Physics A - PHY 2048C

Circular Motion

09/18/2019

My Office Hours:
Thursday 2:00 - 3:00 PM

212 Keen Building

Warm-up Questions

1. In uniform circular motion (constant speed), what is the direction of the acceleration?
2. A rock is thrown from a bridge at an angle 30° below horizontal. Immediately after the rock is released, what is the magnitude and direction of its acceleration?
3. If a force is exerted on an object, is it possible for that object to be moving with constant velocity?

   If so, give an example.

The Monkey or Criminal ...

Consider the situation depicted here. A gun is accurately aimed at a dangerous criminal hanging from the gutter of a building. The target is well within the gun's range, but the instant the gun is fired and the bullet moves with a speed v_0, the criminal lets go and drops to the ground.

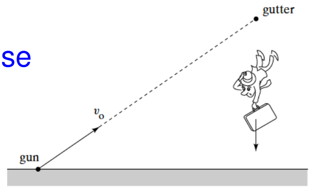
Let us assume you would like to hit the dangerous criminal. Where would you aim?

A Accurately at the dangerous criminal.

B Slightly above the criminal.

C Slightly below the criminal because he is falling downward.

D It depends on the mass of the criminal.

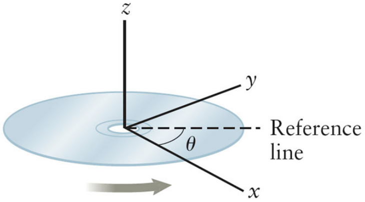


Rotational Motion

So far, objects have been treated as point particles:

- Newton's Laws apply to point particles as well as all other types of particles (extended objects).
- The size and shape of the object will have to be taken into account.

Perspective view of a CD in the x–y plane. The rotation axis is along z.



Need to define rotational quantities:

1. Angular position
2. Angular velocity
3. Angular acceleration

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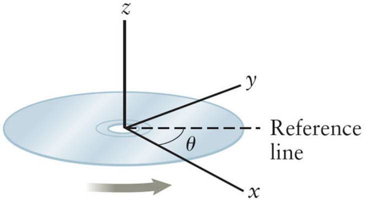


The z-axis is the axis of rotation:

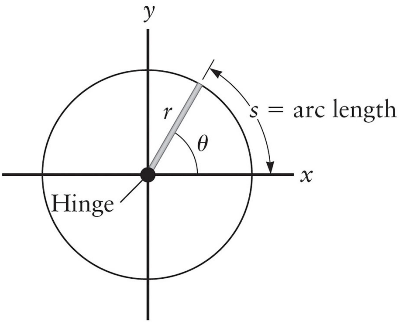
- The angular position, θ, is specified by the angle the reference line makes with the x-axis.

Radian

The end of the rod sweeps out a circle of radius r.

Assume the end of the rod travels a distance s along the circular path (in the figure):

- At the same time, the rod sweeps out an angle θ.



Radian

The end of the rod sweeps out a circle of radius r.

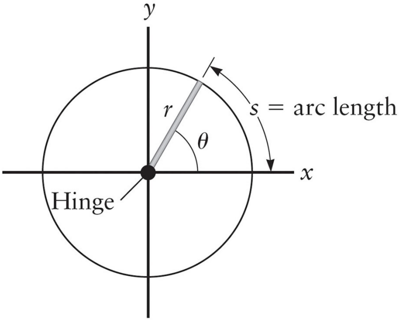
Assume the end of the rod travels a distance s along the circular path (in the figure):

- At the same time, the rod sweeps out an angle θ.
- Distance s and angle θ are related:

$$\theta = \frac{s}{r}$$

- Angles can be measured in two ways:
  - In degrees [°] or radians

$$360^\circ = 2\pi \text{ [rad]} = \frac{2\pi r}{r} \text{ [rad]}$$



Question

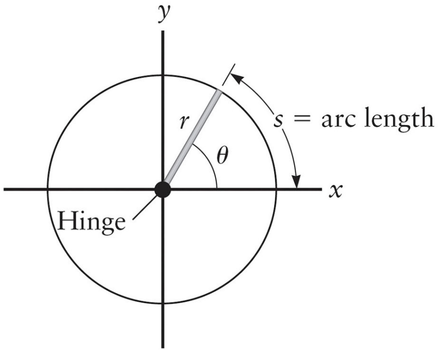
An angle has a value of 180°. What is the angle in radians?

