Physics A - PHY 2048C

Rotational Motion and Torque

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11/06/2019

My Office Hours:
Thursday 2:00 - 3:00 PM

212 Keen Building

Warm-up Questions

1. Did you read Chapter 12 in the textbook on Rotational Motion?
2. Must an object be rotating to have a moment of inertia? Briefly explain.
3. Can torque trigger rotational motion while the net force on the system is zero?

Outline

Forces

It is important to distinguish between internal and external forces:

- Internal forces act between the particles of the system:

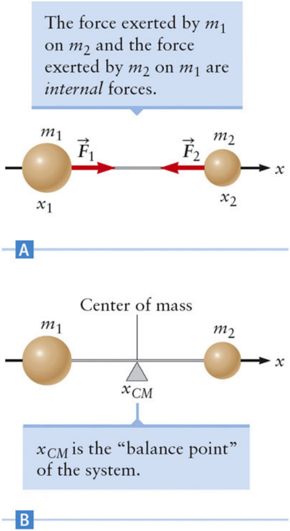
$$\sum F_{\text{int}} = 0$$

action-reaction pairs

- External forces come from outside the system:

$$\sum F_{\text{ext}} = M_{\text{total}}\, a_{\text{c.m.}}\ ,$$

where "c.m." stands for center of mass.



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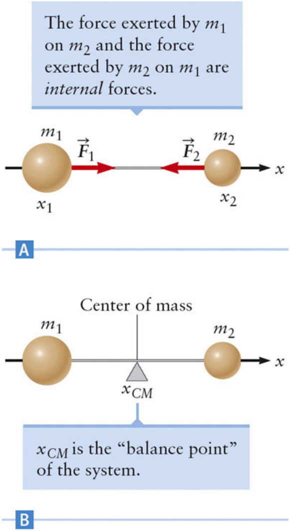
Center of Mass

What is the center of mass?



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Center of Mass

What is the center of mass?

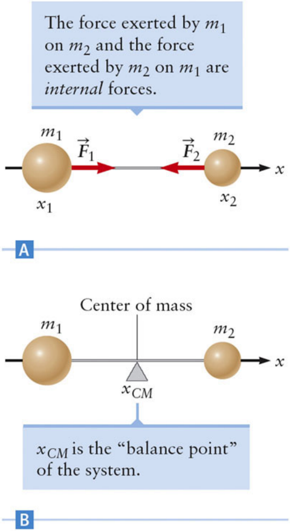
The center of mass can be thought of as the balance point of the system:

$$x_{\text{c.m.}} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{\sum_i m_i x_i}{M_{\text{tot}}}$$

$$y_{\text{c.m.}} = \frac{\sum_i m_i y_i}{\sum_i m_i} = \frac{\sum_i m_i y_i}{M_{\text{tot}}}$$

In three dimensions also:

$$z_{\text{c.m.}} = \frac{\sum_i m_i z_i}{\sum_i m_i} = \frac{\sum_i m_i z_i}{M_{\text{tot}}}$$



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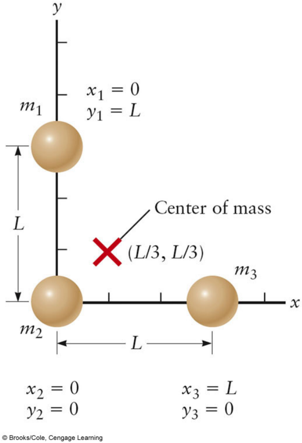
Center of Mass: Example

All the point particles must be included in the center of mass calculation:

- For a symmetric object, the center of mass is the center of symmetry of the object.
- The center of mass need not be located inside the object.

