Study of $\phi \to \pi^+ \pi^- \pi^0$ with CMD-2 detector

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Abstract
The cross section of the process $e^+e^- \to \pi^+\pi^-\pi^0$ has been measured in the c.m. energy range 984–1060 MeV with the CMD-2 detector at the VEPP-2M collider. The obtained value of $\frac{\text{Br}(\phi \to e^+e^-)}{\text{Br}(\phi \to \rho \pi)} = (4.51 \pm 0.16 \pm 0.11) \times 10^{-5}$ is in good agreement with the previous measurements and has the best accuracy. Analysis of the Dalitz plot was performed. The contributions of the dominant $\phi \to \rho \pi$ mechanism as well as of a small direct $\phi \to 3\pi$ amplitude were determined.

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1. Introduction

A study of $e^+e^-$ annihilation into hadrons at low energies has a long history, but despite decades of experiments, new precise measurements are still interesting and can provide important information about interactions of light quarks and spectroscopy of their bound states.

This work is devoted to a study of the process $e^+e^- \to \pi^+\pi^-\pi^0$ within the $\phi$-meson energy range with the CMD-2 detector [1,2] at the VEPP-2M $e^+e^-$ collider [3] in the Budker Institute of Nuclear Physics (Novosibirsk).

It was suggested long time ago by Gell-Mann, Sharp and Wagner [4] that $\phi \to \pi^+\pi^-\pi^0$ decay proceeds via the $\rho\pi$ intermediate state. First evidence of $\rho\pi$ dominance was obtained in [5]. Later, experiments with the CMD-2 [6] and SND [7] detectors confirmed this conclusion and set upper limits on the non-$\rho\pi$ amplitude. However, some phenomenological models, for example, the effective Lagrangian approach [8–10], HLS [11] predict a contact term in this decay. Recently new results on $\phi \to \pi^+\pi^-\pi^0$ from the KLOE experiment [12] as well as preliminary results from the CMD-2 detector [13] were reported. The final results of the latter experiment are presented here.
were based on fully reconstructed 3π events. The cross section measurement as well as Dalitz plot studies were based on fully reconstructed 3π events. Partially reconstructed 3π events were used to determine corrections to the detection efficiency. The main data sample contains events with two charged particles (one positive and one negative) and two or more reconstructed photon clusters selected by the following criteria: All charged particles are required to hit the detector within the solid angle limited by the polar angle |θ – θ0| < 0.67 radians to provide high efficiency track reconstruction both in Z-chamber and DC. The same criterion was applied to the polar angles of γ-quanta to avoid edge effects for the detection efficiency in the CsI calorimeter. For charged particles:

- For each track the spread of the hits from the optimal helix in the (R–φ) plane σR < 0.1 cm and in the (R–Z) plane σZ < 3 cm (to be compared with the average spatial resolution of the DC: σR < 0.025 cm and σZ < 0.4 cm in the transverse and longitudinal directions).
- Tracks should be acollinear in the (R–φ) projection |φ2 – φ1| > 0.1 to reject Bhabha events and a space angle between tracks should be 0.1 < θ < 3.0 to reject events of γ conversion in the wall of the beam pipe.
- The closest approach of each track to the beam axis should be Rmin < 0.2 cm in the (R–φ) projection while the distance from a track to the interaction point along the beam direction should be |Z| < 10 cm.
- The momentum of each track is required to be Pπ < 500 MeV/c.
- To reject events with initial state radiation (ISR) of a hard photon we apply a cut on the absolute value of the 3π system total momentum |Pπ+ + Pπ– + Pγ0| < 100 MeV/c.
- Specific ionization losses in the DC should be dE/dx < 2(dE/dx)MIP to suppress charged kaons.

A neutral pion was identified by two photons with the energy of Eγ > 30 MeV each and invariant mass in the range of 80 < Mπγ < 170 MeV/c². If more than two photons were detected, we selected events where only one π0 candidate was found among all γγ combinations, Nπ0 = 1. Fig. 2 shows the Pπ+ versus Pπ– scatter plot of the selected experimental events at Ebeam = 509.5 MeV.

2. CMD-2 detector

The layout of CMD-2 (Cryogenic Magnetic Detector) is shown in Fig. 1. This general-purpose detector combines features of a spectrometer for detection of charged particles with good calorimetry for photons.

