•Motion Definitions & Relationships

Position vector $\vec{r}$, $r = |\vec{r}|$, and $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

$d \equiv$ Distance traveled $\qquad s \equiv$ speed $\qquad <s> \equiv \dfrac{d}{\Delta t} \qquad s \equiv |\vec{v}| \equiv \left|\dfrac{d\vec{r}}{dt}\right|$

$$<\vec{v}> \equiv \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} \equiv \frac{d\vec{r}}{dt} \qquad \Delta x = \int_{t=t_i}^{t_f} v_x\, dt = \text{area under } v_x \text{ vs. t graph}$$

$$<\vec{a}> \equiv \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} \equiv \frac{d\vec{v}}{dt} \qquad \Delta v_x = \int_{t=t_i}^{t_f} a_x\, dt = \text{area under } a_x \text{ vs. } t \text{ graph}$$

•1-D Vectors

$$\vec{r} = r_x\,\hat{i} = x\,\hat{i} \quad \text{and} \quad x = r_x = |\vec{r}|$$

$$\Delta\vec{r}_{1\to 2} \equiv \vec{r}_2 - \vec{r}_1 = r_{2x}\,\hat{i} - r_{1x}\,\hat{i} = x_2\,\hat{i} - x_1\,\hat{i} = (x_2 - x_1)\,\hat{i}$$

$$|\Delta\vec{r}_{1\to 2}| \equiv \Delta r_{1\to 2} = |r_{2x} - r_{1x}| = |x_2 - x_1|$$

•2-D Vectors

$$\vec{r} = r_x\,\hat{i} + r_y\,\hat{j} = x\,\hat{i} + y\,\hat{j} \text{ and } r = |\vec{r}|$$

$$\Delta\vec{r}_{1\to 2} \equiv \vec{r}_2 - \vec{r}_1 = \left(r_{2x}\,\hat{i} + r_{2y}\,\hat{j}\right) - \left(r_{1x}\,\hat{i} + r_{1y}\,\hat{j}\right) = (r_{2x} - r_{1x})\,\hat{i} + (r_{2y} - r_{1y})\,\hat{j} = \Delta r_{x,1\to 2}\,\hat{i} + \Delta r_{y,1\to 2}\,\hat{j}$$

$$|\Delta\vec{r}_{1\to 2}| \equiv \Delta\vec{r}_{1\to 2} = \sqrt{\left(\Delta r_{x,1\to 2}\right)^2 + \left(\Delta r_{y,1\to 2}\right)^2}$$

•Forces

$$\vec{a} = \frac{\sum \vec{F}_{\to object}}{m_{object}} \qquad \vec{F}_{type,A\to B} = -\vec{F}_{type,B\to A} \qquad \left|\vec{F}_{g,\,earth\to object}\right| = mg$$

$$f_s \le \mu_s n \qquad f_k = \mu_k n \qquad f_r = \mu_r n$$

•Circular Motion

$\vec{a} = \vec{a}_{rad} + \vec{a}_{\tan}$ $\Delta\theta = \theta_f - \theta_i$ $d = r\Delta\theta$ $\Delta\theta = \omega\Delta t + \frac{1}{2}\alpha(\Delta t)^2$

$|\vec{a}_{rad}| = \frac{{v_{\tan}}^2}{r}$ $\omega = \frac{\Delta\theta}{\Delta t}$ $v_{\tan} = r\omega$ $\omega = \omega_0 + \alpha\Delta t$

$|\vec{a}_{\tan}| = dv/dt$ $\alpha = \frac{\Delta\omega}{\Delta t}$ $a_{\tan} = r\alpha$ $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

$\vec{a}_{\tan} = \frac{\vec{F}_{\tan}^{Net}}{m}$ $\vec{a}_{rad} = \frac{\vec{F}_{rad}^{Net}}{m}$ $T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$ θ measured in radians, CCW is +

•Momentum and Impulse

$\vec{p} \equiv m\vec{v}$ $\vec{J}_{agent \to object} \equiv \int \vec{F}_{agent \to object}\, dt = \left\langle \vec{F}_{agnet \to object} \right\rangle \Delta t$