A $\pi/2$

B π

C 2π

D 180π



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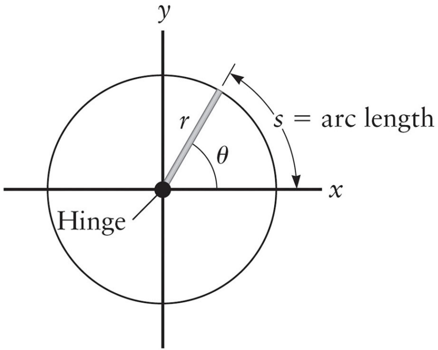
$$\theta = \frac{s}{r}$$

$$180^\circ = \frac{2\pi r}{2r} = \pi \text{ [rad]}$$

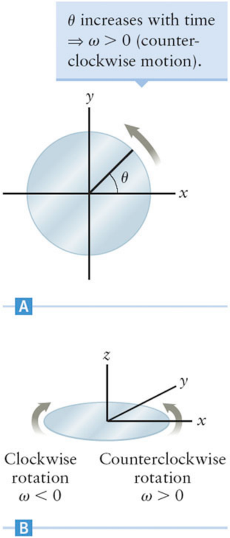


Angular Velocity

The angular velocity, ω, describes how the angular position is changing with time.

Imagine that the end of the rod is moving with a linear velocity, v, along its circular path:

- The linear velocity can describe only the motion of the end of the rod.
- The angular velocity can be used to describe the motion of the entire rod.



Angular Velocity

The angular velocity, ω, describes how the angular position is changing with time.

For some time interval, Δt, the *average angular velocity* is:

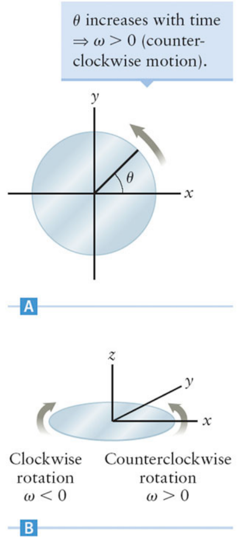
$$\omega_{\text{ave}} = \frac{\Delta\theta}{\Delta t}$$

The *instantaneous angular velocity* is:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

Units are rad/s:

- May also be rpm. (revolutions / minute)



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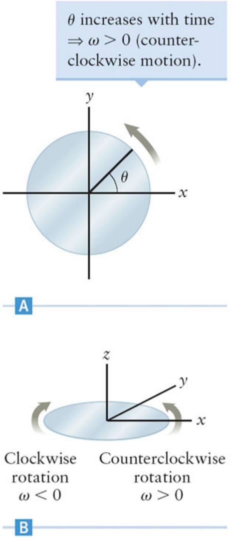
Example: degrees $\rightarrow$ radians

$$360^\circ = 2\pi$$

$$\theta\,[^\circ] = \theta\,[\text{rad}] \times \frac{180}{\pi}$$



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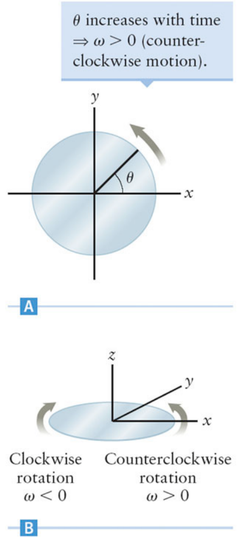
Example: 100 rpm $\rightarrow$ rad/s

$$\omega = 100\,\frac{\text{rev}}{\text{min}} \times \frac{2\pi\ \text{rad}}{60\ \text{s}} \approx 10\ \text{rad/s}$$



Angular Velocity

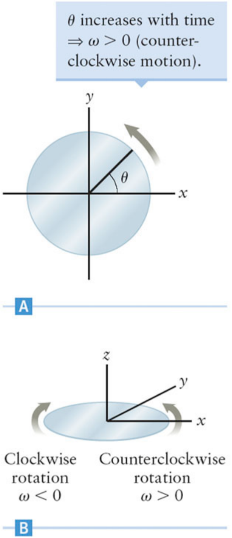
The angular velocity, ω, describes how the angular position is changing with time.