The tracking part of the detector consists of a cylindrical drift chamber (DC) and double-layer multiwire proportional chamber (Z-chamber). Outside the superconducting solenoid with a 1 T magnetic field a barrel electromagnetic calorimeter based on CsI scintillation crystals and muon range system are placed. To keep good energy resolution of the barrel calorimeter, the solenoid design was optimized to have a small thickness, 0.38 X0. The endcap calorimeter is made of BGO scintillation crystals. Together, the barrel and endcap calorimeters cover a solid angle of 0.38 steradians. This general-purpose detector combines features of a spectrometer for detection of charged particles with good calorimetry for photons.

One more important issue of this work is a precise measurement of the cross section σ3π (E), the φ → π⁺π⁻π⁰ branching ratio as well as parameters of φ → ω mixing. This information is important for the precise evaluation of the hadronic contribution to the muon anomalous magnetic moment (g – 2)μ [14] and for a test of various mixing models [15].

3. Selection of π⁺π⁻π⁰ events

The cross section measurement as well as Dalitz plot studies were based on fully reconstructed 3π events. Partially reconstructed events were used to determine corrections to the detection efficiency. The main data sample contains events with two charged particles (one positive and one negative) and two or more reconstructed photon clusters selected by the following criteria: All charged particles are required to hit the detector within the solid angle limited by the polar angle |θ – θ0| < 0.67 radians to provide high efficiency track reconstruction both in Z-chamber and DC. The same criterion was applied to the polar angles of γ-quanta to avoid edge effects for the detection efficiency in the CsI calorimeter. For charged particles:

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4. Background

The background for the studied decay mode can originate from true $e^+e^-$ interactions or from cosmic particles and beam interactions with the residual gas. This contribution was evaluated considering events from the sideband region $10 < |Z_{uk+}| < 20$ cm, and found to be negligible (less than 0.1%). MC simulation showed that the total background contamination is about 1% at the $\phi$-meson peak and increases up to 20% at the edge of the studied energy range. To evaluate the number of background events coming from $e^+e^-$ processes, we studied the $\gamma\gamma$ invariant mass distribution of $\pi^0$ candidates. Signal events group around the $\pi^0$ mass, while the $\gamma\gamma$ distribution of background events is flat for all background processes excluding $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ and $e^+e^- \rightarrow \phi \rightarrow \eta\gamma$, $\eta \rightarrow \pi^+\pi^-\pi^0$ processes (the contribution of the latter was found to be negligible). To extract the number of signal events at each energy point the $\gamma\gamma$ invariant mass distribution was approximated by a sum of a logarithmic Gaussian and a constant term. The mean value, width, asymmetry of logarithmic Gaussian as well as the fraction of signal events with wrong reconstructed $\pi^0$ having a flat $\gamma\gamma$ distribution, were fixed at their values found from the approximation at the $\phi$-meson peak, where background is small ($\sim 1\%$) and can be found from MC. The fraction of $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ events having a peak in

Fig. 2. The $P_+$ vs. $P_-$ scatter plot of the data at $E_{beam} = 509.5$ MeV. Also shown are: the $\pi^+\pi^-\pi^0$ allowed kinematic region, the calculated curve of $P_+(P_-)$ dependence for $K_LK_S$ events and two lines indicating location of $K^+K^-$ events.

Clearly seen are events of three types: $\pi^+\pi^-\pi^0$ inside the allowed kinematic region; $K_LK_S$, where $K_S \rightarrow \pi^+\pi^-$ events are along the calculated curve of $P_+(P_-)$ dependence; $K^+K^-$ events along two lines with $P(K^\pm) = 107$ MeV. Charged particles from the $K^\pm \rightarrow \mu^\pm\nu\mu$ decay can have momenta in the large range up to the 310 MeV/c. Two lines show the location of the events with one kaon decaying to lighter particles. Finally, we selected events above the lower boundary of the $3\pi$ allowed kinematic region to suppress events of $\phi \rightarrow K_SK_L$ decay, and applied a cut on the track momentum $P_\pi > 120$ MeV/c to reject events of $\phi \rightarrow K^+K^-$. With the above criteria, 104 849 events were selected in the c.m. energy range $\sqrt{s} = 984–1060$ MeV.