$\vec{J}^{Net}_{\to system} = \int \vec{F}^{Net}_{\to object}\, dt = \Delta\vec{p}_{\to system}$ $\sum_{system} \vec{p}_f = \sum_{system} \vec{p}_i$

•Potential Energy, Kinetic Energy, Work, Power

$KE = ½\, mv^2$ $U_g = mgy$ $F_{spring} = -k\,\Delta x$ $U_{spring} = ½\, k\,(\Delta x)^2$

$dW = \mathbf{F} \cdot d\mathbf{x}$ $W = \int \mathbf{F} \cdot d\mathbf{x}$ $\Delta KE = \Sigma_i\, W_i$ (work-energy theorem)

For conservative forces: $W_i = -\,\Delta U_i$ $F_x = -dU/dx$

$P = dW/dt = \mathbf{F} \cdot \mathbf{v}$ $P_{average} = \Delta W/\Delta t$

•Rotations of a Rigid Body, equilibrium

$\mathbf{R}_{cm} = \Sigma_i\, m_i \mathbf{r}_i / (\Sigma_i\, m_i)$ $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ $|\,\mathbf{r} \times \mathbf{F}\,| = rF\sin(\theta_{rF})$

$I = \Sigma_i\, m_i r_i^2 = \int r^2\, dm$

$I_{solid\ cylinder} = I_{disk} = ½\, MR^2$ $I_{hoop} = I_{hollow\ cylinder} = MR^2$ $I_{solid\ sphere} = 2/5\, MR^2$
$I_{beam} = 1/3\, ML^2$ (around end) $I_{beam} = 1/12\, ML^2$ (around cm)

Equilibrium: $\Sigma_i\, \mathbf{F}_i = \mathbf{0}$ $\Sigma_i\, \boldsymbol{\tau}_i = \mathbf{0}$

$\mathbf{L} = \mathbf{r} \times \mathbf{p}$ (point object) $\mathbf{L} = I\,\omega$ (rigid body) $\boldsymbol{\tau} = d\mathbf{L}/dt$

$KE_{rot,cm} = ½\, I\,\omega^2$ $V_{cm} = \omega R$ (rolling motion)

• **Newton's theory of gravity**

$F_{on\ m} = - GMm/r^2$, directed along the line from M to m;
if either mass is spherically symmetric, r is the distance to its center

$U_g = - GMm/r$

• **Oscillations**

$\omega = (k/m)^{1/2}$ $\omega = (g/l)^{1/2}$

$d^2x/dt^2 + \omega^2 x = 0$ $x(t) = A \cos(\omega t + \phi) = C \cos(\omega t) + S \sin(\omega t)$

$U_{max} = KE_{max} = \frac{1}{2} kA^2$

• **Constant Velocity Model**

$\Delta x = v_{0,x} \Delta t$ $v_x = v_{0,x}$ $a_x = 0$ m/s² using Newton 1 or 2 => $\sum \vec{F}_{\rightarrow object} = 0$ m/s²

• **Constant Acceleration Model**

$$v_x = v_{0x} + a_x \Delta t \qquad \Delta x = v_{0x}\Delta t + \frac{1}{2} a_x (\Delta t)^2 \qquad v_x^2 = v_{0x}^2 + 2a_x \Delta x$$

$$a_x = \frac{\sum \vec{F}_{\rightarrow object}}{m_{object}} = \text{constant} \neq 0 \text{ m/s}^2$$

• **Uniform Circular Model**

$$\vec{a} = \vec{a}_r = \left(\frac{v_t^2}{r},\ towards\ center\ of\ the\ circle\right) = \frac{\vec{F}^{Net}}{m} \qquad a_t = 0\,m/s^2 \qquad \Delta\theta = \omega_0\ \Delta t$$

• **Thermodynamics**

$P V = n R T$

$U = (3/2) N k_B T = (3/2) n R$ (monatomic gas) (5/2 for a diatomic gas)

$dW = P\,dV$ $W = \int P\,dV$

$Q_{added\ to\ system} = W_{done\ by\ system} + \Delta U_{of\ the\ system}$