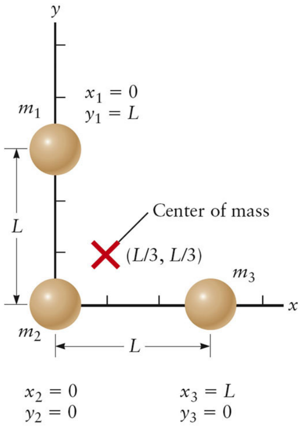


$$x_{\text{c.m.}} = \frac{m_1\,x_1 + m_2\,x_2 + m_3\,x_3}{m_1 + m_2 + m_3} = \frac{m_1 \cdot 0 + m_2 \cdot 0 + m_3 \cdot L}{m_1 + m_2 + m_3} = \frac{m \cdot L}{3\,m} = L/3$$

Center of Mass: Example

All the point particles must be included in the center of mass calculation:

- For a symmetric object, the center of mass is the center of symmetry of the object.
- The center of mass need not be located inside the object.



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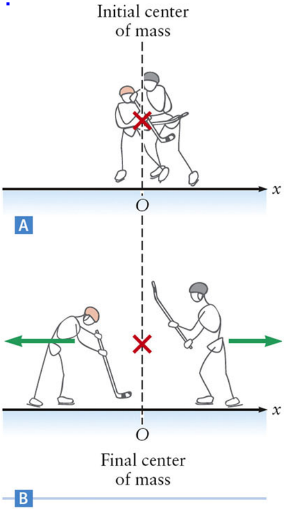
$$y_{\text{c.m.}} = \frac{m_1\,y_1 + m_2\,y_2 + m_3\,y_3}{m_1 + m_2 + m_3} = \frac{m_1 \cdot L + m_2 \cdot 0 + m_3 \cdot 0}{m_1 + m_2 + m_3} = \frac{m \cdot L}{3\,m} = L/3$$

Motion of the Center of Mass

The two skaters push off from each other:

- No friction, so momentum is conserved.
- The center of mass does not move although the skaters separate.
- Center of mass motion is caused only by the external forces acting on the system.



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Question 1

An hourglass timer is first weighed when all the sand is in the lower chamber. The hourglass is then turned over and placed on the scale. While the sand falls, but before sand hits the bottom, the balance will show

A more weight than before.

B the same weight as before.

C less weight than before.

D unpredictable results.

Question 2

An hourglass timer is first weighed when all the sand is in the lower chamber. The hourglass is then turned over and placed on the scale. While the sand falls from the upper chamber in a steady stream and hits the bottom, the balance will show

A more weight than before.

B the same weight as before.

C less weight than before.

D unpredictable results.

Question 3

An hourglass timer is first weighed when all the sand is in the lower chamber. The chamber is then turned over and placed on the scale. When the upper chamber runs out of sand, but sand is still hitting the bottom, the balance will show

A more weight than before.

B the same weight as before.

C less weight than before.

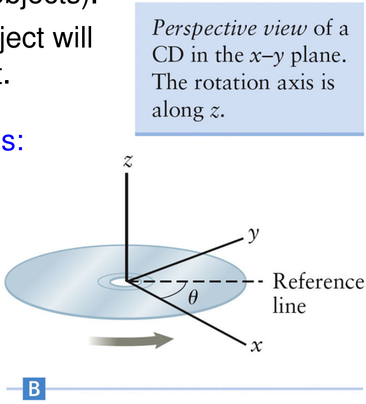
D unpredictable results.

Outline

Rotational Motion

So far, objects have been treated as point particles:

- Newton's Laws apply to point particles as well as all other types of particles (extended objects).
- The size and shape of the object will have to be taken into account.



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Need to define rotational quantities:

1. Angular position
2. Angular velocity
3. Angular acceleration

Angular Velocity

The angular velocity, ω, describes how the angular position is changing with time.

For some time interval, Δt, the *average angular velocity* is:

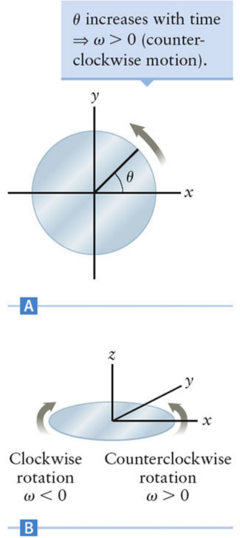
$$\omega_{\text{ave}} = \frac{\Delta\theta}{\Delta t}$$

The *instantaneous angular velocity* is:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

Units are rad/s:

- May also be rpm. (revolutions / minute)



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Angular Velocity

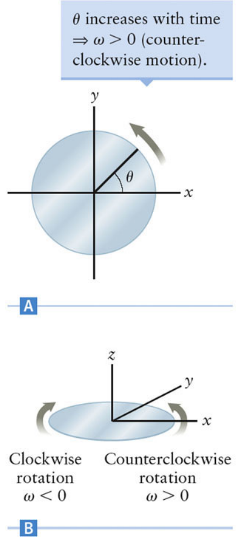
The angular velocity, ω, describes how the angular position is changing with time.