Since angular velocity is a vector quantity, it must have a direction:

- If θ increases with time, then ω is positive.

Therefore:

1. A counterclockwise rotation corresponds to a positive angular velocity.
2. Clockwise would be negative.



Angular Acceleration

The angular acceleration, α, is the rate of change of the angular velocity.

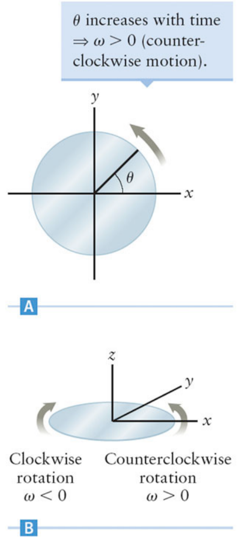
For some time interval, Δt, the *average angular acceleration* is:

$$\alpha_{\text{ave}} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular acceleration is:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

Units are rad/s^2.



Angular Acceleration

Angular acceleration and centripetal acceleration are different.

As an example, assume a particle is moving in a circle with a constant linear velocity:

- The particle's angular position increases at a constant rate, therefore its angular velocity is constant.

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As an example, assume a particle is moving in a circle with a constant linear velocity:

- The particle's angular position increases at a constant rate, therefore its angular velocity is constant.
- Its angular acceleration is 0.

Angular Acceleration

Angular acceleration and centripetal acceleration are different.

As an example, assume a particle is moving in a circle with a constant linear velocity:

- The particle's angular position increases at a constant rate, therefore its angular velocity is constant.
- Its angular acceleration is 0.
- Since it is moving in a circle, it experiences a centripetal acceleration of:

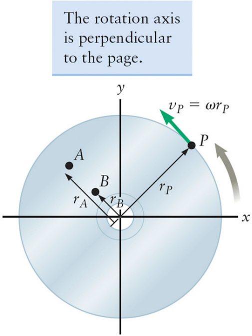
$$a_c = \frac{v^2}{r}$$

- The centripetal acceleration refers to the linear motion of the particle.
- The angular acceleration is concerned with the related angular motion.

Angular and Linear Velocities

When an object is rotating, all the points on the object have the same angular velocity:

- Makes ω a useful quantity for describing the motion.
- The linear velocity is not the same for all points. (depends on distance from rotational axis)



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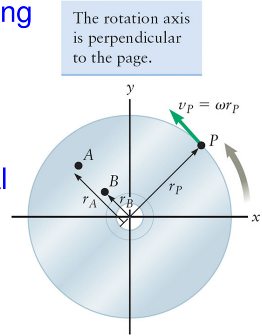
The linear velocity of any point on a rotating object is related to its angular velocity by:

$$v = \omega r ,$$

where r is the distance from the rotational axis to the point. For $r_A > r_B$:

$$v_A > v_B$$



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The linear velocity of any point on a rotating object is related to its angular velocity by:

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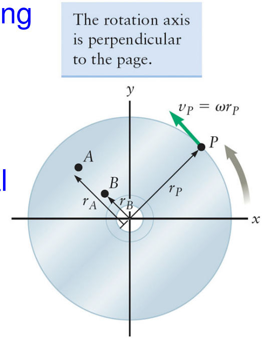
where r is the distance from the rotational axis to the point. Similarly:

$$a = \alpha r$$



Period

When an object is rotating, all the points on the object have the same angular velocity:

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Period of Rotational Motion:

- One revolution of an object corresponds to 2π radians.
- The object will move through $\omega/2\pi$ complete revolutions / s.
- The time required to complete one revolution is the *period:*

$$T = \frac{2\pi}{\omega}$$