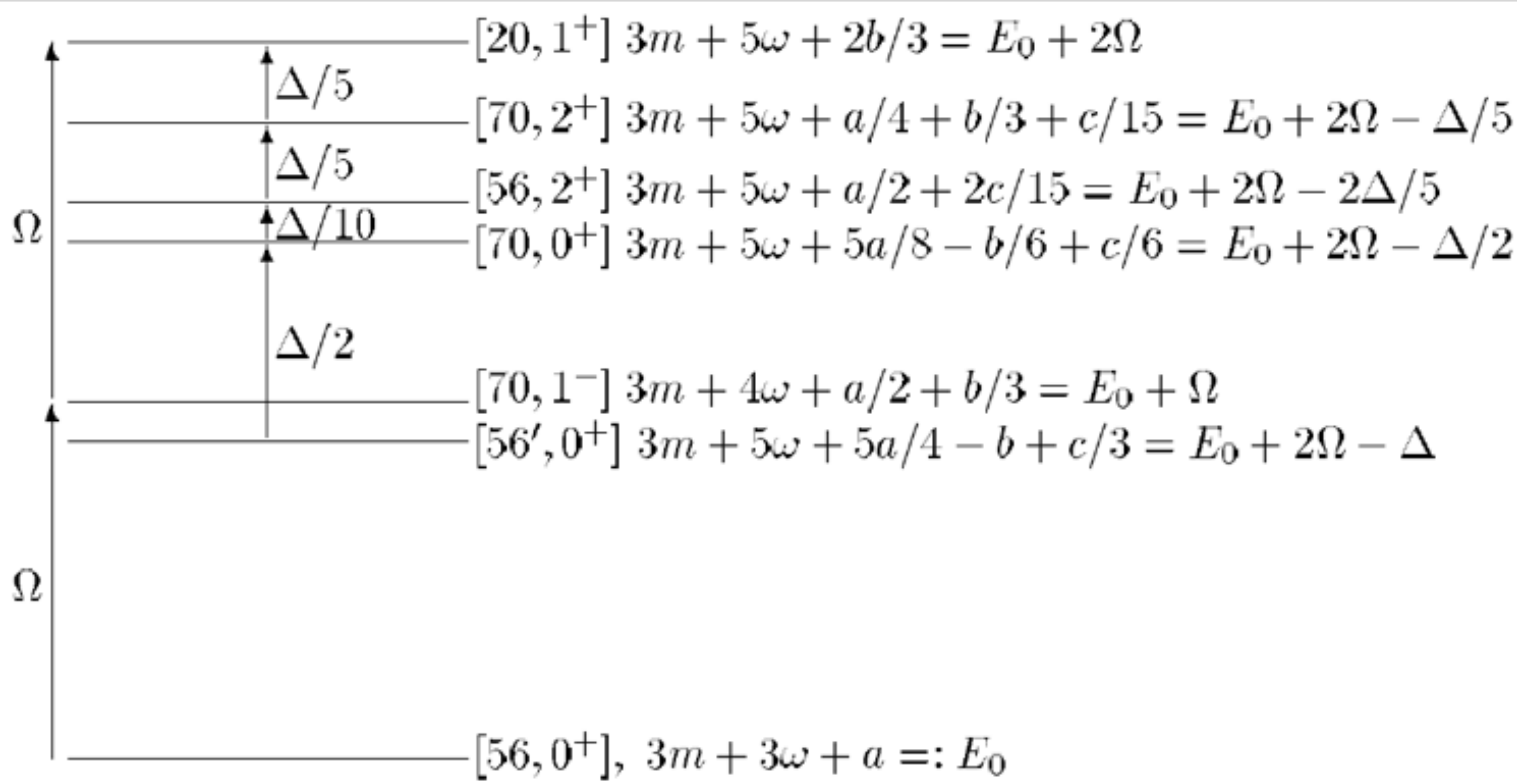
$$\langle \frac{1}{\sqrt{2}} (\phi^\rho_N \psi^\rho_{1M'} + \phi^\lambda_N \psi^\lambda_{1M'}) | \sum_{i<j} U_{ij} | \frac{1}{\sqrt{2}} (\phi^\rho_N \psi^\rho_{1M} + \phi^\lambda_N \psi^\lambda_{1M}) \rangle$$
$$= \frac{3}{2} \delta_{M'M} \left\{ \langle \psi^\rho_{1M} | U(\sqrt{2}\rho) | \psi^\rho_{1M} \rangle + \langle \psi^\lambda_{1M} | U(\sqrt{2}\rho) | \psi^\lambda_{1M} \rangle \right\}$$
$$= \frac{b}{3} + \frac{a}{2}$$

