Rotational Dynamics

Purpose: Investigate rotational dynamics – moment of inertia, torque and conservation of angular momentum and energy

Apparatus: Rotary sensor with Airlink, rod with two equal masses, pulley, mass with hooks (10 g, 20 g and 50 g), Capstone software, triple-beam balance, Vernier caliper and ruler.

Note: DO NOT drop the rotary sensor or the Airlink.

Introduction

To set an object into motion, a net force must be applied to the object. This is Newton's Second Law of Motion.

$$\vec{F}_{net} = m\vec{a} \tag{1}$$

To set an object into rotation, a net torque must be applied to the object. The analogous Newton's Second Law for rotation is

$$\vec{\tau}_{net} = I\vec{\alpha} \tag{2}$$

Torque is defined as the lever arm times the force. For example, the torque about the center of the disk in the Figure 2 is $\tau = R_{\perp}F$ where $R_{\perp}$ is the lever arm and, F is the applied force. The symbol I is the moment of inertia. It is a measure of "resistance" to the change in rotational speed. The moment of inertia for a point mass m that is at a distance R from the axis of rotation is

$$I = mR^2 \tag{3}$$

For several point masses, the total moment of inertia is the sum of the individual moment of inertia. In the linear case, an object of mass m moving with velocity $\vec{v}$ has a linear momentum, $\vec{p} = m\vec{v}$. Similarly, an object that is rotating has an angular momentum given by $\vec{L} = I\vec{\omega}$ where ω is the angular velocity. Just as linear momentum is conserved in a collision in the absence of an external force, so too is angular momentum when there is no net external torque. For example, if a second object is dropped onto an object that is spinning, conservation of angular momentum says that

$$\vec{L}_i = \vec{L}_f \quad or \quad I_i\vec{\omega}_i = I_f\vec{\omega}_f \tag{4}$$

An object that is moving has kinetic energy. The kinetic energy associated with rotation is given by

$$KE_{rot} = \frac{1}{2} I \omega^2 \quad [5]$$

Equation [2] says that when a net torque is applied to an object, its angular velocity changes, i.e., it has an angular acceleration. For this experiment, you will be applying a net torque to various objects and measure the angular velocity. The slope of the angular velocity vs time plot gives the angular acceleration. Then the moment of inertia can be calculated using equation [2]. You will also verify that angular momentum is conserved in a collision.

Procedure

Activity 1 – Moment of inertia of a rod and two equal masses

To measure the moment of inertia of an object, a torque must be applied. Here the torque is applied to the rotary sensor. By measuring the angular velocity of rotation as a function of time, the angular acceleration, α, can be calculated. Then equation [2] is used to determine the moment of inertia, I. For this part, you will measure the moment of inertia of a rod with the axis of rotation in the middle of the rod. Then you will attach two equal masses to the rod at equidistant from the axis of rotation. The aim is to verify that the moment of inertia for a point mass is given by $I = mR^2$. The rotary sensor you are using has very low friction, so in all the following activities, the frictional torque is neglected. Also, the two pulleys have moments of inertia much smaller than the objects you will be measuring so their contributions are negligible.

1. Use the triple beam balance and measure the mass of the rod (m_{rod}) and the two small masses. Use a ruler and measure the length, L, of the rod.
2. Attach the Airlink to the rotary sensor and turn it on. Next, start the Capstone software and link the sensor by the Airlink ID. Create an angular velocity vs time plot on Capstone. Set the units to be rad/s for angular velocity and the sampling rate to 40 Hz.
3. Mount the rotary sensor as shown in Figure 1. For this part mount the rod *without the two masses*. Hang a 20 g mass on the free end of the thread that is attached to the three-step pulley. Wrap the thread a few times around the middle wheel of the three-step pulley and then drape the thread over the super pulley. Make sure the *pulley is aligned so that the thread lines up with the groove of the super pulley*. Also, make sure that the thread connecting the two pulleys is horizontal. Hold the mass

