Experiment V: Conservation of Linear Momentum

Goals

- Study the conditions required for the conservation of linear momentum
- Verify the law of conservation of linear momentum in a collision of two objects with the same mass
- Use the law of conservation of linear momentum to determine the ratio of the masses of two objects in a collision

Introduction and Background

***Conservation of Momentum*:** The linear momentum (which we will simply refer to as momentum below), **P**, of a mass m moving with velocity **v** is defined as **P** = m**v**. For a system consisting of multiple masses, the total momentum of the system is the vector sum $\mathbf{P} = \mathbf{P_1} + \mathbf{P_2} + \mathbf{P_3} + \ldots$, where $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3} \ldots$ are the momentum of individual masses. Newton's second law tells us that if, during a time interval Δt, a net force **F** acts on the system, the total momentum of the system will be changed by an amount $\Delta\mathbf{P} = \mathbf{F}\Delta t$. Consequently, so long as the net force on the entire system is zero, the total momentum of the system remains constant (conserved). Because **F** and **P** are vectors, if the sum of the forces along a particular direction is zero, the component of the total momentum along that direction will be conserved. In other words, it is possible to have a situation in which the total momentum of a system is conserved along some directions but not conserved along others, as in the case of this lab.

***Collisions*:** Conservation of momentum is often applied to the analysis of collisions of two (or more) masses. This is particularly useful since the actual collision forces and the resulting accelerations may be impossible to measure directly; but since the collision forces are internal, the momentum is conserved as long as the net external force is zero. To apply momentum conservation in a given direction, *x*, to a system of two masses undergoing a collision, it is necessary that there be a time interval that includes the collision process but there is no net external force on the system in that direction. Within this time interval, the *x*-component of the total momentum of the system would remain constant although the individual components P_{1x} and P_{2x} are changed during collision.

In this lab, we will study glancing collisions between two balls, one of which is initially at rest. We will obtain data that will allow us to check for momentum conservation in the horizontal plane in which there is no external force.

Experimental Setup

***Equipment*:** Inclined chute, two steel balls and one plastic ball stored in a hollow metal block, large sheets of white wrapping paper, carbon paper, plumb bob, string, meter stick, pan balance, table clamp.

***Setup*:** The experimental setup is shown schematically in Figure 5-1. In an experiment, a target ball is placed on top of a post in front of the inclined chute, while a projectile ball is released from the top of the chute and gain a certain velocity just before the collision. After the collision, the two balls undergo projectile motion with *zero initially velocity in the vertical direction* and they are expected to hit the floor at the same time. Their landing positions are recorded by placing carbon papers on top of the white wrapping paper. The displacements of the landing points from the origin are directly proportional to the momentum of the two balls after collision.

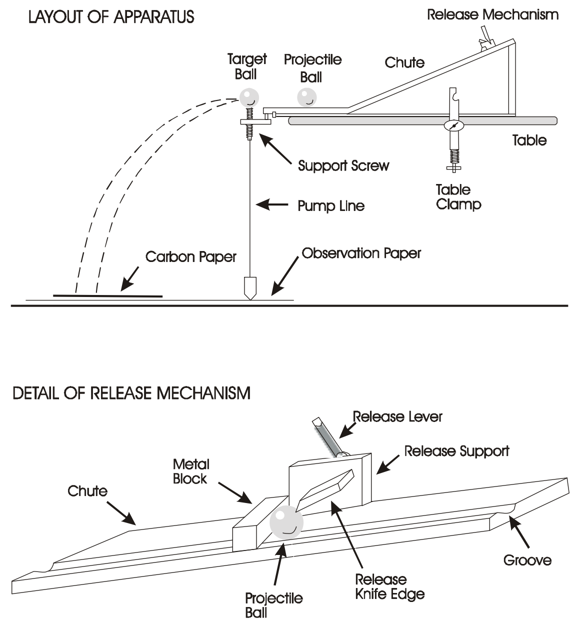

Figure 5.1 – Conservation of linear momentum apparatus.

To understand the design of this experiment, it is absolutely necessary that you understand the answers to the following questions and you need to answer them in your report:

1. After the projectile ball has rolled down the chute and is just about to strike the target ball, what is the direction of the **total** momentum of the system of two balls?

2. What is the expression for the total momentum of the two balls just **before** collision in terms of their masses and velocities?

3. What is the expression for the total momentum of the two balls just **after** collision in terms of their masses and velocities?

4. If the screw holding the target ball exerts a very small horizontal force on the balls during the collision, should the total momentum of the two balls be conserved in a horizontal plane from before to after the collision? Why is this small force negligible?