5. Detection efficiency

The value of the $\pi^+\pi^-\pi^0$ detection efficiency, $\varepsilon_{3\pi}^{MC} = (4.71 \pm 0.02)\%$, was obtained using a large MC sample of $10^6$ generated $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ events at the c.m. energy $\sqrt{s} = 1019.5$ MeV. However, Monte Carlo simulation cannot reproduce all details of the detector response. The corrections $\delta_{\mu\nu}(s)$ were introduced to take into account the difference between the track efficiency in Monte Carlo and the experimental one. These values were determined by comparing the numbers of selected events in the sample with one track and one $\pi^0$ and with two tracks and $\pi^0$. Similar procedure was used to determine $\pi^0$ efficiency corrections $\delta_{\mu\nu}^{MC}(s)$. The details of these procedures can be found in [17]. The total correction $\Delta^{MC}$ at each energy point, determined by the formula:

$$
(1 - \Delta^{MC}) = (1 - \delta_{\mu\nu}^{MC})(1 - \delta_{\nu\mu}^{MC})(1 - \varepsilon_{3\pi}^{MC}).
$$

is shown in Fig. 3.

6. Cross section

A visible cross section of the process $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ is calculated according to the formula:

$$
\sigma_{vis} = \frac{N_{3\pi}}{\mathcal{L}\varepsilon_{trig}\varepsilon_{3\pi}^{MC}(1 + \delta_{\mu\nu})(1 - \Delta^{MC})} = (1 + \delta_{rad})\sigma_B,
$$

where: $N_{3\pi}$—number of $3\pi$ events, $\mathcal{L}$—integrated luminosity, $\varepsilon_{trig}$—trigger efficiency, $\varepsilon_{3\pi}^{MC}$—$3\pi$ MC detection efficiency, $\Delta^{MC}$—efficiency correction, $\delta_{rad}$—takes into account the effect of the beam energy spread ($\sigma_E = 300$ keV), $\sigma_B$—$3\pi$ Born cross section, $\delta_{rad}$—correction due to the initial state radiation. Table 1 shows the values of $\sigma_{vis}$ and $\sigma_B$ for different c.m. energies. The value of $\varepsilon_{trig}$ is determined from the experimental data sets,
triggered by a signal from one of the two independent subsystems (CsI or DC), and by a signal from both subsystems. It was found to be higher than 98% for all c.m. energies. The procedure is discussed in detail in [17].
The important issue for the evaluation of $\sigma_B$ is an accurate account of the radiative corrections. In this work the approach developed in [18] was used which assumes that both electron and positron emit photon jets in the forward direction. In this case the visible cross section is related to $\sigma_B$ as

$$\sigma_{\text{vis}}(s) = \int \frac{1}{s} e^x_1 dx_1 \int \frac{1}{x_2} D(x_1, s) D(x_2, s) \sigma_B(s) \epsilon(x_1, x_2),$$

where: $D(x_1, s)$—the probability function for initial $e^\pm$ to emit a $\gamma$-quantum jet carrying $x_1, 2$ part of $e^\pm$ energy $\sqrt{s}/2$, $\sigma_B(s')$—the Born cross section at $s' = s(1 - x_1)(1 - x_2)$, $\epsilon(x_1, x_2)$—efficiency function, which is defined as a detection efficiency for a boosted (due to $\gamma$-quantum radiation) $\pi^+\pi^-\pi^0$ system normalized to that at $x_1 = x_2 = 0$ and calculated using $3\pi$ MC simulation of $10^6$ events with initial state radiation. The Born cross section is dominated by the $\omega$ and $\phi$ contributions:

$$\sigma_B(s) = \frac{F_{3\pi}(s)}{s} A_\omega + A_\phi e^{i\delta_{\phi\omega}} + A_{\text{add}}^2,$$

where: $A_\omega, A_\phi$—$\omega$ and $\phi$ meson amplitudes, $\delta_{\phi\omega}$—$\phi-\omega$ interference phase, and the constant term $A_{\text{add}}$ takes into account the contributions of the higher mass vector resonances (such as $\omega', \omega''$) around the $\phi$-meson. The detailed description of the $A_\omega, A_\phi$ parametrisation can be found elsewhere [6,15,17].

