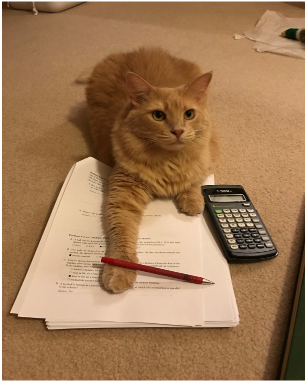
Physics A - PHY 2048C

Potential Energy and Hooke's Law



10/23/2019

My Office Hours:
Thursday 2:00 - 3:00 PM

212 Keen Building

Warm-up Questions

1. How do you determine the direction of kinetic energy and what is the meaning of negative kinetic energy?
2. Define (in your own words) a perfectly elastic collision.
3. If a particle's speed increases by a factor of 3, by what factor does its kinetic energy change?

Outline

Review: Kinetic Energy

The kinetic energy of an object can be changed by doing work on the object. This is called the *Work-Energy theorem:*

$$W = \Delta KE$$

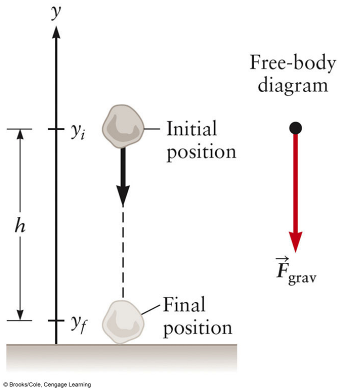
Acceleration can be expressed in terms of velocities:

(Remember: $v_f^2 = v_i^2 + 2a\,\Delta x$)

$$a\,\Delta x = \frac{v^2 - v_i^2}{2} \quad \text{and thus}$$

$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

Term $\frac{1}{2} mv^2$ is *kinetic energy.*



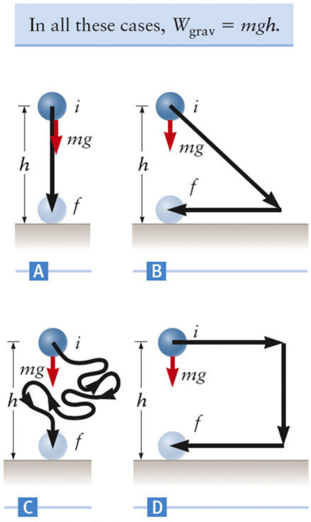
Potential Energy

When an object of mass *m* follows any path moving through a vertical distance *h*, the work done by the gravitational force is always equal to:

$$W = F\Delta x$$
$$= mg\,h$$

An object near the Earth's surface has *potential energy* (PE) depending only on the object's height, *h*.

The work done by the gravitational force as object moves from its initial position to its final position is indeed independent of the path taken.



Mechanical Energy

The sum of the potential and kinetic energies in a system is called the mechanical energy.

Since the sum of the mechanical energy at the initial location is equal to the sum of the mechanical energy at the final location, the energy is conserved.

Conservation of Mechanical Energy:

$$KE_i + PE_i = KE_f + PE_f$$

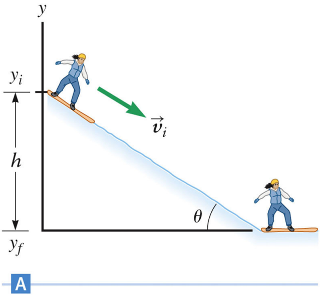
This is true as long as all forces are *conservative forces.* These are forces that are associated with a potential energy function. They can be used to store energy as potential energy.

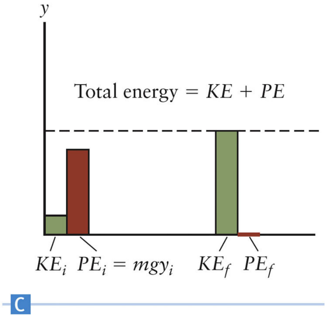
Examples of non-conservative forces: air drag and friction.

Conservation of Energy

The snowboarder is sliding down a frictionless hill:

- Gravity and the normal forces are the only forces acting on the board.
- The normal is perpendicular to the object and so does not work on the boarder.



