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## Evidence for a Scalar Glueball

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## Abstract

We show that the newly discovered scalar meson  $f_0(1500)$  at LEAR has properties compatible with the lightest scalar glueball predicted by lattice QCD and incompatible with a  $Q\overline{Q}$  state. We suggest that decays of glueballs are into pairs of glueballs (including  $\eta$ ,  $\eta'$  or  $(\pi \pi)_S$ ) or by mixing with nearby  $Q\overline{Q}$  states. The partial widths of  $f_0(1500)$  are in accord with this hypothesis, tests of which include characteristic radiative decays to  $\gamma\phi$ ,  $\gamma\omega$ ,  $\gamma\rho$  and the prediction of a further scalar state,  $f'_0(1500 - 1800)$  which couples strongly to  $K\overline{K}$ ,  $\eta\eta$  and  $\eta\eta'$ .

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Glueballs are a major missing component of the standard model. For two decades, experimental searches for bound states of gluons have produced only controversial signals. Whereas the gluonic degrees of freedom expressed in  $L_{QCD}$  have been established beyond doubt in high momentum data, their dynamics in the strongly interacting limit epitomised by hadron spectroscopy remain obscure.

In this letter we show that if intuition based on lattice QCD is a reliable guide, the emerging properties of the scalar meson  $f_0(1500)$  discovered at LEAR are consistent with those of a glueball mixed with nearby members of the quark model scalar nonet. This hypothesis may be tested in forthcoming experiments.

There are now clear signals for the  $f_0(1500)$  in a range of production processes that are traditionally believed to favour glueballs, namely [\[1](#page-8-0)]

- 1. Radiative  $J/\psi$  decay:  $J/\psi \rightarrow \gamma + G$  [[2\]](#page-8-0)
- 2. Collisions in the central region away from quark beams and target:  $pp \to p_f(G)p_s$  $|3, 4|$  $|3, 4|$  $|3, 4|$ .
- 3. Proton-antiproton annihilation where the destruction of quarks creates opportunity for gluons to be manifested [\[5](#page-8-0)]-[[10](#page-8-0)]. The  $f_0(1500)$  was observed to decay into  $\pi^0 \pi^0$ ,  $\eta\eta$  and  $\eta\eta'$  in  $\overline{p}p \to 3\pi^0$  [[5, 6\]](#page-8-0),  $\pi^0\eta\eta$  [\[7](#page-8-0), [8](#page-8-0)] and  $\pi^0\eta\eta'$  [\[9](#page-8-0)].

By contrast, there are no significant sightings of  $f_0(1500)$  in processes where glueballs are not expected to be enhanced.

This prima facie evidence for a scalar glueball is particularly interesting now that studies of lattice-QCD appear to be coming to a concensus that the lightest glueball is indeed a scalar in this region of mass[[11](#page-8-0), [12](#page-8-0)]. In these circumstances it is natural to speculate that the  $f_0(1500)$  and the "primitive" (i.e. quenched approximation) scalar glueball of lattice-QCD are intimately related. The purpose of this letter is to evaluate and to propose further tests of this hypothesis. A detailed discussion may be found in ref.[[13\]](#page-8-0).

Lattice QCD is not yet able to make detailed quantitative statements about the partial decay widths of glueballs against which we could evaluate those of  $f_0(1500)$ . However, qualitative features may be abstracted from flux-tube models which are probably the nearest we can presently get to simulating the strong coupling lattice formulation of QCD. In this simulation[[14](#page-8-0)] mesons consist of a quark and antiquark connected by a tube of coloured flux whereas glueballs  $G_0$  consist of a loop of flux. These eigenstates are perturbed by two types of interaction [\[15\]](#page-8-0)

- 1.  $V_1$  which creates a Q and a Q at neighbouring lattice sites, together with an elementary flux-tube connecting them. When  $V_1$  acts on conventional  $Q\overline{Q}$  mesons it causes their decays (fig. [1a](#page-10-0)), and is thereby known to be significant. Fig. [1b](#page-10-0) shows that when  $V_1$  acts on a glueball  $G_0$ , in leading order it causes glueball - $Q\bar{Q}$  mixing.
- 2.  $V_2$  which creates or destroys a unit of flux around any plaquette, where a plaquette is an elementary square with links on its edges: in leading order this causes glueballs to decay to pairs of glueballs (fig. [1c](#page-10-0)).