$$b := 3 \frac{\alpha^5}{\pi^{\frac{3}{2}}} \int d^3\boldsymbol{\rho} \rho^2 U(\sqrt{2}\rho) e^{-\alpha^2\rho^2}$$

$$c := 3 \frac{\alpha^7}{\pi^{\frac{3}{2}}} \int d^3\boldsymbol{\rho} \rho^4 U(\sqrt{2}\rho) e^{-\alpha^2\rho^2}$$

Positive-parity excited-state spectrum

Define $\Omega = \omega - a/2 + b/3$, $\Delta = -5a/4 + 5b/3 - c/3$



Roper-like states below negative-parity: $\Omega \approx \Delta \approx 440$ MeV, but $\omega = 250$ MeV, so first-order perturbation theory not justified

Positive-parity excited-state spectrum

Can then apply your favorite flavor and spin-dependent short-range interaction: Isgur and Karl used one-gluon exchange (OGE), minus spin-orbit interactions from both OGE, and Thomas precession in confining potential

Break SU(6) spin-flavor symmetry, so all of the states mix up

Because of the multiplicity of states and near-denegeracies:

- Lots of strong mixing
- L and S no longer good quantum numbers
- Beware of any model/algebra that identifies states by their L and S

How is this modified when we add strange quarks?

NOT advantageous to use a basis where the wave f'n (minus color) is symmetric under exchange of {u,d} and s

$m_s-(m_u+m_d)/2$ substantial compared to quark momenta, $SU(3)_f$ symmetry broken

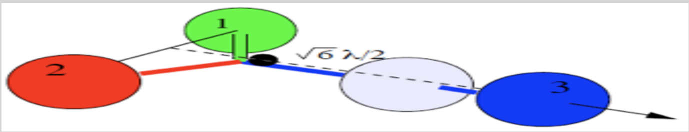
E.g. ground state Λ wave f'n not same as that of p and n

Use 'uds' basis: don't symmetrize s with u and d

$$\phi_{\Lambda^0} = \frac{1}{\sqrt{2}}(ud-du)s$$

$$\phi_{\Sigma^{+,0,-}} = uus, \frac{1}{\sqrt{2}}(ud+du)s, dds$$

(12) AS



$$\phi_{\Omega^-} = sss$$

(12) S

How is this modified when we add strange quarks?

Sums now built symmetric only under (12) exchange

$$\Psi = C_A \sum \psi \, \chi \, \phi$$

Break symmetry between ρ and λ oscillators in wave f'ns

$$\psi_{00} = \frac{\alpha_\rho^{\frac{3}{2}} \alpha_\lambda^{\frac{3}{2}}}{\pi^{\frac{3}{2}}} e^{-(\frac{\alpha_\rho^2}{2}\rho^2 + \frac{\alpha_\lambda^2}{2}\lambda^2)}$$