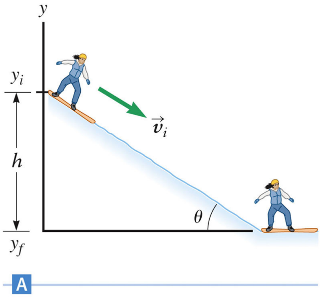


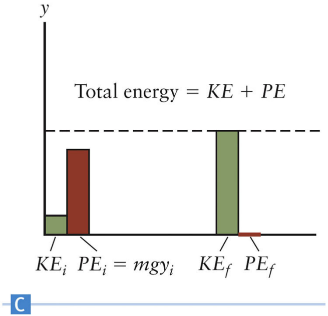
Conservation of Energy

The snowboarder is sliding down a frictionless hill:

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2} m v_i^2 + m g y_i = \frac{1}{2} m v_f^2 + m g y_f$$



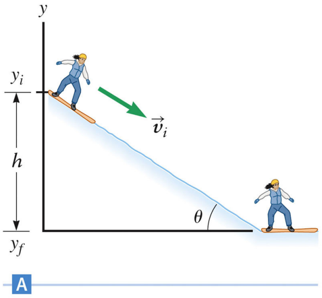


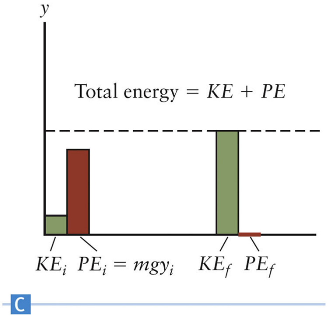
Conservation of Energy

Final velocity depends on the height of the hill, not the angle:

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2} m v_i^2 + m g y_i = \frac{1}{2} m v_f^2 + m g y_f$$



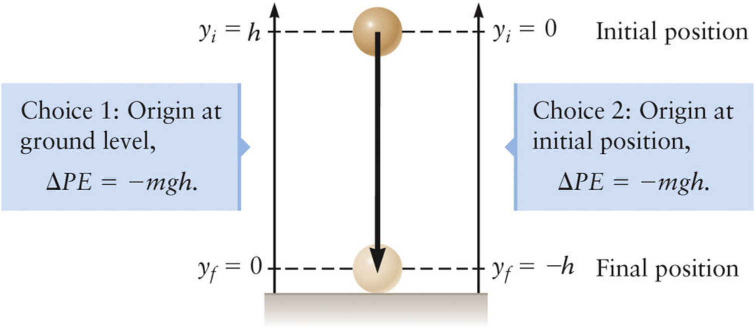


Outline

Changes in Potential Energy

The change in potential energy is the same in both cases:

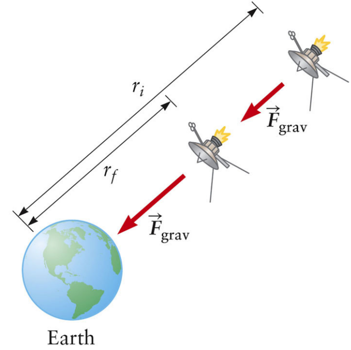
- It is the change in potential energy that is important!
- The change in potential energy does not depend on the choice of the origin.



Gravitational Potential Energy

Potential energy function associated with gravity:

$$F_{\text{grav}} = -\frac{G\,m_1\,m_2}{r^2}$$ (negative sign indicates attraction)



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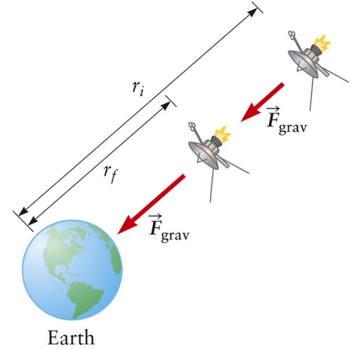
$$F_{\text{grav}} = -\frac{G\, m_1 m_2}{r^2} \qquad \text{(negative sign indicates attraction)}$$