Since angular velocity is a vector quantity, it must have a direction:

- If θ increases with time, then ω is positive.

Therefore:

1. A counterclockwise rotation corresponds to a positive angular velocity.
2. Clockwise would be negative.



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Angular Acceleration

The angular acceleration, α, is the rate of change of the angular velocity.

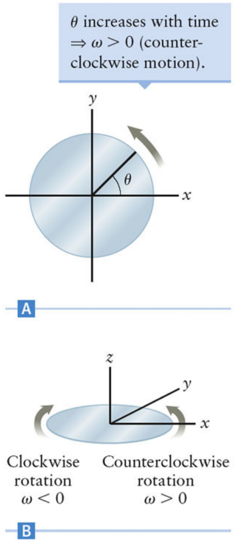
For some time interval, Δt, the *average angular acceleration* is:

$$\alpha_{\text{ave}} = \frac{\Delta \omega}{\Delta t}$$

Instantaneous angular acceleration is:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$$

Units are rad/s^2.



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Angular Acceleration

Angular acceleration and centripetal acceleration are different.

As an example, assume a particle is moving in a circle with a constant linear velocity:

- The particle's angular position increases at a constant rate, therefore its angular velocity is constant.
- Its angular acceleration is 0.
- Since it is moving in a circle, it experiences a centripetal acceleration of:

$$a_c = \frac{v^2}{r}$$

- The centripetal acceleration refers to the linear motion of the particle.
- The angular acceleration is concerned with the related angular motion.

Angular and Linear Velocities

When an object is rotating, all the points on the object have the same angular velocity:

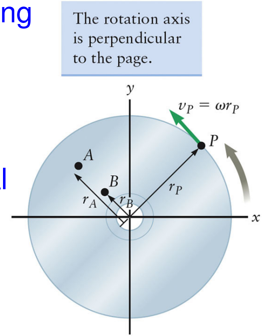
- Makes ω a useful quantity for describing the motion.
- The linear velocity is not the same for all points. (depends on distance from rotational axis)

The linear velocity of any point on a rotating object is related to its angular velocity by:

$$v = \omega r\,,$$

where r is the distance from the rotational axis to the point. For $r_A > r_B$:

$$v_A > v_B$$



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The linear velocity of any point on a rotating object is related to its angular velocity by:

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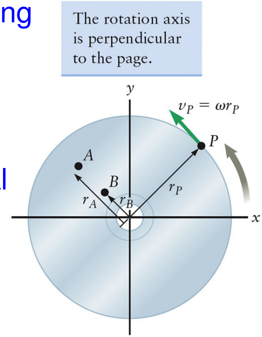
where r is the distance from the rotational axis to the point. Similarly:

$$a = \alpha r$$



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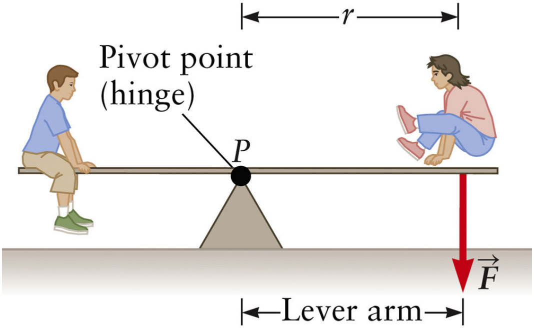
Outline

Torque

A connection between force and rotational motion is needed. Specifically, how forces give rise to angular accelerations.