The experimental $\sigma_{\text{vis}}(s)$ values were approximated by the function given by Eq. (3). Table 2 shows the results of two fits. For both fits the $\phi$-meson mass $m_\phi$, its width $\Gamma_\phi$ and $3\pi$ peak cross section $\sigma_{3\pi}$ were free parameters. However, our sensitivity is not sufficient to keep free both $\phi_{\omega\phi}$ and $A_{\text{add}}$. So $\phi_{\omega\phi}$ is free in the Fit 1 and fixed at $180^\circ$ in the Fit 2. All the other parameters were fixed at their world average values [19] within their uncertainties. Fig. 4 demonstrates the visible cross section for experimental data with an optimal curve for the Fit 1.

From Table 2 good fit quality can be seen for both approximations. The Fit 1 was chosen as the main one, with both statistical (first) and systematic (second) uncertainties shown for its optimal parameters.

To determine the $3\pi$ Born cross section, the radiative correction was calculated according to Eq. (3) and then applied to the experimental values of the visible cross section

$$\sigma_B(s) = \sigma_{\text{vis}}(s)/(1 + \delta_{\text{rad}}(s)).$$

### Table 2

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Fit 1</th>
<th>Fit 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_\phi, \text{MeV}/c^2$</td>
<td>$1019.30 \pm 0.02_{\text{stat}} \pm 0.10_{\text{syst}}$</td>
<td>$1019.33 \pm 0.03$</td>
</tr>
<tr>
<td>$\Gamma_\phi, \text{MeV}$</td>
<td>$4.30 \pm 0.06_{\text{stat}} \pm 0.17_{\text{syst}}$</td>
<td>$4.26 \pm 0.06$</td>
</tr>
<tr>
<td>$\phi_{\omega\phi}$</td>
<td>$167^\circ \pm 14^\circ_{\text{stat}} \pm 10^\circ_{\text{syst}}$</td>
<td>$180^\circ$-fixed</td>
</tr>
<tr>
<td>$\sigma_{3\pi}, \text{nb}$</td>
<td>$637 \pm 23_{\text{stat}} \pm 16_{\text{syst}}$</td>
<td>$658 \pm 7$</td>
</tr>
<tr>
<td>$A_{\text{add}}, \sqrt{\text{nb}/\text{MeV}^2}$</td>
<td>$0$-fixed</td>
<td>$22 \pm 8$</td>
</tr>
<tr>
<td>$\chi^2/\text{Ndf}$</td>
<td>$57.0/50$</td>
<td>$51.8/50$</td>
</tr>
<tr>
<td>$P(\chi^2)$, %</td>
<td>$23$</td>
<td>$40$</td>
</tr>
</tbody>
</table>

Fig. 4. Visible cross section with an optimal curve for the Fit 1 (see Table 1).

To calculate an additional uncertainty related to the radiative corrections, we performed toy MC simulation of the Born cross section shape determined by the fit parameters.

### 7. Analysis of $\phi \rightarrow \pi^+\pi^-\pi^0$ dynamics

For this analysis the Dalitz plot in $X = \frac{E_{\pi^+} - E_{\pi^-}}{\sqrt{s}}$ and $Y = \sqrt{s} - E_{\pi^-} - E_{\pi^+} - m_{\pi^0}$ variables was used. The kinematically allowed region was divided into 198 square bins of $20 \times 20 \text{ MeV}$ size. 79 577 experimental $3\pi$ events from the c.m. energy range $\sqrt{s} = 1017$–1021 MeV were selected for this analysis. In this c.m. energy region background is about 1%, and its influence was found to be negligible. In addition to the selection criteria mentioned in Section 3, a constrained fit taking into account total energy–momentum conservation was applied.