$$PE_{\text{grav}} = -\frac{G\, m_1 m_2}{r} = F\, r$$



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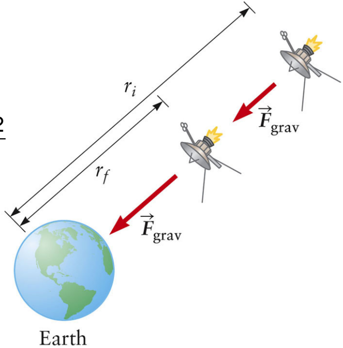
The change in potential energy is:

$$\Delta PE_{\text{grav}} = -\frac{G\,m_1\,m_2}{r_f} + \frac{G\,m_1\,m_2}{r_i}$$

$$\Delta PE_{\text{grav}} = \begin{cases} < 0 & E \text{ released} \\ > 0 & E \text{ invested} \end{cases}$$



Escape Velocity

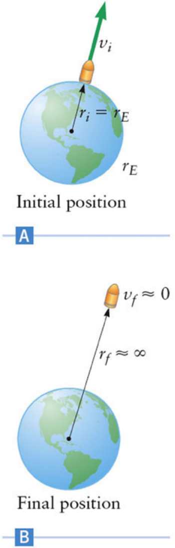
The escape velocity of a satellite is the speed needed for it to escape from the Earth's gravitational pull.

Measure distances from the center of the Earth with $r_f = \infty$:

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2}mv_i^2 - \frac{G\,M_{\text{Earth}}\,m}{r_i} = \frac{1}{2}mv_f^2 - \frac{G\,M_{\text{Earth}}\,m}{r_f}$$

- Initial distance, r_i, is radius of Earth.
- Final distance, r_f, is ∞, so $PE_f = 0$.
- Final speed, v_f, is 0.



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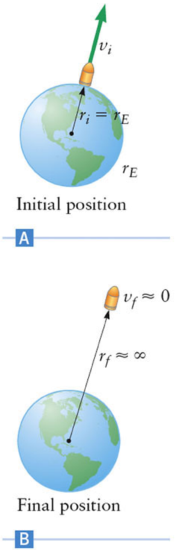
For the Earth:

$$v_i = \sqrt{\frac{2G M_{\mathrm{Earth}}}{r_{\mathrm{Earth}}}} \approx 11.2\ \mathrm{km/s}$$



Why Air Sticks Around

Competition between gravity and heat

All gas molecules are in constant random motion:

- Temperature of any gas is direct measure of this motion
  The hotter the gas, the faster the molecules are moving.
- The rapid movement of heated molecules creates pressure tending to oppose the force of gravity.

Escape speed

$$\text{escape speed (in km/s)} = 11.2 \cdot \sqrt{\frac{\text{mass of body (in Earth masses)}}{\text{radius of body (in Earth radii)}}}$$

Examples:

If mass of planet is quadrupled, the escape speed doubles.

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Examples: (Moon)

escape speed = $11.2 \cdot \sqrt{0.012\,/\,0.27}$ = 2.36 km/s

Why Air Sticks Around

To determine whether or not a planet will retain an atmosphere, we must compare the planet's escape speed with average speed of the gas particles making up the atmosphere:

$$\text{average molecular speed (in km/s)} = 0.157 \cdot \sqrt{\frac{\text{gas temperature (K)}}{\text{molecular mass (hydrogen atom masses)}}}$$

Example: (Oxygen (O_2) on Earth)

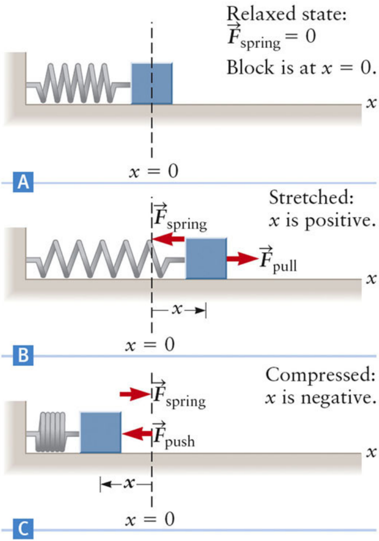
- $T = 300$ K and mass = 32 hydrogen masses

$$v_{\mathrm{av}} = 0.157 \cdot \sqrt{300/32} \approx 0.5\ \mathrm{km/s}$$

Springs

There is a potential energy associated with springs and other elastic objects. When there is no force applied to its end, the spring is relaxed:

- B Assume you exert a force to stretch the spring. The spring itself exerts a force that opposes the stretching.
- C You could also compress the spring. The spring again exerts a force back in the opposite direction.