Ground states

$$\begin{aligned}
\psi^\rho_{1\pm1} &= \mp \frac{\alpha_\rho^{\frac{5}{2}} \alpha_\lambda^{\frac{3}{2}}}{\pi^{\frac{3}{2}}} \rho_\pm e^{-(\frac{\alpha_\rho^2}{2}\rho^2 + \frac{\alpha_\lambda^2}{2}\lambda^2)}, \\
\psi^\lambda_{1\pm1} &= \mp \frac{\alpha_\rho^{\frac{3}{2}} \alpha_\lambda^{\frac{5}{2}}}{\pi^{\frac{3}{2}}} \lambda_\pm e^{-(\frac{\alpha_\rho^2}{2}\rho^2 + \frac{\alpha_\lambda^2}{2}\lambda^2)}, \\
\psi^\rho_{10} &= \frac{\alpha_\rho^{\frac{5}{2}} \alpha_\lambda^{\frac{3}{2}}}{\pi^{\frac{3}{2}}} \sqrt{2}\rho_0 e^{-(\frac{\alpha_\rho^2}{2}\rho^2 + \frac{\alpha_\lambda^2}{2}\lambda^2)}
\end{aligned}$$

Orbitally excited states

Similarly for positive-parity excited states

Highly-excited states

It is not a good idea to try to symmetrize the basis if you plan to extend it; e.g. Karl and Obryk did S_3 group theory (“fairly tediously” appears several times in their paper)

$N = 3$

$$\psi_{3,3;S} = -Y_+^\lambda Y_+^\lambda Y_+^\lambda + 3\,Y_+^\rho Y_+^\rho Y_+^\lambda$$

$$\psi_{3,3;M} = [Y_+^\rho Y_+^\rho Y_+^\lambda + Y_+^\lambda Y_+^\lambda Y_+^\lambda,\ Y_+^\rho Y_+^\rho Y_+^\rho + Y_+^\lambda Y_+^\lambda Y_+^\rho]$$

$$\psi_{3,3;A} = Y_+^\rho Y_+^\rho Y_+^\rho - 3\,Y_+^\lambda Y_+^\lambda Y_+^\rho$$

$$\psi_{3,2;M} = [(Y_+^\lambda Y_0^\rho - Y_+^\rho Y_0^\lambda)\,Y_+^\rho,\ -(Y_+^\lambda Y_0^\rho - Y_+^\rho Y_0^\lambda)\,Y_+^\lambda]$$

$$\psi_{3,1;S} = (\varrho^2 - \lambda^2)\,Y_+^\lambda + 2\lambda\cdot\varrho\,Y_+^\rho$$

$$\psi_{3,1;A} = (\varrho^2 - \lambda^2)\,Y_+^\rho - 2\lambda\cdot\varrho\,Y_+^\lambda$$

$$\psi^1_{3,1;M} = [(\varrho^2 + \lambda^2)\,Y_+^\lambda,\ (\varrho^2 + \lambda^2)\,Y_+^\rho$$

$$\psi^2_{3,1;M} = [2\lambda\cdot\varrho\,Y_+^\rho - (\varrho^2 - \lambda^2)\,Y_+^\lambda,\ (\varrho^2 - \lambda^2)\,Y_+^\rho + 2\lambda\cdot\varrho\,Y_+^\lambda]$$

Highly-excited states

$N = 4$

$$\psi_{4,4;S} = (Y_+^\rho Y_+^\rho + Y_+^\lambda Y_+^\lambda)(Y_+^\rho Y_+^\rho + Y_+^\lambda Y_+^\lambda)$$

$$\psi^1_{4,4;M} = [(-Y_+^\lambda Y_+^\lambda Y_+^\lambda + 3\,Y_+^\rho Y_+^\rho Y_+^\lambda)\,Y_+^\lambda,\ (-Y_+^\lambda Y_+^\lambda Y_+^\lambda + 3Y_+^\rho Y_+^\rho Y_+^\lambda)\,Y_+^\rho]$$

$$\psi^2_{4,4;M} = [(Y_+^\rho Y_+^\rho + Y_+^\lambda Y_+^\lambda)(Y_+^\rho Y_+^\rho - Y_+^\lambda Y_+^\lambda),\ (Y_+^\rho Y_+^\rho + Y_+^\lambda Y_+^\lambda)\cdot 2\,Y_+^\lambda Y_+^\rho]$$

$$\psi_{4,3;A} = (Y_+^\lambda Y_0^\rho - Y_+^\rho Y_0^\lambda)(Y_+^\rho Y_+^\rho + Y_+^\lambda Y_+^\lambda)$$

