Experiment II: Determination of g from Newton's Laws

Goals

- Experimentally determine the acceleration (a) of a two-body system, and the friction force (f)
- Calculate g, the acceleration due to gravity, from a and f using Newton's laws of motion

Introduction and Background

The acceleration due to gravity, g, is an important quantity that is most commonly used to calculate the weight (W) of an object from its mass (m): W = mg. The value of g depends on the location. For example, the g value on the Moon is much smaller than that on Earth. Even on the surface of the Earth g varies slightly from location to location, although it is a good approximation to use g = 9.80 m/s^2 as an average. In this lab, however, we will pretend that we know nothing about the value of g, and we will experimentally determine the value of g by measuring the acceleration of a two-mass system and the friction force associated with the motion.

***Theory*:** The set-up for the experiment is shown schematically in Fig. 2-1. Obviously the acceleration of the two masses depends on the weight of the mass m_1, which in turn depends on the value of g. Mass m_2 is a cart, able to roll on the lab table with a small but finite friction. m_2 is connected to the hanging mass m_1 by a light (negligible mass) string passing over a pulley. When the system is released from rest, m_1 will accelerate downward and m_2 will accelerate forward with an acceleration of the same magnitude since the string is unstretchable. This acceleration can be derived by applying Newton's second law individually to both masses. The free-body force diagrams for both masses are shown in Figure 2-2.

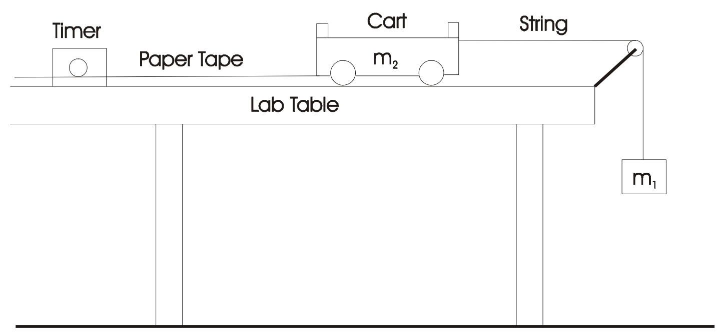

Figure 2.1 – Experimental arrangement.

Now **derive *g* using Newton's second law and the force diagrams and show the derivation in your lab report**. You should arrive at the equation:

$$g = \left(\frac{m_1 + m_2}{m_1}\right)a + \frac{f}{m_1} \tag{2-1}$$

Therefore, we can determine the value of g if we can experimentally measure a and f since m_1 and m_2 are easily determined.

***Measuring a*:** The acceleration of the two-body system can be regarded as constant if the friction is constant. This is one of the assumptions we will make in this experiment. For motion with constant acceleration we have

$$v = v_0 + at \qquad (2\text{-}2)$$

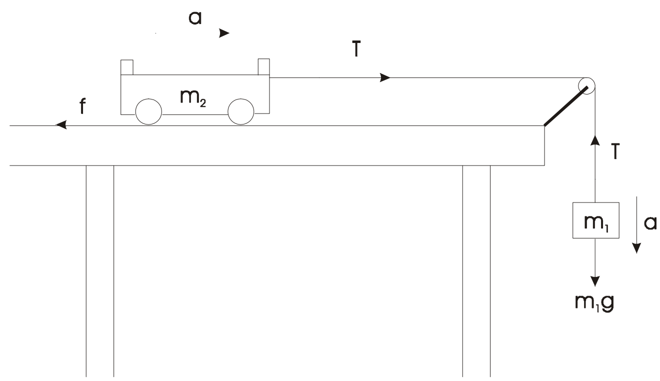

Figure 2.2 – Force analysis.

where v is the instantaneous velocity at time t and v_0 is the initial velocity. You should recognize that Equation 2-2 is if the form $y = mx + b$, so a graph of v versus t should be a straight line whose slope is the acceleration a. 2-3 illustrates this. Therefore, we need to generate a set of (v, t) data points to determine the acceleration a. This is accomplished by attaching a piece of paper tape to the cart and running the tape through a timer. The position of the cart is recorded on the tape every 1/30 of a second (1/60 s in case of a spark timer). After m_1 is released, the system accelerates and a series of dots are generated on the tape with increasing distance between adjacent dots, as depicted in Figure 2-4. The average velocity in between two adjacent dots can be calculated from

$$\bar{v} = \frac{\Delta D}{\Delta t} \qquad (2\text{-}3)$$

Note several important points:

i) Δt is always 1/30 (1/60) seconds;

ii) Any point can be chosen as the $t = 0$ point since we are looking for the slope (a) and the intercept (v_0) is not important;

iii) Because of the linear relation between v and t, the average velocities you have determined are also the *instantaneous* velocities midway through the time intervals between two adjacent marks.

This way, a series of (v, t) data points can be obtained and the slope of the linear plot of v versus t yields the acceleration a.

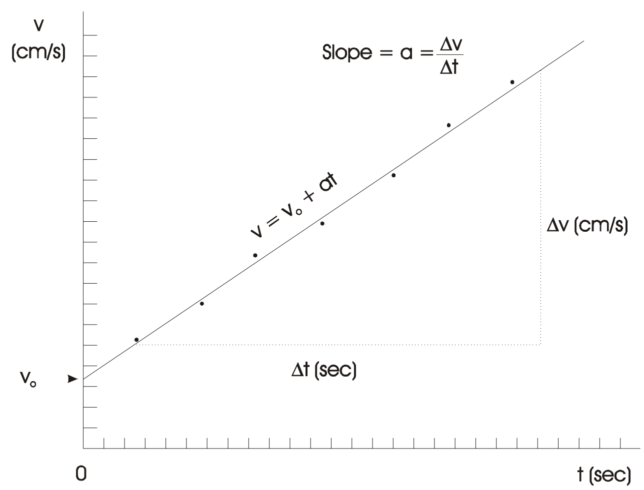

Figure2.3 – Finding acceleration.