To fit the Dalitz plot distribution, a model incorporating the $\rho\pi$ mechanism and non-$\rho\pi$ contribution described by a contact amplitude was used. The expected number of events in the bin number $k$ is given by expression

$$N_k^{\text{th}} = N_0 \int dX dY |\vec{P}_+ \times \vec{P}_-|^2 A_{\text{Rain}} e^{i\phi} + A_{\rho\pi}|^2,$$

where: $N_0$—normalization parameter proportional to the total number of $3\pi$ events, $A_{\rho\pi}$—$\rho\pi$ contribution to the amplitude determined from formula

$$A_{\rho\pi} = \frac{1}{D_{\rho^+}(Q^2_\rho)} + \frac{1}{D_{\rho^-}(Q^2_\rho)} + \frac{1}{D_{\rho}(Q^2_\rho)},$$

where the $\rho^i$-meson ($i = +, -, 0$) propagator is written in the form

$$\frac{1}{D_{\rho^i}(Q^2_\rho)} = \frac{Q^2_\rho}{M_{\rho^i}^2} - i \frac{\sqrt{Q^2_\rho} f_{\rho^i}(Q^2_\rho) + M_{\rho^i}^2}{M_{\rho^i}^2}.$$
The non-$\rho\pi$ amplitude includes: a normalization coefficient $A_0=7.52$ determined by expression
\[
\int dXdY |\vec{P}_+ \times \vec{P}_-|^2 |A_{\rho\pi}|^2
\]
for the Dalitz plot distribution,
\[
= |A_0|^2 \int dXdY |\vec{P}_+ \times \vec{P}_-|^2,
\] (9)
an absolute value of the normalized contact amplitude $a$ and a phase of the contact amplitude $\varphi$. The calculated number of events in the $i$th cell is given by expression:
\[
N_{i}^{\text{calc}} = \varepsilon_{ik}N_{i}^{\text{th}},
\] (10)
where $\varepsilon_{ik}$ is $198 \times 198$ matrix of the detector apparatus function. Due to the non-ideal reconstruction and finite resolution of the detector, a $3\pi$ event initially produced in the bin number $k$ can be found in the bin number $i$, so along with the detection efficiency for every bin, $\varepsilon_{ik}$ provides a matrix of bin-to-bin transition probabilities. It also takes into account the effect of Dalitz plot distortion due to the initial state radiation. Full $3\pi$ MC simulation with initial state radiation was used to extract $\varepsilon_{ik}$.

To approximate the Dalitz plot distribution we minimize the $\chi^2$ function
\[
\chi^2 = \sum_{i=1}^{198} \frac{(N_{i}^{\exp} - N_{i}^{\text{calc}})^2}{\sigma_i^2(N^{\text{calc}})},
\] (11)
where: $N_{i}^{\exp}$ — number of experimental events, $N_{i}^{\text{calc}}$, $\sigma_i(N^{\text{calc}})$ — calculated number of events and its error. Free parameters of the fit are: $N_0$, $a$ and $\varphi$. Fig. 5 shows cuts along the $Y$ axis for different $X$ values of the experimental Dalitz plot distribution (points) and fit result (histograms). The obtained optimal parameters are given in Table 3. Also shown are the values of the contact term found by KLOE\(^1\) [12], SND [7], and CMD-2 [6] groups.

Table 3

<table>
<thead>
<tr>
<th>Group</th>
<th>$a$ (stat)</th>
<th>$\varphi$ (stat)</th>
<th>$\chi^2$/Ndf</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMD-2</td>
<td>$0.101 \pm 0.044$</td>
<td>$-2.91 \pm 0.14$</td>
<td>0.95</td>
<td>3.3σ</td>
</tr>
<tr>
<td>This work</td>
<td>$0.144 \pm 0.01$</td>
<td>$-2.47 \pm 0.08$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>KLOE (2003)</td>
<td>$0.104 \pm 0.01$</td>
<td>$-2.47 \pm 0.08$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SND (2002)</td>
<td>$-0.06 &lt; a &lt; 0.06$</td>
<td>$\varphi = 0$-fixed</td>
<td>90% CL</td>
<td></td>
</tr>
<tr>
<td>CMD-2 (1998)</td>
<td>$-0.15 &lt; a &lt; 0.10$</td>
<td>$\varphi = 0$-fixed</td>
<td>90% CL</td>
<td></td>
</tr>
</tbody>
</table>

\(^1\) In [12, p. 8] $A_{\rho\pi} = a_\rho e^{i\phi_\rho}$ should be read as $A_{\rho\pi} = a_\rho e^{-i\phi_\rho}$ — private communication with Dr. C. Bini.