$$\psi_{4,3;M} = [(Y_+^\lambda Y_0^\rho - Y_+^\rho Y_0^\lambda)2\,Y_+^\rho Y_+^\lambda,\ -(Y_+^\lambda Y_0^\rho - Y_+^\rho Y_0^\lambda)(Y_+^\rho Y_+^\rho - Y_+^\lambda Y_+^\lambda)]$$

$$\psi^1_{4,2;S} = (\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2)(Y_+^\rho Y_+^\rho + Y_+^\lambda Y_+^\lambda)$$

$$\psi^2_{4,2;S} = (\boldsymbol{\rho}^2 - \boldsymbol{\lambda}^2)(Y_+^\rho Y_+^\rho - Y_+^\lambda Y_+^\lambda) + 4(\boldsymbol{\lambda}\cdot\boldsymbol{\rho})\,Y_+^\rho Y_+^\lambda$$

$$\psi_{4,2;A} = 2(\boldsymbol{\rho}^2 - \boldsymbol{\lambda}^2)\,Y_+^\rho Y_+^\lambda - 2\boldsymbol{\lambda}\cdot\boldsymbol{\rho}\,(Y_+^\rho Y_+^\rho - Y_+^\lambda Y_+^\lambda)$$

$$\psi^1_{4,2;M} = [(\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2)(Y_+^\rho Y_+^\rho - Y_+^\lambda Y_+^\lambda),\ (\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2)2\,Y_+^\rho Y_+^\lambda]$$

$$\psi^2_{4,2;M} = [(\boldsymbol{\rho}^2 - \boldsymbol{\lambda}^2)(Y_+^\rho Y_+^\rho + Y_+^\lambda Y_+^\lambda),\ 2\boldsymbol{\lambda}\cdot\boldsymbol{\rho}\,(Y_+^\rho Y_+^\rho + Y_+^\lambda Y_+^\lambda)]$$

$$\psi^3_{4,2;M} = [4\boldsymbol{\lambda}\cdot\boldsymbol{\rho}\,Y_+^\rho Y_+^\lambda - (\boldsymbol{\rho}^2 - \boldsymbol{\lambda}^2)(Y_+^\rho Y_+^\rho - Y_+^\lambda Y_+^\lambda),\ 2\boldsymbol{\lambda}\cdot\boldsymbol{\rho}\,(Y_+^\rho Y_+^\rho - Y_+^\lambda Y_+^\lambda) + (\boldsymbol{\rho}^2 - \boldsymbol{\lambda}^2)\cdot 2\,Y_+^\rho Y_+^\lambda]$$

$$\psi_{4,1;A} = (\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2)(Y_+^\lambda Y_0^\rho - Y_+^\rho Y_0^\lambda)$$

$$\psi_{4,1;M} = [2\boldsymbol{\lambda}\cdot\boldsymbol{\rho}\,(Y_+^\lambda Y_0^\rho - Y_+^\rho Y_0^\lambda),\ -(\boldsymbol{\rho}^2 - \boldsymbol{\lambda}^2)(Y_+^\lambda Y_0^\rho - Y_+^\rho Y_0^\lambda)]$$

$$\psi^1_{4,0;S} = (\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2)(\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2)$$

$$\psi^2_{4,0;S} = (\boldsymbol{\rho}^2 - \boldsymbol{\lambda}^2)^2 + 4(\boldsymbol{\lambda}\cdot\boldsymbol{\rho})^2$$

$$\psi^1_{4,0;M} = [(\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2)(\boldsymbol{\rho}^2 - \boldsymbol{\lambda}^2),\ (\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2)2\boldsymbol{\lambda}\cdot\boldsymbol{\rho}]$$

$$\psi^2_{4,0;M} = [4(\boldsymbol{\lambda}\cdot\boldsymbol{\rho})^2 - (\boldsymbol{\rho}^2 - \boldsymbol{\lambda}^2)^2,\ 4(\boldsymbol{\lambda}\cdot\boldsymbol{\rho})(\boldsymbol{\rho}^2 - \boldsymbol{\lambda}^2)]$$