<span id="page-3-0"></span>Note that in this approach  $G_0$  does not decay to  $\pi\pi$  or  $K\overline{K}$  in leading order since gluons are isoscalar; decays to  $\eta$  and  $\eta'$  are allowed to the extent that these have nonzero overlap with glue.

The mixing with  $Q\bar{Q}$  is likely to play a significant role in glueball phenomenology if a quarkonium nonet of the same  $J^{PC}$  is nearby. As a result of extensive data from the Crystal Barrel collaboration at LEAR it is now clear that there is a scalar nonet in the 1.5 GeV region, as well as the  $f_0(1500)$ .

(i) The discovery of the  $I = 1$  state  $a_0(1450) \rightarrow \eta \pi$  [[16](#page-8-0)] sets the natural scale of masses and widths for the other members of the scalar nonet. In quark models such as ref.[[15](#page-8-0), [17](#page-8-0)] the widths of the scalar  $Q\overline{Q}$  are qualitatively ordered as  $\Gamma(\overline{n}n) > \Gamma(s\overline{s}) > \Gamma(a_0) \ge \Gamma(K^*)$ . Empirically  $\Gamma(a_0) = 270 \pm 40$  MeV,  $\Gamma(K_0^*) = 287 \pm 23$  MeV which are consistent with quark model expectations and lead one to expect for their partners  $\Gamma(\overline{n}n) \sim 400$ -700 MeV and  $\Gamma(s\overline{s}) \sim 300\text{-}500 \text{ MeV}$  [\[13](#page-8-0), [15, 17, 18\]](#page-8-0).

(ii) The Crystal Barrel data show clear signals for an independent scalar meson  $f_0(1370)$  in  $\pi\pi$  and  $\eta\eta$  whose mass is consistent with it being the  $n\bar{n}$  partner of the  $a_0(1450)$  and whose ratio of partial widths to  $\pi\pi$  and  $\eta\eta$  also is consistent with it being the  $n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$  state of a nonet [\[5](#page-8-0), [6](#page-8-0), [8, 16\]](#page-8-0). The total width of  $f_0(1370)$  is not yet well determined, 200-700 MeV being possible [\[5](#page-8-0), [8, 6\]](#page-8-0) depending on the theoretical model used in the analysis and in accord with the  $n\bar{n}$  hypothesis. The  $\gamma\gamma$  width in this regionis also consistent with it containing the  $n\bar{n}$  state [[19](#page-8-0), [20\]](#page-8-0). By contrast, the  $f_0(1500)$ widthis  $116 \pm 17$  $116 \pm 17$  $116 \pm 17$  MeV [6, [9, 8](#page-8-0)] and is clearly out of line with the scalar nonet, being even smaller than the  $K^*$  and  $a_0$  widths.

The properties of the  $f_0(1500) - f_0(1370)$  system are incompatible with them both belonging to a  $QQ$  nonet. The  $f_0(1370)$  appears to be dominantly  $n\bar{n}$  and on mass grounds we expect that the  $s\bar{s}$  would lie some 200-300 MeV higher than this and the  $a_0(1450)$ . The strong coupling of  $f_0(1500)$  to pions implies that it is not primarily an ss state; the decoupling from  $KK$  further distances it from the nonet. Specifically [\[13\]](#page-8-0) after correcting for phase space and minor effects of form factors, the Crystal Barrel data imply for the ratios of partial widths

$$
R_1 \equiv \frac{\gamma^2 (f_0(1500) \to \eta \eta)}{\gamma^2 (f_0(1500) \to \pi \pi)} = 0.27 \pm 0.11,\tag{1}
$$