Hooke's Law

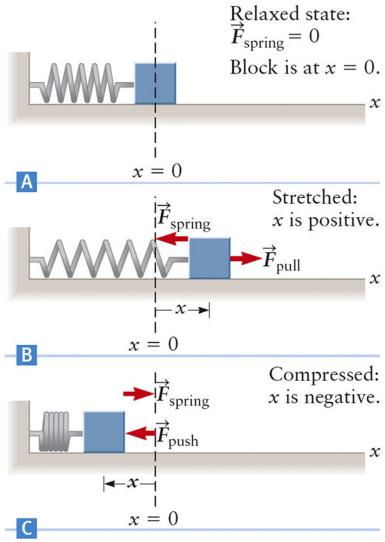
The force exerted by the spring has the form:

$$F_{\text{spring}} = -k\,x$$

At equilibrium, $x = 0$:

- x is the amount the end of the spring is displaced from equilibrium position.
- k is called the *spring constant* with unit N/m.

This is known as Hooke's Law. A potential energy function can be associated with the force.



Hooke's Law

Since the force is not constant, the work is found by looking at the area under the curve of the force-displacement curve.

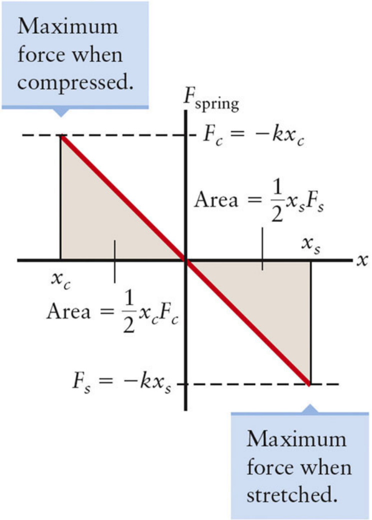
Area of triangles is:

$$A = \frac{1}{2} F x$$

With the force of $F = -k x$:

$$W = -\frac{1}{2} k x^2$$

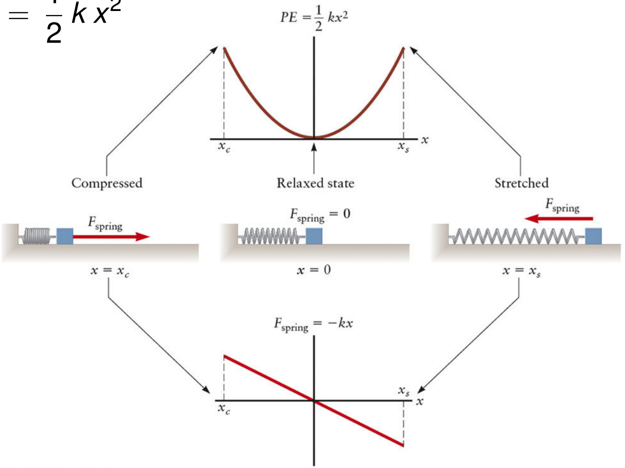
The negative sign confirms the force and displacement are in opposite directions.



Hooke's Law

From the work, an expression for potential energy can be found:

$$PE_{\text{spring}} = \frac{1}{2} k x^2$$



Power

Time enters into the ideas of work and energy through the concept of power.

The average power is defined as the rate at which the work is being done:

$$P_{\mathrm{ave}} = \frac{W}{t} = \frac{F\,\Delta x}{t} = F\,v_{\mathrm{ave}}$$

Units of power are watts: 1 W = 1 J/s.

For a given power:

- The motor can exert a large force while moving slowly.
- The motor can exert a small force while moving quickly.