Extending the basis to highly-excited states

Don't antisymmetrize in u and d for Δ and N flavor wave f'ns

Δ^{++} = uuu, {Δ^{+},p} = uud, {Δ^{0},n} = ddu, Δ^{-} = ddd

$$\Psi = C_A \phi \sum \chi \psi$$

Require only (12) symmetry (N,Δ,Σ,Ξ) or antisymmetry (Λ) in sums, and good angular momentum

Build a basis large enough to ensure convergence of expansion of wave functions; harmonic oscillator is convenient, i.e. easily Fourier transformed

Extending the basis to highly-excited states

Solve $H\Psi=E\Psi$ (doesn't have to be non-relativistic) by diagonalizing H matrix formed by expanding Ψ in large basis Ψ_a (hundreds of sub-states)

Variational calculation (separate for each eigenstate; Hylleraas-Undheim theorem) in oscillator parameter(s)

u,d symmetry of H reflected in eigenfunctions

Costs?

Have to look at wave functions to decide if a state is Δ ($\sum \chi\psi$ is S) or N ($\sum \chi\psi$ is M^λ)

No longer have $\sum_{i<j} \langle\psi|H_{ij}|\psi\rangle = 3\langle\psi|H_{12}|\psi\rangle$ (Moshinsky)

Momentum and position-dependence in H

If have basis easy to Fourier transform, can deal with momentum and position-dependent terms in H

E.g. kinetic energy $\langle\psi_a|\sum_i \sqrt{p_i^2+m_i^2}|\psi_b\rangle$

simply evaluate in momentum space

Effects of spinor normalization in, e.g. OGE contact interaction

$$\left(\frac{m_i m_j}{E_i E_j}\right)^{\frac{1}{2}+\epsilon_{\rm cont}} \frac{8\pi}{3}\alpha_s(r_{ij})\frac{2}{3}\frac{\mathbf{S}_i\cdot\mathbf{S}_j}{m_i m_j}\left[\frac{\sigma_{ij}^3}{\pi^{\frac{3}{2}}}e^{-\sigma_{ij}^2 r_{ij}^2}\right]\left(\frac{m_i m_j}{E_i E_j}\right)^{\frac{1}{2}+\epsilon_{\rm cont}}$$

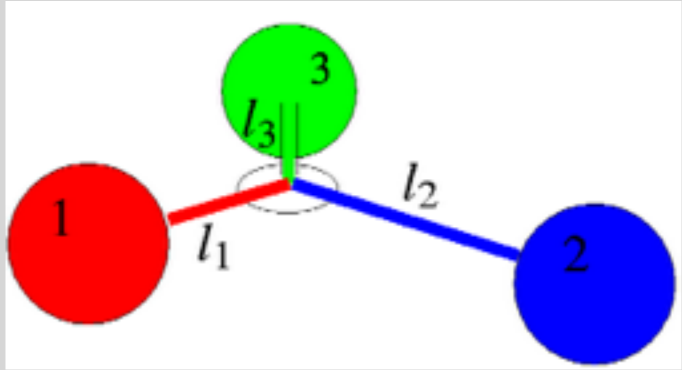
insert (nominally) complete sets of states, multiply matrices

$$\sum_{cd}\langle\psi_a|f(p_i)|\psi_c\rangle\langle\psi_c|V(r_{ij})|\psi_d\rangle\langle\psi_d|f(p_i)|\psi_b\rangle$$

Sample spectrum (SC & N. Isgur)