$$
R_2 \equiv \frac{\gamma^2 (f_0(1500) \to \eta \eta')}{\gamma^2 (f_0(1500) \to \pi \pi)} = 0.19 \pm 0.08,\tag{2}
$$

while a bubble chamber experiment leads to the (95% C.L.) upper limit[[21](#page-8-0)]

$$
R_3 \equiv \frac{\gamma^2 (f_0(1500) \to K\bar{K})}{\gamma^2 (f_0(1500) \to \pi\pi)} < 0.1. \tag{3}
$$

In the  $Q\overline{Q}$  hypothesis it is possible to fit two ratios but not all three for the  $f_0(1500)$ . This is shown in fig. [2](#page-10-0) [\[22\]](#page-9-0): The roughly equal couplings to  $\eta\eta$  and  $\eta\eta'$  together with the dominance of  $\pi\pi$  (eqn. 1 and 2) imply that  $f_0(1500)$ , if  $Q\overline{Q}$ , is nearly  $\overline{n}n$ . However, for an  $\overline{m}n$  state,  $\gamma^2(K\overline{K}) = 1/3$   $\gamma^2(\pi\pi)$ , in contradiction with data (eqn. 3). Furthermore, the <span id="page-4-0"></span>strong affinity of  $f_0(1370)$  for  $\pi\pi$  and its branching ratios and widths suggest that this state is strongly  $n\bar{n}$  which creates further problems for a  $QQ$  interpretation of  $f_0(1500)$ . This remains true for any reasonable breaking of  $SU(3)_f$  symmetry [\[13\]](#page-8-0). These results are stable against form factor choice[[13, 18\]](#page-8-0) and contrast the experience with other known nonets [\[23\]](#page-9-0).

However, the observed decay branching ratios and in particular the suppression of  $KK$ for the  $f_0(1500)$  are natural for a scalar glueball which is in the vicinity of the  $Q\overline{Q}$  nonet and mixes with the two nearby  $Q\overline{Q}$  isoscalars, one  $(n\bar{n})$  with mass below 1.5 GeV and the other (ss) above. The  $f_0(1370)$  is a natural candidate for the  $n\bar{n}$ ; this hypothesis requires that a (mainly)  $s\overline{s}$  state lies in the 1600 MeV region.

We demonstrate this more quantitatively by considering the effect of  $V_1$  on a primitive glueball  $G_0$ . For this first look we assume flavour blindness at the fundamental level such that  $\langle s\bar{s}|V_1|G_0\rangle \equiv \langle d\bar{d}|V_1|G_0\rangle$ . The quarkonium mixing into the glueball state is then

$$
N_G|G\rangle = |G_0\rangle + \xi \{\sqrt{2}|n\bar{n}\rangle + \omega|s\bar{s}\rangle\} \equiv |G_0\rangle + \sqrt{2}\xi|Q\overline{Q}\rangle
$$
\n(4)

where  $N_G$  is the normalisation  $\equiv \sqrt{1 + \xi^2 (2 + \omega^2)}$ ,  $\xi$  is the dimensionless mixing parameter

$$
\xi \equiv \frac{\langle d\bar{d}|V_1|G_0\rangle}{E_{G_0} - E_{n\bar{n}}},\tag{5}
$$

and

$$
\omega \equiv \frac{E_{G_0} - E_{n\bar{n}}}{E_{G_0} - E_{s\bar{s}}}
$$
\n
$$
\tag{6}
$$

is the ratio of the mass gap of  $G_0$  and the  $n\bar{n}$  and  $s\bar{s}$  intermediate states in old fashioned perturbation theory (fig. [3](#page-10-0)).