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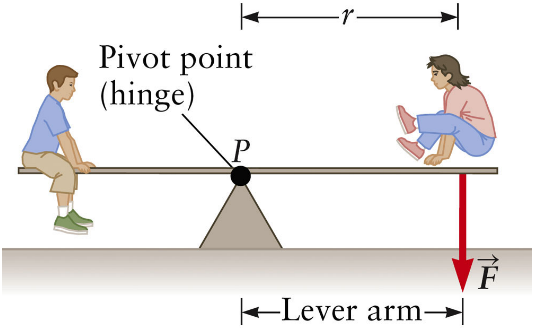
Torque

A connection between force and rotational motion is needed. Specifically, how forces give rise to angular accelerations.

Torque is the product of an applied force and the distance it is applied from the support point. It is denoted by τ.



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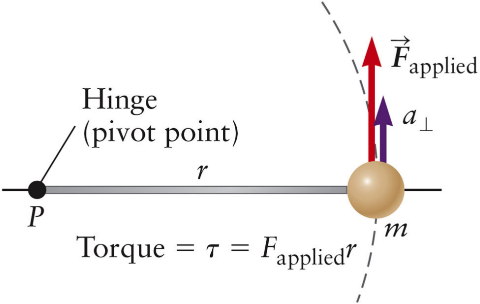
Torque

Torque is the product of an applied force and the distance it is applied from the support point. It is denoted by τ.

When the force is perpendicular to a line connecting its point of application to the pivot point, the torque is given by:

$$\tau = F\,r \qquad \text{Unit [N m]}$$



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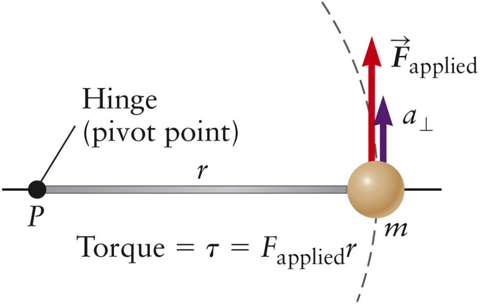

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In general:

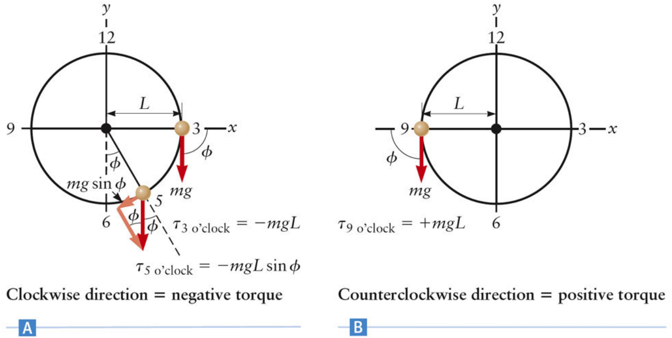
$$\vec{\tau} = \vec{F} \times \vec{r}$$

$$|\vec{\tau}| = |\vec{F}|\,|\vec{r}|\sin\theta$$

Torque and Direction

For a single rotation axis, the direction of the torque is specified by its sign:

1. A positive torque is one that would produce a counterclockwise rotation.
2. A negative torque would produce a clockwise rotation.



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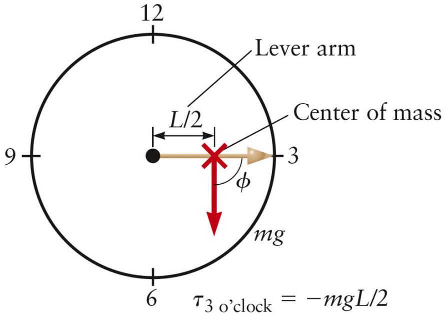
Torque and Direction

We could imagine breaking the clock hand up into many infinitesimally small pieces and finding the torques on each piece.

A more convenient approach is to use the **center of gravity** of the hand. The clockwise torque will be negative.



