Experiment VI: Static Equilibrium of Rigid Bodies: Torques

Goals

- Study the relationship between force, lever arm, and torque; study how torques add

- Study the conditions for static equilibrium of a rigid body

- Study the concept of "center of gravity"

Introduction and Background

Torques: The torque τ created by a force F with respect to a certain axis of rotation is
$$\tau = F \times R_\perp \tag{6-1}$$
where $R_\perp$ is the so-called lever arm, which is the perpendicular (shortest) distance between the axis of rotation and the line of force. Torques are vectors. Strictly speaking, the direction of a torque should be perpendicular to (either into or out of) the plane defined by the force and its lever arm. However, for simplicity, we will define the direction of a torque as either clockwise (CW) or counterclockwise (CCW) depending which way the torque would make the object rotate about the axis of rotation chosen. Either CW or CCW can be chosen as the "+" direction as long as consistency is maintained. We will choose CW as the "+" direction here. The resultant of multiple torques can then be obtained as the algebraic sum of the individual torques.

Static Equilibrium: In order for a rigid body to stay in equilibrium, the net force and the net torque on the object must both be zero. We usually only deal with motion of objects in two dimensions, in which case the conditions for static equilibrium are
$$\sum F_x = 0, \qquad \sum F_y = 0 \tag{6-2}$$
$$\sum \tau = 0 \tag{6-3}$$
The second condition should hold for any choice of the axis of rotation.

Center of Gravity: For a rigid body with finite size, the force of gravity acts on all parts of the body. But for the purpose of studying the translational motion of the body as a whole or the static equilibrium of the body, we can assume that the entire weight of the body acts at a single point. This point is the *center of gravity*. When we draw the force diagram for a rigid body, we can put a single force of gravity at the center of gravity.

Experimental Setup

Equipment: Meter stick, fulcrum, clamps, weights, pan balance

Setup: The experimental setup is shown schematically in Figure
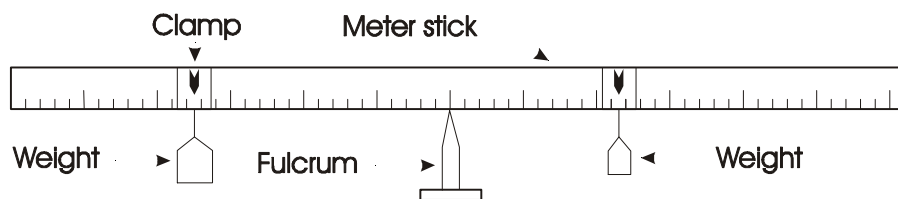

Figure 6.1 – Arrangement for balancing torques.

6-1. The apparatus consists of a meter stick suspended from a fulcrum. Weights may be hung from the meter stick at various positions along the stick by means of special clamps in order to apply torques.

Experimental Procedure and Data Analysis

A. Torques

1. Adjust the location of the fulcrum so that the meter stick, with no weights hanging on it, is in static equilibrium (balance) in a horizontal position. The position of the fulcrum is roughly the center of gravity of the meter stick. Note that this position is **not** necessarily in the middle of the stick (50 cm mark).

2. With a clamp, hang a 200-g mass on the stick 20 cm from the **fulcrum**. Hang another 200-g mass with a clamp on the opposite side of the stick. Find a position so that the meter stick is again in equilibrium in a horizontal position.
    Record the position of the fulcrum, the masses added to each side of the stick and their positions to the nearest millimeter mark.
    Keep the first 200-g fixed and repeat the procedure changing the second mass successively to values of 500 g, 400 g, 300 g, 250 g, 150 g, and 100 g. In each case adjust the position of the second mass to achieve equilibrium and record the data.

    Remember that you need to include the mass of the clamps! Weigh them with a balance and include them in the masses you record.

3. Calculate the weight (W) from the mass.
    Calculate the lever arm (L) from the positions of the mass and the fulcrum.
    Start the Excel template named "Torque" and input the values of W and L into it.

    Since you have kept the added torque on one side of the stick fixed, from the second condition of static equilibrium you should realize that the torque produced by any weight on the other side must be equal in magnitude and opposite in direction. Check whether your results are consistent with this conclusion by analyzing WL versus W with the template, including a "Linear Regression Fit" and a graph for WL versus W.

    -   What value of the slope do you obtain from your computer analysis? What is the expected value? What is the +/- uncertainty in the slope? Is your slope the same as the expected value within the uncertainty?
    -   How does the intercept compare with the fixed torque on the other side of the stick?

4. Now perform a "Linear Regression Fit" and plot a graph for W versus 1/L

    -   What should the slope of this plot give you? What is its value?
    -   What is the expected value for the intercept? What did you obtain?
    -   From the results can you any conclusion as to how the force required to balance a fixed torque depend on its lever arm?

B. *Resultant of Multiple Torques*

1. Maintain the fulcrum at the same position and balance the meter stick now with three weights: Put a single weight on one side and the other two on the other side. Record the masses and the positions. Calculate the magnitude of various torques around the fulcrum and find their vector sum.
   - Does your vector sum have the value you expect?

C. *Center of Gravity*:

1. So far the fulcrum has been placed only at the center of gravity. Now move the fulcrum to a point between the 10 cm and 25 cm marks. Balance the meter stick by applying a single mass between 100 to 500 g to the shorter side. Record the mass, its position, and the fulcrum position. Weigh the stick (without clamp) and record its mass.

   Although the meter stick has two sections on either side of the fulcrum, according to the concept of center of gravity we can still regard the entire weight acting at a single point, center of gravity, and it is the torque generated by this entire weight acting at center of gravity that is balanced by the torque from the hanging weight on the other side. Use the second condition for equilibrium to calculate the distance from the center of gravity to the fulcrum, and thus the position of the center of gravity, and compare this point with the previously determined position of center of gravity.

2. Draw a free-body force diagram for the meter stick for the last experimental situation and answer the following:

   - What must be the force exerted *by the fulcrum on the meter stick*? Show your calculation.
   - Remember the second condition for static equilibrium should hold for any choice of axis of rotation. Now take the 0 cm mark as the axis of rotation and calculate the net torque about it. Is your result as expected within experimental error?

Conclusions

Briefly discuss whether you have accomplished the goals listed at the beginning.