Only in the particular case  $\omega = 1$ , where  $E_{n\bar{n}} \equiv E_{s\bar{s}}$ , does the underlying flavour blindness naturally survive at hadron level as  $G_0$  mixes into the flavour singlet

$$
|Q\overline{Q}\rangle \equiv |u\bar{u} + d\bar{d} + s\bar{s}\rangle/\sqrt{3}.
$$
 (7)

However, when  $\omega \neq 1$ , as will tend to be the case when  $G_0$  is in the vicinity of a  $Q\overline{Q}$ nonet with the same  $J^{PC}$  quantum numbers (as in the  $0^{++}$  case of interest here) significant distortion from naive flavour singlet can arise: the mass breaking,  $\Delta m \equiv m_s - m_d$ , which is usually regarded as a small perturbation in hadron dynamics, is here magnified.

There are three data that suggest a self consistency with this hypothesis.

1. The suppression of KK in the  $f_0(1500)$  decays suggests a destructive interference between  $n\bar{n}$  and  $s\bar{s}$  such that  $\omega \approx -1$ . This arises naturally if the primitive glueball mass is equidistant between those of  $n\bar{n}$  and the primitive  $s\bar{s}$  - a situation not inconsistent with lattice QCD. As the mass of  $G_0 \to m_{n\bar{n}}$  or  $m_{s\bar{s}}$ , the KK remains suppressed though non-zero. Specifically from eqn. 4 and  $SU(3)_f$  we obtain for  $G \equiv f_0(1500)$ 

$$
\langle K \overline{K} | V_1 | G \rangle = \frac{1 + \omega}{2} \langle \pi \pi | V_1 | G \rangle \tag{8}
$$

<span id="page-5-0"></span>per unit charge combination for  $K\overline{K}$  and  $\pi\pi$ . The upper limit  $R_3$  (eqn. [3](#page-3-0)) leads to the constraint

$$
-0.27 < \frac{\langle K\bar{K}|V_1|G\rangle}{\langle \pi\pi|V_1|G\rangle} < 0.27 \tag{9}
$$

which then gives the range

$$
-1.5 < \omega < -0.5. \tag{10}
$$

Thus eventual quantification of the  $K\bar{K}$  signal may be used to constrain  $m_{G_0}$ .

- 2. Lattice QCD suggests that the primitive scalar glueball  $G_0$  lies at or above 1500 MeV, hence above the  $I = 1$  QQ state  $a_0(1450)$  and the (presumed) associated  $n\bar{n}$  $f_0(1370)$ . Hence  $E_{G_0} - E_{n\bar{n}} > 0$  in the numerator of  $\omega$ , eqn. 6.
- 3. The allowed range for  $\omega$  (eqn. 10) enables predictions of the mass of the  $\Psi_s$  state. Using eqn. [6](#page-4-0) and  $m(G) = 1509 \pm 10$  MeV,  $m(\Psi_n) = 1360 \pm 40$  MeV, which dominates the error, we find [\[13\]](#page-8-0)

$$
1580[m(\Psi_n) = 1400] < m(\Psi_s) < 1890[m(\Psi_n) = 1320] \text{ MeV} \tag{11}
$$

This is consistent with naive mass estimates whereby the  $\Delta m = m_{s\bar{s}} - m_{n\bar{n}} \approx$ 200 − 300 MeV

If this were the whole story, the mixing eqn. [4](#page-4-0) would suppress not just  $K\bar{K}$  but also  $\eta$ . However, this is where the effect of the perturbation  $V_2$  comes into prominence: At  $O(V_2)$  the glueball decays directly into pairs of glueballs or  $(gg)$  continuum and thereby into mesons whose Fock states have strong overlap with gg. To the extent that there is significant gg coupling to  $\eta$ ,  $\eta'$  or to  $\sigma \equiv (\pi \pi)_s$  (e.g.  $\psi' \to \psi gg \to \psi \eta$  and  $\psi(\pi \pi)_s$  have large intrinsic couplings notwithstanding the fact that they are superficially OZI violating) one may anticipate  $\eta\eta$  and  $\eta\eta'$  in the two-body decays of scalar glueballs. Note that the perturbation  $V_2$  triggering  $G_0 \to G_0G_0$ ,  $G_0(gg)$  or  $(gg)(gg)$  will not directly feed exclusive two-body flavoured states such as  $\pi\pi$  nor KK since gluons are isoscalar. However, from the above  $\psi$  phenomenology, one may anticipate  $(qq)(qq) \rightarrow (\pi \pi)_s (\pi \pi)_s$  in the decay of  $G(0^{++})$  and analogously for  $0^{-+}$  glueballs one may anticipate  $\eta\sigma$  or  $\eta'\sigma$  decays.