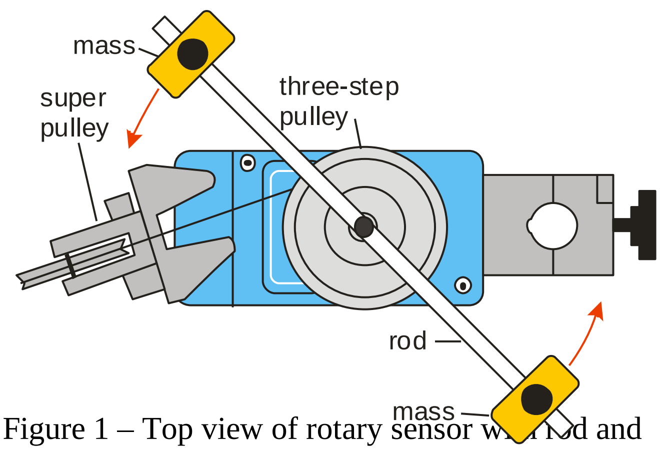

Figure 1 – Top view of rotary sensor with rod and two small masses.

steady. Have your partner click on the "Record" button. Release the mass. Once the thread has completely unwound from the pulley, click the "Stop" button.

4. On the angular velocity vs time plot, identify the region which corresponds to the mass falling freely. Find the slope of the region using the Capstone line-fitting tool. The slope of an angular velocity vs time plot gives the angular acceleration, α. Record your data in an excel spreadsheet table as shown below. Calculate the torque ($\tau = mR_{pulley}(g - \alpha R_{pulley})$) where m is the hanging mass and R_{pulley} = 1.45 cm is the radius of the middle-step pulley of the three-step pulley (see * below). Next use equation [2] to determine the measured I_{rod}. Record your value in column 4 of the spreadsheet table.

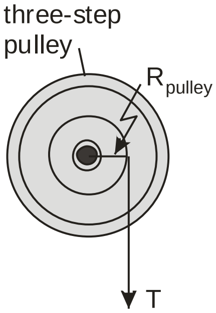

Figure 2 - Torque on the three-step pulley.

Hanging mass m (g)	Angular acc. α (rad/s^2)	Torque, $\tau = mR_{pulley}(g - \alpha R_{pulley})$ (N·m)	Measured I_{rod} (kg·m^2)	Calculated I_{rod} (kg·m^2)
20				

Next, use the mass (m_{rod}) and length (L) of the rod that you obtained in step 1 to calculate the theoretical moment of inertia of the rod ($I_{rod} = \frac{1}{12} m_{rod} L^2$). Does your measured I_{rod} agree with the calculated I_{rod}? Explain any discrepancy, if any.

5. Next, you want to verify that the moment of inertia for a point mass is determined by equation 3. Mount two masses at the ends of the rod as shown in figure 1. Using a ruler measure the *distance from the axis of rotation to the center of mass of each mass*. Record this value as distance *R* in the spreadsheet below. Make sure that the masses are at the same distance from the axis of rotation. Now repeat steps 3 and 4. However, in step 3, use a 50 g mass instead of the 20 g mass.
6. Repeat step 5 four more times by changing the distance of the masses. Suggested distances are 14, 12, 8 and 4 cm from the axis of rotation. (Make sure the masses do not hit the super pulley in the 10 to 12 cm range.) For smaller distances (8 cm or less) use a 20 g hanging mass instead of the 50 g.
7. For each run, find the slope of the angular velocity vs time plot for the region that corresponds to the free falling of the hanging mass. Record your slope as angular acceleration α in an excel spreadsheet. Recall that the torque is $mR_{pulley}(g - \alpha R_{pulley})$.

Run	Hanging mass m (g)	Distance R (m)	Angular acc. α (rad/s^2)	Torque (N·m)	Inertia $I_{measured}$ (kg·m^2)	I_{masses} (kg·m^2)	Calculated I_{masses} (kg·m^2)
1	50						

2							
…							

The moment inertia of the masses, I_{masses}, is the moment of inertia that you measured (column 6) minus the moment of inertia of the rod that you found in step 4. Use equation [3] for the calculated moment of inertia of the masses (column 8).

8. Verify graphically that the measured moment of inertia of the masses depends on the square of the distance from the axis of rotation. Comment on whether your results agree with the calculated values.

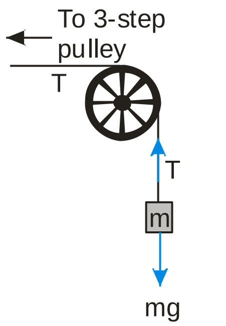

Figure 3 - Force diagram for hanging mass, m.