***Measuring f*:** Since friction is not negligible in this experiment, the kinetic friction force, f, must be measured. To do so we make use of Newton's first law: if the weight on the string produces a force equal in magnitude to the kinetic friction, then the cart ought to move with constant velocity once it is set in motion. The mass of the hanging mass, m, yields the kinetic friction force:

$$f = mg \qquad (2\text{-}4)$$

In principle one can plug the measured a and f into Equation 2-1 and calculate g. However, one should remember that we are looking for g in this lab; we cannot simply use $g = 9.8\ m/s^2$ to calculate f. We need to plug a and Equation 2-4 into Equation 2-1 and do some algebra to get

$$g = \left(\frac{m_1 + m_2}{m_1 - m}\right)a \qquad (2\text{-}5)$$

This is the equation we will use to calculate g.

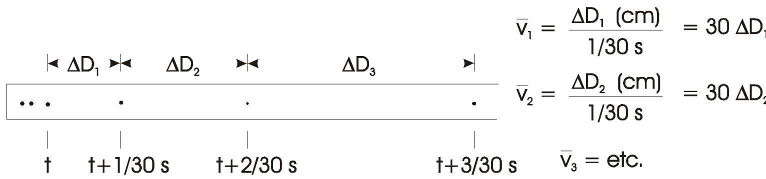

Figure 2.4 – Paper tape marker.

Experimental Setup

Equipment**:** Cart, pulley with mounting clamps, string, timer, spark timer, timing tape, weights and sandbags.

Setup**:** The experimental setup is shown schematically in Figure 2-1 and has been described in previous section.

Experimental Procedure

1. Set up the equipment as shown in Figure 2-1. Adjust for at least 65 cm of travel for the cart and the hanging mass. Fasten the tape to the bottom of the cart with a thumb tack or masking tape. Run the tape through the timer. **To obtain good quality data you must observe the following precautions**:
   i) Be sure that the timer is mounted so that the rim of the rotating wheel moves by the tape in the opposite direction to the motion of the tape. This prevents slack from developing in the tape when the timer is started. (If you have a spark timer, ignore this part).
   ii) Be sure that the pulley height is adjusted so that the string from the pulley to the cart is horizontal.
   iii) Be sure that the pulley, the cart, and the timer are all in one line with the cart aimed straight at the pulley.

2. Hold the cart in a steady position with m_1 just below the pulley. Have your partner take any slack out of the timing tape and then have him/her start the timer. Wait an instant for the timer to come up to speed and then abruptly release the cart.
   Practice the above once or twice before taking any data. As soon as you obtain a tape,
   i) write on it the values for m_1, m_2, and m so that the tapes do not get mixed up;
   ii) examine the distances between adjacent dots to make sure that the distance increase with time. Occasionally the bad points appear because of unevenness of the table top or lock of the wheels of the cart. You have to solve this problem if it occurs.

   Take three motion records:
   i) m_1 = 1.000 kg, m_2 = cart mass;
   ii) m_1 = 0.750 kg, m_2 = cart mass;
   iii) m_1 = 1.000 kg, m_2 = (cart + sandbag) mass.

3. To measure the kinetic friction, hang a small mass (m) on the string. m = 0.020 kg is a good place to start. Give the cart a nudge and observe the motion. Increase or decrease m as

needed to produce a constant velocity. You need to do this for empty cart and cart with a sandbag.

Data Analysis

1. For each tape, measure the distance between successive adjacent points. Use a plastic ruler to do this and do it carefully. Since you can use any point as the starting point, begin with a distance of at least 0.5 cm and measure at least 10 intervals.
   Make up a data table for each of your tapes in the following format. Remember Δt is always 1/30 seconds (1/60 seconds for spark timer).

m_1 = m_2 = m =		
Total time t (s)	ΔD (cm)	$v = \Delta D/\Delta t$ (cm/s)
0.0333		
0.0667		
0.1000		
…		
…		

2. Make a linear plot of v versus t for each tape, similar to the one shown in Figure 2-3. Use a plastic ruler to draw a best-fit straight line to your data. Note that your objective in drawing a line through your data should be to minimize deviations of the data points from the line. You should have nearly equal number of data points more or less randomly distributed on either side of the line. You should not require that the line pass through the beginning or end points since these are no more accurate than any others.

3. Determine the slope (a) for each of your graphs, as shown in Figure 2-3. *Use two points far apart on the line, not original data points that are not on the line.* Each value will, of course, be different.
   Calculate the value for g for each of your runs using Equation 2-5 and your values for m_1, m_2, m and a.
   Calculate the percentage difference between each of your results and the value of 980 cm/s^2 (9.80 m/s^2) expected at this location.

4. Generate a table as shown below.

	Run 1	Run 2	Run 3
m_1 (kg)			
m_2 (kg)			
m (kg)			
a (cm/s^2)			
g (cm/s^2)			
% difference			

In lab IV in two weeks you will need this data table. Remember to bring it with you.

Discussions and Questions

1. When you were measuring the friction force, why did you have to give the cart a nudge and then observe whether the velocity is constant? If you simply increase the hanging mass until the cart starts moving, will it travel with constant velocity after it starts moving?

2. If you could do this experiment on the Moon, under the same conditions would you expect the acceleration of m_2 to be greater or smaller than that on Earth? Why?

Conclusions

Briefly discuss whether you have accomplished the goals listed at the beginning.