The manifestation of this mechanism in final states involving the  $\eta$  or  $\eta'$  mesons depends on the unknown overlaps such as  $\langle qg|V|q\bar{q}\rangle$  in the pseudoscalars. Chiral symmetry suggests that the coupling of glue to  $\eta$  or  $\eta'$  vanishes in the limit  $m_q \to 0$  and hence occurs dominantly through their  $s\bar{s}$  component, thereby favouring the  $\eta'$ :

$$
\frac{\langle gg|V|\eta'\rangle}{\langle gg|V|\eta\rangle} = \frac{\langle s\bar{s}|\eta'\rangle}{\langle s\bar{s}|\eta\rangle} \sim -\frac{4}{3}.\tag{12}
$$

This appears to be consistent with the decays of  $f_0(1500)$  as we now show. Combining the  $\eta\eta$  amplitudes from both the  $QQ$  and  $G_0$  components we have, in the approximation that  $\eta$  and  $\eta'$  are approximately 50 : 50 mixtures of  $n\bar{n}$  and  $s\bar{s}^{-1}$ 

$$
\frac{\langle \eta \eta | V | G \rangle}{\langle \pi \pi | V | G \rangle} = \frac{\langle \eta \eta | V | G_0 \rangle}{N_G \langle \pi \pi | V | G \rangle} + (\frac{1 + \omega}{2}) = \pm (0.90 \pm 0.20) \tag{13}
$$

from  $R_1$  (eqn. [1\)](#page-3-0) and

$$
\frac{\langle \eta \eta' | V | G \rangle}{\langle \pi \pi | V | G \rangle} = \frac{\langle \eta \eta' | V | G_0 \rangle}{N_G \langle \pi \pi | V | G \rangle} + (\frac{1 - \omega}{2}) = \pm (0.53 \pm 0.11) \tag{14}
$$

from  $R_2$  (eqn. [2](#page-3-0)). Hence using the range eqn. [10](#page-5-0) for  $\omega$  one finds with the  $+$  sign in eqn. 13 and the − sign in eqn. 14

$$
-1.8 < \frac{\langle \eta \eta' | V | G_0 \rangle}{\langle \eta \eta | V | G_0 \rangle} < -1.5. \tag{15}
$$

A small breaking of chiral symmetry is consistent with this.

An upper limit for  $\xi^2$  can be obtained by comparing the ratio of amplitudes (eqn. 13) for  $f_0(1500)$  decay to that for  $f_0(1370)$ . We find [\[13](#page-8-0)]:  $\xi^2$  < 0.2. The magnitudes of the partial widths of 1500/1370 also fit with the  $G-Q\overline{Q}$  mixing scenario. The above analysis shows that the decay amplitudes from the  $Q\bar{Q}$  component of are all at  $0(\xi)$  and so we expect

$$
\frac{\gamma^2(f(1500) \to \pi\pi)}{\gamma^2(f(1370) \to \pi\pi)} \simeq O(2\xi^2). \tag{16}
$$

Empirically this ratio is smaller than 0.4 and hence we again find  $\xi^2 \leq 0.2$ .

