# Experiment III: Centripetal Force

## Goals

- Learn the procedure for determining the exponents in a power law
- Experimentally determine the relationship between the centripetal force and the mass, radius, and speed for an object in uniform circular motion

## Introduction and Background

***Centripetal Force*:** Those of you who have tied an object to a string and whirled it in a horizontal circle no doubt have noticed that you have to pull on the string, and therefore, on the object in a direction toward the center of the circle in order to keep the object in the circular path. If you do not pull on the string, the object will simply fly off. This pull or force, as you learned in class, is called a centripetal force. You may also have noticed that you have to pull harder if the mass increases, or the speed increases, or the radius decreases, which is consistent with theoretical relationship between centripetal force ($F$) and the mass ($M$), speed ($V$), and radius ($R$) you have learned in class. In this lab, however, we will pretend that we know nothing about the relationship and try to determine it through some designed experiments.

***Determination of a Power Law*:** A power law is a functional form as shown below:

$$y = C \times x^n \tag{3-1}$$

where $C$ is a constant and $n$ is the exponent; namely, $y$ is proportional to the $n$th power of $x$. We can take the logarithmic of both sides of Equation 3-1:

$$\log y = n \times \log x + \log C \tag{3-2}$$

Therefore, a plot of log $y$ versus log $x$ should be a straight line whose slope is the exponent $n$. Many relationships in physics take some form of a power law and very often we need to determine the exponents in these power laws. To do this experimentally we would generate a set of data points of (y, x) and plot log($y$) versus log($x$) on linear scales to determine the slope and thus the exponent.

In case multiple variables are involved, ie

$$y = C \times x_1^a \times x_2^b \times x_3^c \tag{3-3}$$

we would determine the exponents one at a time. For example, to determine $b$, we would keep $x_1$ and $x_2$ constant in the experiment and we would have

$$\log y = b \times \log x_2 + \log(C \times x_1^a \times x_3^c) \tag{3-4}$$

where the last term is just a constant and the slope of log $y$ versus log $x_2$ yields the exponent $b$.

***Exponents in Centripetal Force*:** Even if we know nothing about the relationship for centripetal force, it is reasonable to assume a power law relationship for $F$ and $M$, $V$, $R$. Although one generally sees $F$ expressed as a function of $M$, $V$, and $R$, in this experiment it is not convenient to set given values of $M$, $V$ and $R$ and then measure the required $F$. Rather, it is much easier to set values of $F$, $M$, and $R$ and measure the corresponding speed $V$ needed for uniform circular motion. Therefore, we will assume a power law:

$$V = C \times F^a \times M^b \times R^c \tag{3-5}$$

We will measure *V* in a series of experiments in which only one variable is changed at a time with the other two remaining fixed:

A. Measure *V* as *F* is varied and *M* and *R* kept constant;
B. Measure *V* as *M* is varied and *F* and *R* kept constant;
C. Measure *V* as *R* is varied and *M* and *F* kept constant;

In this way we can acquire data to investigate the functional relationships between *V* and *F*, *V* and *M*, as well as *V* and *R*, one at a time.

**Experimental Setup**

***Equipment*:** Centripetal force apparatus, springs, extenders, stop watch, string, and weights

***Setup*:** The experimental setup is shown schematically in Figure 3-1. The figure shows the apparatus as set up to measure the spring force. The cylindrical bob is suspended by means of a string from a cross arm. The cross arm is supported by a freely turning axle. The distance *R* of the bob from the axis of rotation is adjustable by means of the cross arm set screw. A movable counter weight is provided to balance the system and reduce wobble. The centripetal force is provided by a spring connected between the axle and bob by means of extender and hooks. The radius position indicator may be slid back and forth to the radius of choice. The rod of the position indicator has a spring clip at the top to hold a piece of paper with a narrow "V", which the pointer of the bob is supposed to pass through without hitting either side.

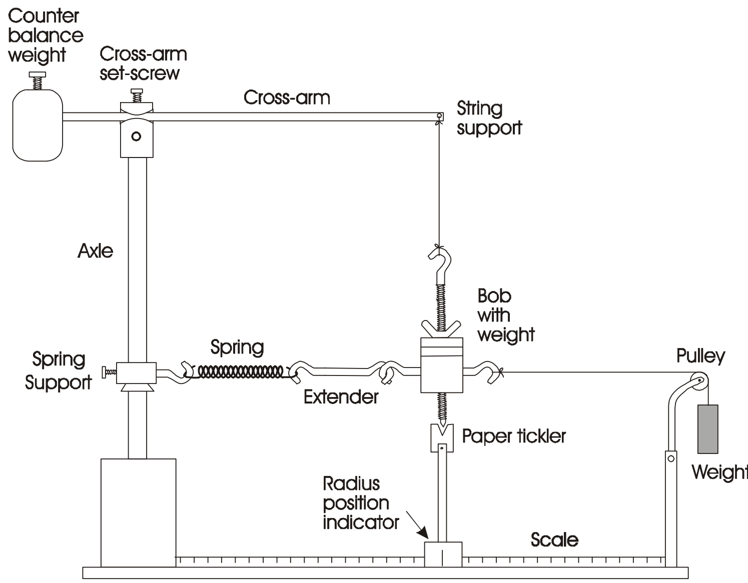


Figure 3.1 – Centripetal force apparatus.