* From figure 2, the torque on the 3-step pulley is $\tau = R_{pulley}T$ where T is the tension in the string and is due to the hanging mass, m (figure 3). When m is free to drop, the equation of motion is

$$mg - T = ma$$

where $a = \alpha R_{pulley}$ is the linear acceleration of m. So, the tension in the string is

$$T = mg - m\alpha R_{pulley}$$

The torque on the 3-step pulley is

$$\tau = mR_{pulley}(g - \alpha R_{pulley})$$

This derivation assumes that the moment of inertia of the super pulley and friction is negligible.

Activity 2 – Moment of inertia of a disk and ring and, conservation of angular momentum

1. Use the triple-beam balance to measure the mass of the disk and the ring. Then use a Vernier caliper to measure the radius of the disk and, the inside and outside radii of the ring. Now, calculate the theoretical moment of inertia of the disk and the ring using $I_{disk} = \frac{1}{2}mR^2$ and, $I_{ring} = \frac{1}{2}m(R_{outer}^2 + R_{inner}^2)$.
2. Remove the rod from the three-step pulley and mount the disk in its place. Use the technique in the last activity to measure the moment of inertia of the disk. For this measurement use a 10 g for the hanging mass. Record your results in a table similar to step 4 of activity 1.
3. Place the ring on top of the disk, making sure that the two tabs on the ring line up with the two holes on the surface of the disk. Measure the moment of inertia of both the disk and the ring. Use this result to determine the moment of inertia of the ring only. Again, record your results in a table similar to the last step.
4. Compare the moment of inertia for the disk and ring with the theoretical values that you calculated in step 1. Are they equal to within experimental errors? If not, explain your discrepancy. By what percent is the experimental off from the theoretical values?

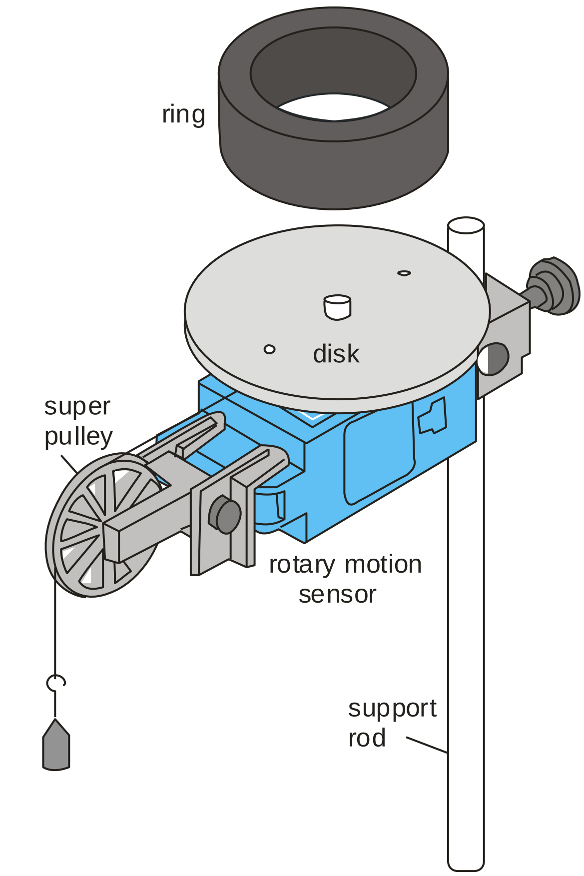

Figure 4 - Angular momentum setup.

5. Now, remove the hanging mass and the ring. Wrap the thread a few times around the middle pulley of the three-step pulley. Pull the thread to set the disk spinning. While the disk is spinning, gently drop the ring onto the disk. Aim it so that the ring is centered about the disk and a few mm above the disk. Practice it a few times so you can drop the ring without it being thrown off the disk. Then "Record" a run starting with just the disk spinning and stopping just after dropping the ring onto the disk.
6. You should observe a sudden change in angular velocity on the angular velocity plot from step 5 when the ring is dropped on the disk. Take the average angular velocities just before and just after the ring is dropped. They represent ω_i and ω_f respectively.
7. Calculate the angular momentum of the disk, and the disk with the ring (equation 4). Is angular momentum conserved? If not, explain any discrepancy.