Fig. 5. Result of the fit. Slices of the Dalitz plot distribution (right half) along the $Y$ axis for different values of $X$ are shown. Points are experimental data, histogram—calculated numbers.
Free parameters of the fit were: \( N_0, a', \) and \( \varphi' \). The obtained values are: 
\[
a' = 0.215 \pm 0.092 \pm 0.036 \quad \text{and} \quad \varphi' = 0.177 \pm 0.132 \pm 0.051, \]
where the first error is statistical and the second is systematic.

8. Systematic uncertainties

The systematic uncertainty on the \( \phi \)-meson mass—\( m_\phi \) is evaluated to be 0.1 MeV dominated by the energy determination procedure. The systematic uncertainties on the total \( \phi \)-meson width—\( \Gamma_\phi \) and \( \phi \rightarrow \omega \) interference phase—\( \varphi_{\phi-\omega} \) were evaluated approximating cross section data for three different energy scans, and found to be 0.17 MeV and 10\(^\circ\), respectively. The dominant contribution to the \( \sigma_{3\pi} \) systematic uncertainty comes from the uncertainty on the integrated luminosity (2\%). The systematic uncertainty on the radiative corrections is determined by the efficiency function \( \epsilon(x_1, x_2) \) error (\( \approx 1\% \)) and theoretical uncertainty of the method itself (0.2\%) [18]. The systematic uncertainty on the detection efficiency consists of a 0.4\% uncertainty due to the limited 3\( \pi \) MC statistics, 0.3\% uncertainty on the track efficiency correction and 0.2\% uncertainty on the \( \pi^0 \) efficiency correction. We assigned a 1\% systematic uncertainty related to the trigger efficiency. The 0.3\% systematic uncertainty due to the background subtraction was found applying two different background subtraction procedures. A total uncertainty of 2.5\% was obtained by adding all the contributions in quadrature.

The systematic uncertainties on the value of the contact amplitude come from the non-uniformity of the detection efficiency over the Dalitz plot—0.017 for \( a \) and 0.07 for \( \varphi \); the uncertainty on the \( \rho \)-meson parameters—0.003 for \( a \) and 0.03 for \( \varphi \). The model uncertainty was evaluated applying two different parametrizations of the \( \rho \)-meson shape—relativistic Breit–Wigner and Gounaris–Sakurai formula [22]. This difference was found to be negligible. The total systematic uncertainty was obtained by adding all the contributions in quadrature.

9. Conclusions

The cross section of the process \( e^+e^- \rightarrow \pi^+\pi^-\pi^0 \) was measured in the c.m. energy range from 984 to 1060 MeV. The mass and width of the \( \phi \)-meson as well as \( \omega-\phi \) mixing phase are shown in the second column of Table 2. They do not contradict to the world average values [19]. Our result on the 3\( \pi \) peak cross section is
\[
\sigma_{3\pi} = (637 \pm 23_{\text{stat}} \pm 16_{\text{syst}}) \text{ nb}.
\]

We calculate the product of the \( \phi \rightarrow \pi^+\pi^-\pi^0 \) branching ratio and \( \phi \rightarrow e^+e^- B_{\phi e} B_{\pi^0} \) according to the relation
\[
B_{ee} B_{\pi^0} = \frac{\sigma_{3\pi} M_\phi^2}{12\pi} = (4.51 \pm 0.16 \pm 0.11) \times 10^{-5}.
\]

Our result is in good agreement with the previous measurements coming from CMD-2 [6], SND [20] and BaBar [21] and has the best total accuracy.

Dalitz plot analysis showed good agreement between the CMD-2 and KLOE results, see Table 3. The determined value of the non-\( \rho\pi \) amplitude is consistent with the theoretical estimations of the contact term [8,9]. Under the assumption that the non-\( \rho\pi \) amplitude is dominated by the \( \rho(1450)\pi \) mechanism the ratio of the hadronic coupling constants was extracted:
\[
\frac{g_{\phi\rho}^e g_{\phi\pi}^e}{g_{\phi\rho}^\pi g_{\phi\pi}^\pi} = 0.215 \pm 0.092 \pm 0.036.
\]

Acknowledgements

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