5. What happens to the total **vertical** component of the momentum of the two balls from just before the collision to the time the balls hit the floor? Is the vertical component of the momentum conserved?

6. How can one compare the momentum before and after collision in the horizontal plane without directly measuring the horizontal velocities of the balls?

The answer to question 6 is a key to this experiment and is based on the following facts about projectile motion: Objects projected horizontally (without any component in initial velocity) from the same height all take *the same amount of time* to hit the floor. If air resistance is negligible, the *horizontal component of their velocity remains unchanged* during the fall and therefore, the *distance they travel horizontally is directly proportional to their horizontal velocity*. Thus by measuring the horizontal distances the balls travel from the collision point to their landing points, and by knowing their masses (or ratio of masses) it is possible to compare the horizontal momenta without obtaining specific numbers for the velocities.

Experimental Procedure

A. Collision of Two Steel Balls

1. Tear off a sheet of white wrapping paper about 75 cm long. Tape the paper to the floor so that the plumb bob hangs centered over one edge of the paper and several cm from the end. The position of the support screw can be adjusted by rotating or pulling when the appropriate screws are loosened. Mark the position of the bob **when the support screw is straight out from the chute and about 1-1/2 ball diameters from the end of the chute**. This is the origin. Now rotate the support screw to one side.

2. Roll a steel ball down the inclined chute in the narrowest track from the release point and note approximately where it lands on the paper. In order to guarantee that the projectile ball is always started from the **same** position on the chute, you should perform the following procedure as shown in the diagram: Place the metal container for the balls flush against the release support with the stepped side pointing towards the top of the chute. Place the projectile ball in the **narrowest** track and against the metal block, and lower the release knife edge onto the top of the ball. Remove the metal block and the ball should stay in place. The ball can now be released by gently raising the knife edge.

   Place a sheet of carbon paper, face down, covering the point where the ball landed. Roll the steel ball down the chute six times and draw a circle that encloses all the points. The distance from the origin to the center of this circle gives a measure of the initial momentum of the projectile ball, and since the target ball is initially at rest, this also represents the total momentum of the two-ball system before collision. The radius of the circle, on the other hand, gives a measure of the uncertainty in the initial total momentum.

   Remove the carbon paper.

3. To obtain good results in this experiment, it is extremely important that you observe the following in adjusting the position of the target ball:

   i) The horizontal distance of the support screw should be adjusted so that the collision takes place just after the projectile ball has completely left the chute, even for most oblique collisions. This means that **when the support screw is straight out from the chute, it should be about 1-1/2 ball diameters from the end of the chute**.

ii) The height of the support screw should be adjusted so that the projectile ball just passes over the screw without hitting it. You may test it by running the projectile ball over the screw without the target ball on top.

Now place the other steel ball on the support screw and roll the projectile ball down the chute to produce a collision. Record the landing positions of the two balls by using two pieces of carbon papers at the appropriate places. Immediately mark on the paper the points according the collision number and whether it is from projectile or target ball (e.g., P_1, T_1, P_2, T_2, etc.), and make sure you mark off unwanted points.

Record at least five collisions at different angles on each side evenly spaced at different angles, but *avoid nearly head-on collisions*. As you work you should see a pattern developing on your paper. You need to do enough collisions so that this pattern is well defined.

You may choose to perform the data analysis for this part before proceeding to Part B.

B. Collision of a Steel Ball and a Plastic Ball

1. Repeat the experiment replacing the steel target ball with the plastic one of the same diameter. You may use the other side of the same piece of wrapping paper.

   Try a head-on collision first to see how long the paper needs to be. Then try a glancing collision to see how much the paper has to be shifted to one side in order to perform collisions with the target ball on one side only. Tape the paper down to the floor, mark the origin, and determine the initial momentum as before.