Activity 3 – Potential energy to kinetic energy

This part explores gravitational potential of a mass being transformed into the rotational kinetic energy of the disk and translational kinetic energy of the mass. The setup for this part is similar to the last activity.

1. Remove the ring from the disk so that only the disk is attached to the rotational sensor. Hang a 50 g mass to the thread and let the thread unwind completely from the three-step pulley. Use a meter stick and measure the height of the mass from the table. Now wind the thread around the middle pulley of the three-step pulley. Again, measure the height of the mass. The potential energy of the mass is $PE = mg\Delta h$.
2. Click on the "Record" button. Release the 50 g mass. When the mass has reached its lowest point and starts to move back up, click on the "Stop" button.
3. Your angular velocity vs time plot should look like the figure 5 below. Identify the point

where the mass is at the lowest point; on the example plot the angular velocity is ω = 21.952 rad/s. At this point, you have two kinetic energies, one from the falling mass and, two the rotating disk.

$$KE_{tran} + KE_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I_{disk}\omega^2 \quad [5]$$

Now the velocity of the hanging mass can be calculated from $v = \omega R$ where R (=1.4 cm) is the radius of the middle pulley of the three-step pulley. Calculate the total kinetic energy and compare it to the potential energy that you calculate in step 1. Is mechanical energy conserved? If not, explain what may have caused the loss in energy.

4. Turn the Airlink off and unplug it from the rotary sensor. (To turn it off, press and hold the ON button until the status LEDs stop blinking.)

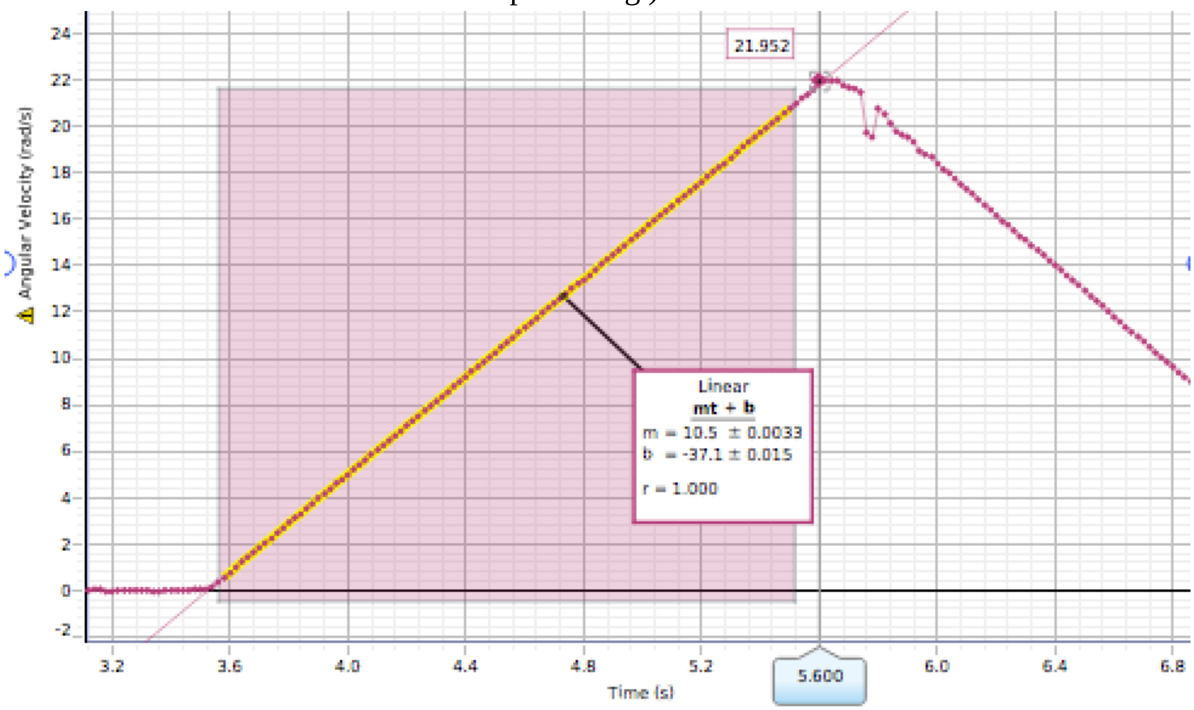

Figure 5 - Gravitational potential energy to kinetic energies.