In an actual experiment, one would increase the mass of the hanging weight until the bob is pulled to a position straight above the position indicator. The tension in the spring would be equal to the weight of the hanging mass. Then the hanging mass would be taken off and the bob spun around until the right speed is reached so that it consistently pass through the paper "V". Since the spring is stretched by the same amount, the tension (pulling force on bob) should be the same as before (i.e. the weight of the hanging mass). This is how the centripetal force is measured. **It is very important to observe the following precautions:**

i) The vertical position of the spring support on the axle should be adjusted so that the spring is horizontal when the bob is hanging vertically;
ii) The bob should be straight vertically when it is right on top of the position indicator.

**Experimental Procedure**

***A. Velocity versus Centripetal Force (mass and radius constant)***

1. Secure two 100-g mass on the bob with their slots pointing away from the center of rotation. Set the position indicator at a radius of 21 cm.
   Move the cross arm so that pointer of the *free hanging* bob is at the center of the paper "V".

2. Hook a spring between the axle and the bob (You will use an extender in most cases). Attach a piece of string to the outside of the bob and pass it over the pulley. Attach weights to the end of the string such that the bob is pulled out to its vertical position and again centered in the paper "V". *Adjust the pulley height so that the string is horizontal*.
   Any time the purpose is to determine the dependence of one variable on another, the wider the range of variables used the better. Hence before measuring the speed for any particular force, it is a good idea to try a variety of spring-extender combinations to see what forces can be obtained. You should be able to produce forces with the hanging mass on the string roughly in the 0.1 kg to 0.5 kg range. Make a list of spring-extender combinations and the forces they yield. Pick out your list of at least five forces with the greatest and the smallest differing by at least a factor of 4 and the other three reasonably spaced in between.

3. Now remove the hanging weight and the string from the bob. Rotate the axle. With a little practice you should be able to rotate it at a constant rate with the bob hanging vertically and its pointer passing right through the center of the paper "V".
   Time 15 revolutions of the bob. Remember that the count is zero, not one, at the instant the timer is started. Do this twice. If two measurements agree within a few percent, average them. If not, perform additional measurements so that if you have an obviously bad measurement it can be discarded and the remainder averaged. Since you know the radius of the circle, the average speed of the bob can be calculated.
   Prepare a data table in the following format:

| *t* for 15 revolutions | *t* average | $v = \dfrac{2\pi R \times 15}{t\ average}$ | *F* (N) | *M* (kg) | *R* (m) |
|---|---|---|---|---|---|
| a)<br>b) | | | | | |
| a)<br>b) | | | | | |
| …<br>… | | | | | |

***B. Velocity versus Mass (force and radius constant)***

1. Set up the apparatus: Keep the radius at 21 cm and choose a spring-extender combination that gave a centripetal force near the middle of the force range in Part A.

2. Start with the bob with no mass added. Rotate the axle to determine the required bob speed. Again time each run at least twice.

3. Add slotted masses equally to the top and bottom of the bob to increase the mass and determine the required speed for at 5 different masses including the largest practicable. Generate a data table similar to the one in Part A.

### *C. Velocity versus Radius (force and mass constant)*

1. Set up the apparatus: Use the same bob mass as in Part A (bob + 200 g) and the same force as in Part B.

2. Without changing the hanging mass, change the spring-extender combination to allow the bob to move to a new radius. **Readjust the cross arm so that the bob hangs vertically and rebalance**. Determine the speed of the bob needed for rotation at the new radius. Again, time the run at least twice.

3. Repeat this procedure, keeping and bob mass and the hanging mass constant, but changing the spring-extender combination to give new radii. Remember you must readjust the cross arm each time to keep the bob hang vertically and balance. Try to get five different radii spanning values between 12 cm and 30 cm.

## Data Analysis

***A.*** Construct a table for the results from each run in Experiment A in the form shown below:

| Experiment A    $M$ =    $R$ = 0.210 m | | | |
|---|---|---|---|
| $F$ (N) | $V$ (m/s) | *Log F* | *Log V* |
| | | | |
| … | … | … | … |

Use your calculator to calculate values of *Log F* and *Log V*.

Plot *Log V* versus *Log F* on a linear graph paper and determine the slope, $a$, of the graph.

***B.*** Repeat the above for Experiment B and determine the exponent $b$.

***C.*** Repeat the above for Experiment C and determine the exponent $c$.

In lab IV in next week you will need these data. Remember to bring them with you.

## Discussions and Questions

1. From what you have learned in class, what are the expected values for the exponents $a$, $b$, and $c$?

2. When setting up the apparatus, why is it important to keep the spring-extender and the string over the pulley horizontal?
   Why must the bob be hanging straight vertically when it is right on top of the paper “V”?

**Conclusions**

Briefly discuss whether you have accomplished the goals listed at the beginning.