2. Perform at least four glancing collisions for various positions of the target ball all on one

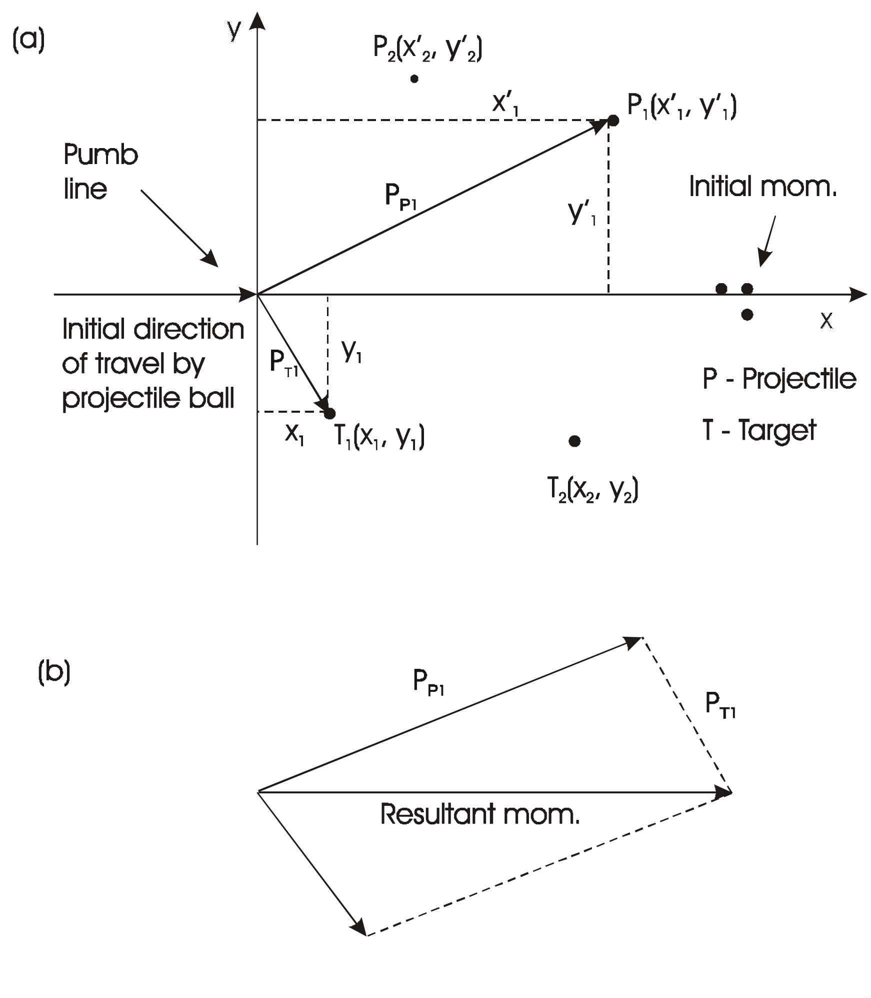

Figure 5.2 (a) Collision sample output and (b) parallelogram analysis.

side of the center. Mark the landing positions of the two balls carefully.

Data Analysis

A. Collision of Two Steel Balls

Since the target and the projectile balls have the same mass in this part of the experiment, the same vector on the paper can be used to represent horizontal *displacement*, *velocity*, and *momentum*, although the magnitude of the three quantities represented will differ by some proportionalities.

1. Graphical Analysis (for one collision only)

As shown in Figure 5-2(b), draw on the paper the vector representing the total momentum for the system of two balls just before collision.
Pick one collision and draw vectors representing the horizontal momentum for each of the two balls after collision. Add these two vectors graphically with the parallelogram method to find the total momentum of the system after collision.
Compare the total momentum before and after collision. Does the total horizontal momentum seem conserved?

2. Quantitative Analysis (for all collisions)

Choose the direction of the initial total momentum as the positive x-axis and set up the x-y coordinate system, as shown in Figure 5-2(a).
Use a meter stick to measure the x- and y-components for every point. For each collision you should have a (x, y) pair for both the projectile and the target ball. Note that the y values can be positive or negative.
Load the template named "momentum" on your computer. Input your data and perform the analysis. The results will include the average of the total momentum in the x-direction after collision for all the collisions. Compare this average final momentum with the initial total momentum. What is the percentage difference?
The results also include the average of the total momentum after collision in the y-direction (perpendicular to the initial momentum). What is the expected result? Compare your result with the expected value.

B. Collision of a Steel Ball and a Plastic Ball

For this part of the experiment, you cannot use the displacement vectors on the paper to represent the momentum because the two balls have different masses. For example, if the plastic ball and the steel ball have the same momentum but the plastic ball has 1/3 as much mass, then the plastic ball would have a displacement vector 3 times as long. Using this fact and assuming momentum conservation, you can deduce from your data the ratio of the mass of the two balls. Explain the principle behind it.

Now measure the *x*- and *y*-displacements for each vector and input them into the spreadsheet template. The analysis should yield an average of the ratio of the *y*-displacements for the two balls after collision.
Measure the masses of the two balls with a pan balance and compare the ratio with the ratio of the *y* displacements.

Discussions and Questions

1. Can you think of any reason that the horizontal momentum might not be exactly conserved in this experiment? Would the fact that the collision is not completely elastic affect your conclusions about momentum conservation?

2. Does the component of the total momentum perpendicular to the initial momentum remain zero after collision? Is the average closer to zero than the values for individual collisions?

3. If you have not answered the questions in other parts of the lab, answer them now.

Conclusions

Briefly discuss whether you have accomplished the goals listed at the beginning.