This hypothesis implies that there are small  $G_0$  admixtures in  $\Psi_{n,s}$ :

$$
\sqrt{1 + \omega^2 \xi^2} |\Psi_s\rangle = |s\bar{s}\rangle - \xi \omega |G_0\rangle
$$
  

$$
\sqrt{1 + 2\xi^2} |\Psi_n\rangle = |n\bar{n}\rangle - \xi \sqrt{2} |G_0\rangle
$$
 (17)

which in turn implies that the sum of the partial widths of the two states is

$$
\Gamma(\Psi_n) + \Gamma(G) \simeq \Gamma(n\bar{n}),\tag{18}
$$

in accord with quark model estimates[[17, 18\]](#page-8-0).

The smallness of  $|\xi|$  then also implies that  $\Psi_s$  decays essentially like an  $s\bar{s}$  state with the branching ratios to  $K\bar{K}$  and  $\eta\eta'$  dominating over  $\eta\eta$  and with  $\pi\pi$  much suppressed. If  ${}^{3}P_{0}$  quark pair creation is important in the decay dynamics, the values of the branching ratios and total width may be strongly mass dependent [\[13](#page-8-0), [15](#page-8-0), [18\]](#page-8-0)

The quantitative predictions of our analysis depend on the apparent suppression of  $f_0(1500)$  decay to  $K\overline{K}$ . Thus detailed study of  $p\bar{p} \to \pi K\overline{K}$  can be seminal (i) in confirming

<sup>&</sup>lt;sup>1</sup>Theactual numbers vary slightly when a mixing angle of  $-17.3^{\circ}$  [[24\]](#page-9-0) is used, see ref. [[13\]](#page-8-0)

the  $K\overline{K}$  suppression, (ii) in confirming the  $K_0^*$  (1430)  $\rightarrow K\pi$  and  $a_0(1450) \rightarrow \eta\pi$  and  $K\overline{K}$ , (iii) in quantifying the signal for  $f_0$  (1370) and  $f_0$  (1500) and (iv) in isolating the predicted ss member of the nonet. Furthermore, study of radiative decays  $f_0(1500) \rightarrow \gamma + \omega(\phi, \rho)$ may probe the flavour content in the  $Q\overline{Q}$  mixing. To the extent that  $\omega \equiv n\overline{n}$  and  $\phi \equiv s\overline{s}$ , the amplitude ratios will be

$$
f_0(1500) \to \gamma \phi : \gamma \omega : \gamma \rho = -\omega : 1 : 3. \tag{19}
$$

It seems clear that new dynamics, beyond simple  $Q\overline{Q}$ , is operating at the  $f_0(1500)$ . The simplest explanation within strong QCD is that a glueball excitation is seeding the phenomena, in particular the clarity of the signals in channels that are believed favourable to glue. It is also tantalising that the signal appears particularly sharp in  $p\bar{p} \to \eta\eta\eta$  [\[25](#page-9-0)] which might be consistent with direct production of a  $0^{-+}$  glueball above 2GeV (in line with lattice predictions) decaying via the  $V_2$  process into  $G_0G_0$  with consequent affinity for  $f_0(1500) + \eta$ . An excitation curve for this process could be interesting.

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## <span id="page-10-0"></span>Figure Captions

Figure 1: The effect of perturbation  $V_1$  causes  $Q\bar{Q}$  decay (a) or  $G_0$  mixing with  $Q\overline{Q}$  (b). The effect of  $V_2$  on  $G_0$  causes decay to two glueballs (c).

Figure 2: Invariant couplings  $\gamma^2$  to two pseudoscalars for an isoscalar  $0^{++}Q\overline{Q}$  meson as a function of nonet mixing angle for a pseudoscalar mixing angle of  $-17.3^{\circ}$  (up to a common arbitrary normalization constant). Full curve:  $\eta\eta$ ; dashed curve:  $\eta\eta'$ ; dotted curve:  $\pi\pi$ ; dashed-dotted curve:  $K\overline{K}$ . Ideal mixing occurs at 35.3° ( $s\overline{s}$ ) or at 125.3° ( $\overline{n}n$ ).

Figure 3: Gluonium- $Q\bar{Q}$  mixing involving the energy denominator  $E_G - E_{Q\bar{Q}}$ 

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