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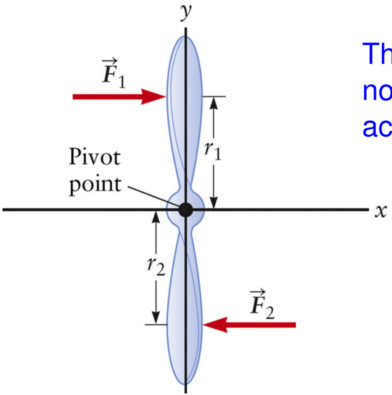
For the purposes of calculating the torque due to a gravitational force, you can assume all the force acts at a single location:

center of gravity
= center of mass.

Rotational Equilibrium

An object can be in equilibrium with regard to both its translation and its rotational motion. Its linear acceleration must be zero and its angular acceleration must be zero.



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The total force being zero is not sufficient to ensure both accelerations are zero:

$$\tau = \vec{F}_1 r_1 + \vec{F}_2 r_2 \neq 0$$

$$F_1 + F_2 = 0$$

Lever Arm and Torque

In linear motion, a force is responsible for a change in the acceleration (according to Newton's Second Law):

$$\vec{F} = m\vec{a}$$

In rotational motion, Newton's Second Law can be written as:

$$\sum \vec{\tau} = I\vec{\alpha}$$

Analogy with translational motion:

translational motion	rotational motion
force, $\vec{F}$	torque, $\vec{\tau}$
mass, m	moment of inertia, I
acceleration, $\vec{a}$	angular acceleration, $\vec{\alpha}$

Moment of Inertia

The moment of inertia enters into rotational motion in the same way that mass enters into translational motion.

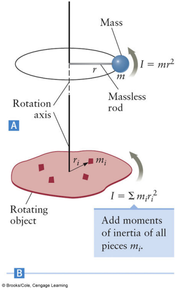
For a point mass, the moment of inertia is:

$$I = m r^2$$

For an object composed of many pieces of mass located at various distances from the pivot point, the moment of inertia of the object is:

$$I = \sum_i m_i r_i^2$$

- I depends on the mass and also on how that mass is distributed relative to the axis of rotation.



Moment of Inertia

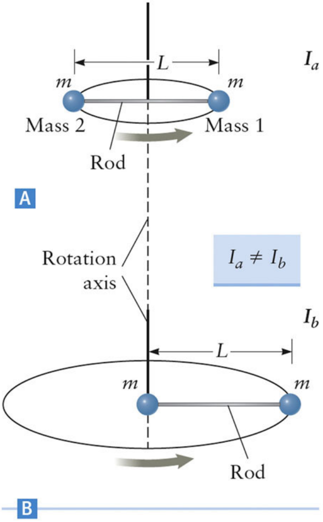
The value of I depends on the choice of rotation axis.

In the figure, m and L are the same:

A $I_A = m(\frac{L}{2})^2 + m(\frac{L}{2})^2 = m\frac{L^2}{2}$

B $I_B = 0 + mL^2 = mL^2 \neq I_A$

Moments of inertia are different due to the difference in rotation axes.



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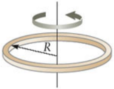
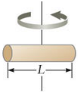
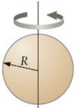
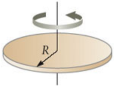
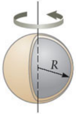
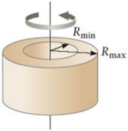
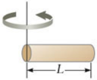
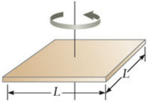
Various Moments of Inertia

Table 8.2 Moment of Inertia for Some Common Objects[a]

Object	Shape	Object	Shape
Hoop $I = mR^2$		Rod pivoted at center $I = \frac{1}{12}mL^2$	
Solid sphere $I = \frac{2}{5}mR^2$		Pulley/cylinder/disc $I = \frac{1}{2}mR^2$	
Spherical shell $I = \frac{2}{3}mR^2$		Wheel or hollow cylinder $I = \frac{1}{2}m(R_{max}^2 + R_{min}^2)$	
Rod pivoted at one end $I = \frac{1}{3}mL^2$		Solid square plate with axis perpendicular to plate $I = \frac{1}{6}mL^2$	

[a]In each case, m is the total mass of the object.