Hamiltonian : relativistic kinetic energy

confining potential $b\Sigma_i\, l_i$
+ associated spin-orbit



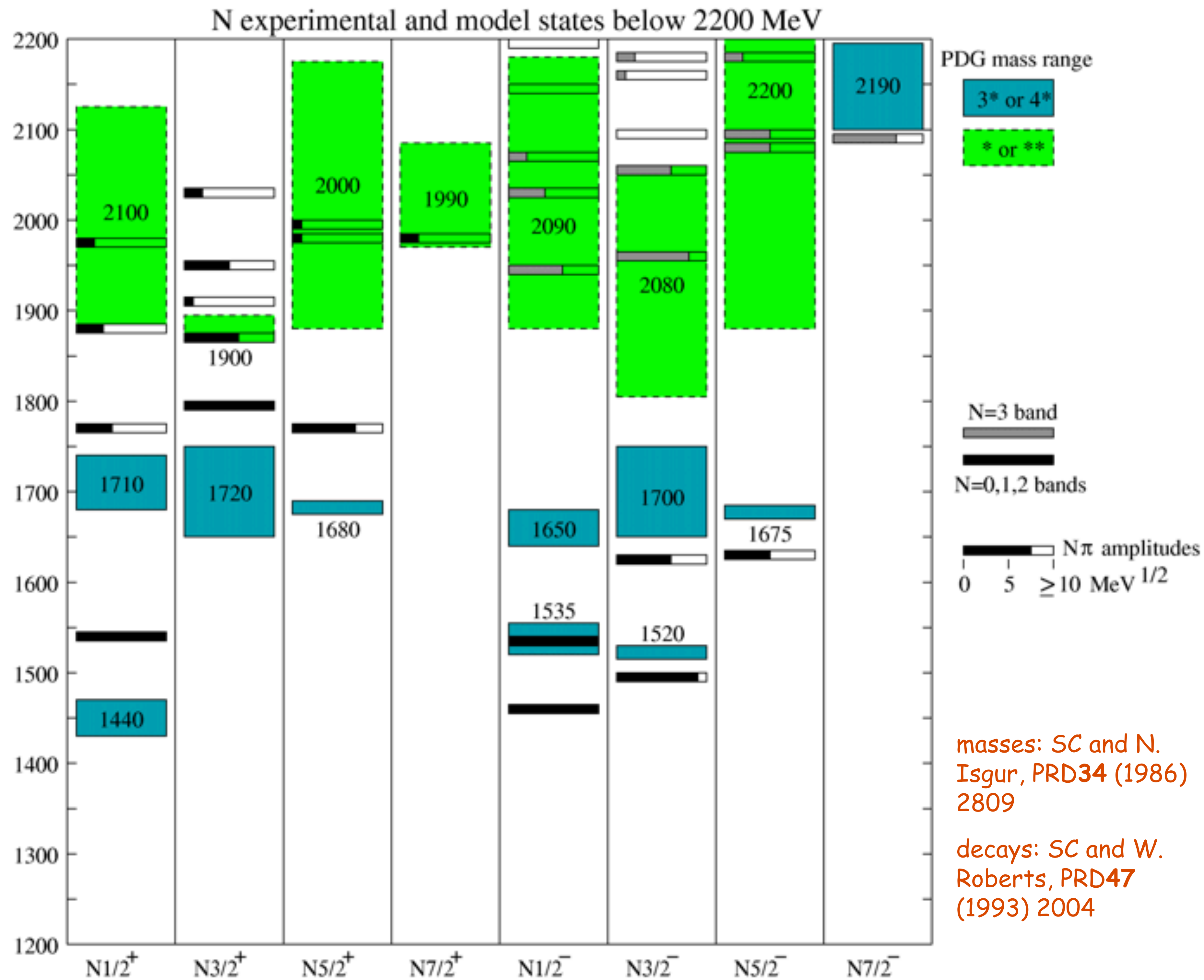
one-gluon exchange (color-Coulomb, contact, tensor, spin-orbit interactions)

smeared quarks, suppression of potentials at high momentum

Large oscillator basis; 8^{th}-order polynomials (positive parity) or 7^{th}-order (negative parity)