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Outline

Kinematic Relationships

Once the total torque and moment of inertia are found, the angular acceleration can be calculated: $\sum \tau = I\alpha$.

For constant angular acceleration:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Table 8.3 Kinematic Relations for Constant Angular Acceleration α and Corresponding Relations for Linear Motion with Constant Acceleration a

Rotational Motion	Equation Number	Translational Motion
$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	8.35	$x = x_0 + v_0 t + \frac{1}{2}at^2$
$\omega = \omega_0 + \alpha t$	8.34	$v = v_0 + at$
$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	8.36	$v^2 = v_0^2 + 2a(x - x_0)$

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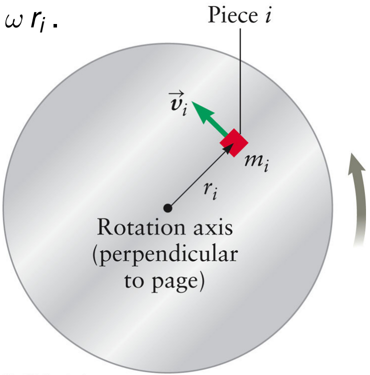
Outline

Kinetic Energy of Rotation

For a single point particle of mass m moving with a linear speed v, the kinetic energy is $KE = 1/2mv^2$.

Rotational motion is concerned with extended objects:

- Each small piece has the KE of a point particle with speed $v_i = \omega\, r_i$.



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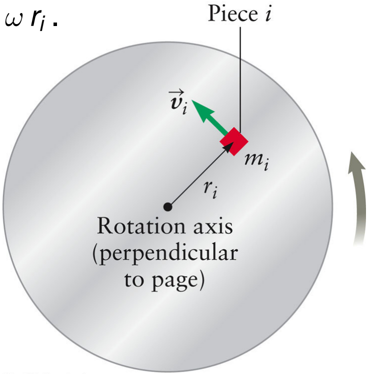
Rotational motion is concerned with extended objects:

- Each small piece has the KE of a point particle with speed $v_i = \omega\, r_i$.

The total KE energy of the object can be found by adding up all the kinetic energies of the small pieces:

$$KE_{\text{rot}} = \sum_i \frac{1}{2} m\, v_i^2 = \sum_i \frac{1}{2} m\, (\omega\, r_i)^2$$



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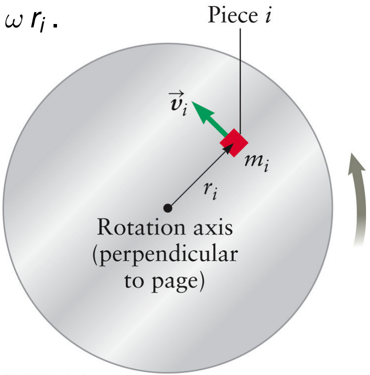

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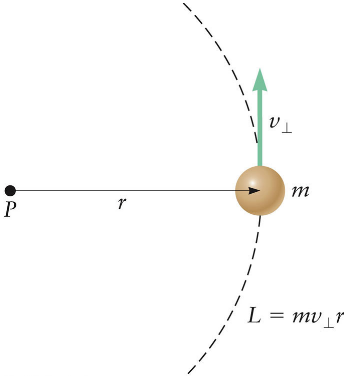
$$KE_{\text{rot}} = \sum_i \frac{1}{2} m\, v_i^2 = \sum_i \frac{1}{2} m\, (\omega\, r_i)^2 = \frac{1}{2} I\, \omega^2$$

Outline

Angular Momentum

Linear momentum was defined as:

$$\vec{p} = m\vec{v}$$



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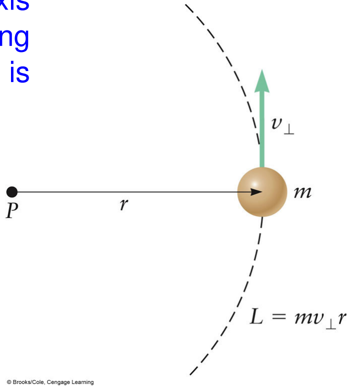
Angular Momentum