Calculate all baryon masses & wave f'ns with one consistent set of parameters

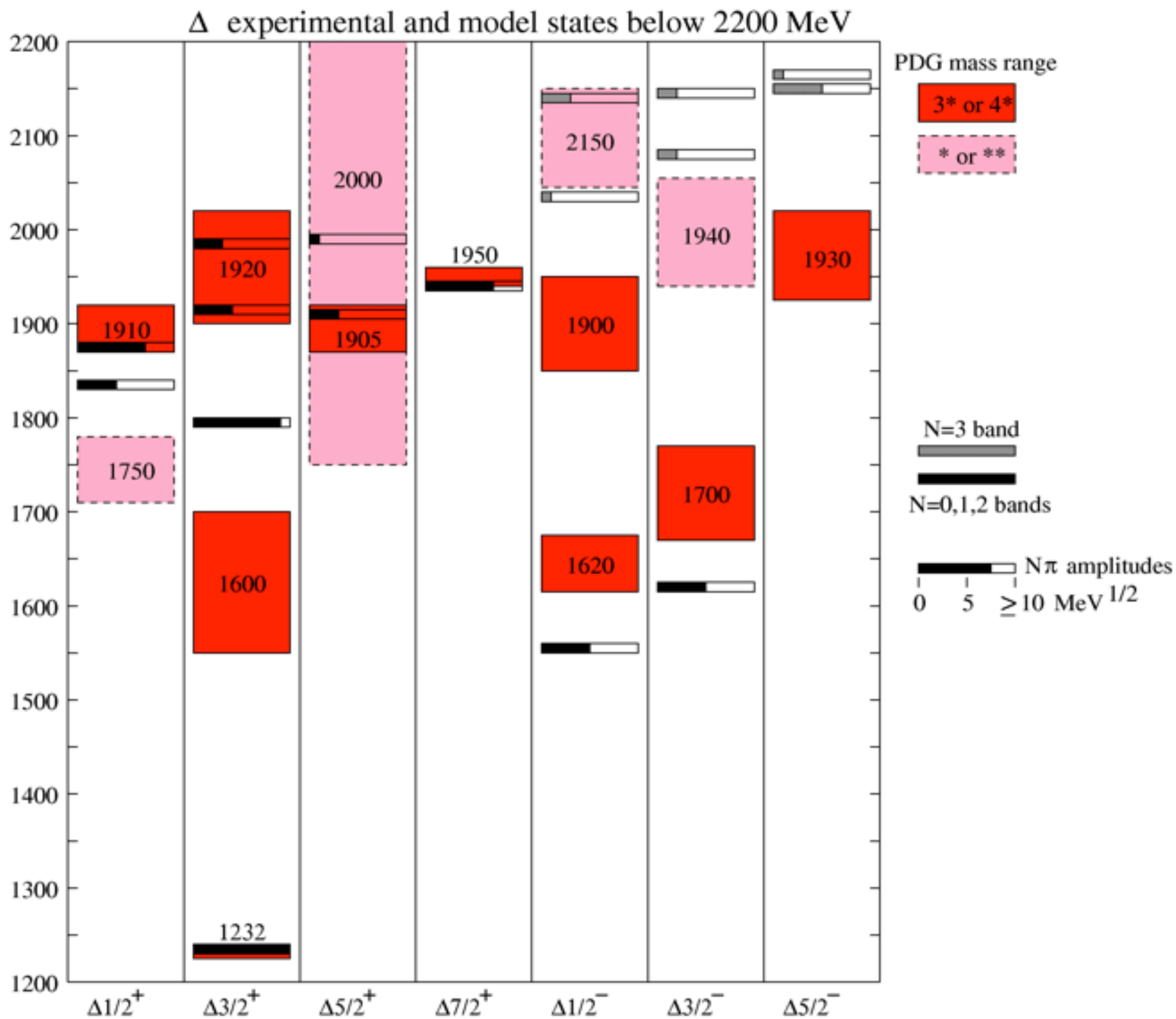
Sample spectrum (SC & N. Isgur)



masses: SC and N. Isgur, PRD**34** (1986) 2809

decays: SC and W. Roberts, PRD**47** (1993) 2004

Sample spectrum (SC & N. Isgur)



Sample spectrum (SC & N. Isgur)

Lowest few non-strange baryons of either parity up to J=11/2 (bars) vs. PDG mass range (boxes)