Linear momentum was defined as:

$$\vec{p} = m\vec{v}$$

In the case of a single rotation axis that does not change direction during the motion, the angular momentum is given by (figure to the right):

$$\begin{aligned}|\vec{L}| &= I\,\omega \\ &= I\,\frac{v_{\text{perpendicular}}}{r} \\ &= m\,r^2\,\frac{v_{\text{perpendicular}}}{r} \\ &= m\,r\,v_{\text{perpendicular}}\end{aligned}$$

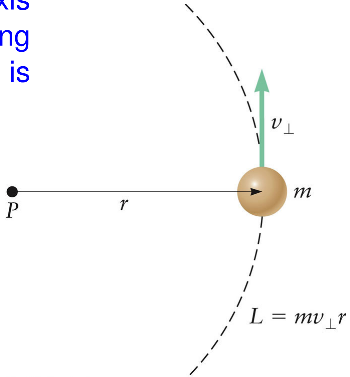


Angular Momentum

A rotating object will maintain its angular momentum provided no external torque acts on it. In this case, the total angular momentum of the object will be conserved.

In the case of a single rotation axis that does not change direction during the motion, the angular momentum is given by (figure to the right):

$$\begin{aligned} |\vec{L}| &= I\,\omega \\ &= I\,\frac{v_{\text{perpendicular}}}{r} \\ &= m\,r^2\,\frac{v_{\text{perpendicular}}}{r} \\ &= m\,r\,v_{\text{perpendicular}} \end{aligned}$$



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Angular Momentum

Example of conservation of angular momentum: $L = I\omega$.

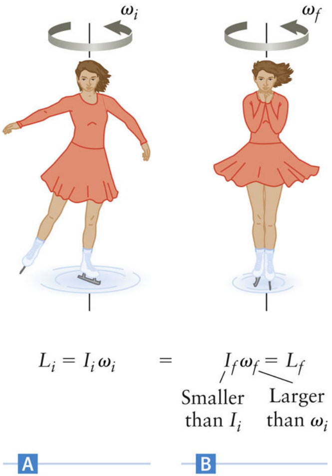
The skater has no external torque acting on her:

- Assume the ice is frictionless.
- The normal force and gravity do not produce torques.

Pulling both her arms and legs in decreases her moment of inertia.

- Her mass is distributed closer to the axis of rotation.

Since her total angular momentum is conserved, her angular velocity increases.



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Angular Momentum

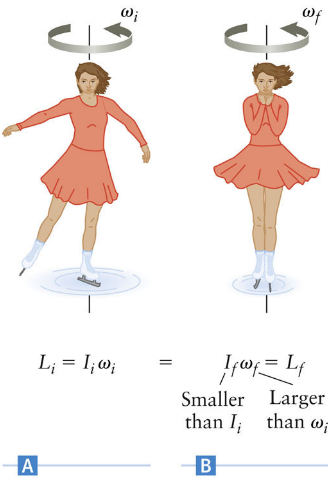
Remember: In inelastic collisions momentum is conserved, but kinetic energy is not.

Even when angular momentum is conserved, kinetic energy may not be conserved. With $L_i = L_f$:

$$KE_i = \frac{1}{2} I_i \omega_i^2 \qquad KE_f = \frac{1}{2} I_f \omega_f^2$$

$$KE_f = \frac{I_i}{I_f} KE_i$$

The skater does work to pull in her arms.



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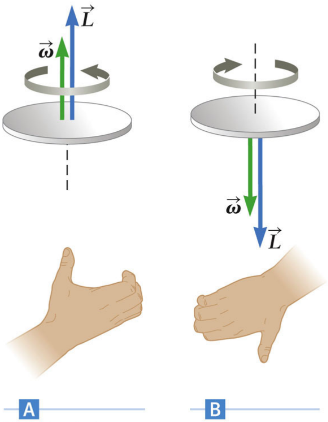
Angular Momentum

In many applications, recognizing the vector nature of rotational quantities is very important.

The right-hand rule provides a way to determine the direction:

- If the fingers of your right hand curl in the direction of motion of the edge of the object, your thumb will point in the direction of the rotational velocity vector.



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