Experiment IX: Simple Harmonic Motion

Goals

- Verify Hooke's law for spring force
- Determine the period of a simple harmonic motion

Introduction and Background

A periodic motion is one that repeats itself in successive equal intervals of time. The time required for one complete repetition of the motion is called the period. The simplest periodic motion is a particle moving back and forth between two fixed points along a straight line. To undergo such a motion, the particle must be subject to a "restoring" force that is opposite to the displacement at least part of the time.

If the net force on the particle in the above periodic motion is such that the magnitude of the force is proportional to the displacement of the particle but the direction of the force is always opposite to that of the displacement (the force is always directed toward the midpoint). Namely,

$$F = -kx \qquad (9\text{-}1)$$

where k is a constant and $x = 0$ is the midpoint. Any object that obeys this relationship is said to obey Hooke's law, and the motion that results from this specific type of net force acting on a particle is called a *Simple Harmonic Motion*. The most common object that obeys Hooke's law on large length scale is a spring. Therefore, the motion of a particle on a spring is a classical example of simple harmonic motion. The diagram below illustrates an instant in such a simple harmonic motion. The points $x = \pm A$ are the endpoints of the motion, where A is called the *amplitude*.

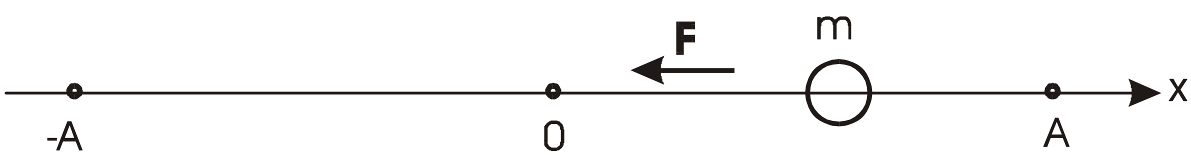

Figure 9.1 – Instantaneous displacement.

In your textbook it is shown that the resulted equation of motion (the equation giving the position x of the particle at time t) is,

$$x = A\sin(\frac{2\pi t}{T}) \qquad (9\text{-}2)$$

when the particle starts at $x = 0$ at time $t = 0$. Analysis shows that the period T of the motion depends on the spring constant k and the mass m in the following fashion:

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad (9\text{-}3)$$

In this lab we will verify that the force from a spring obeys Hooke's law and the period of the motion of a mass on a spring is indeed given by Equation 9-3.

Experimental Setup

***Equipment*:** Spring, meter stick, stopwatch, table clamp, rod and support, weights, balance

***Setup*:** A schematic diagram of the experimental setup is shown in Figure 9-2. Here the motion of the mass on the spring is up and down on a straight line.

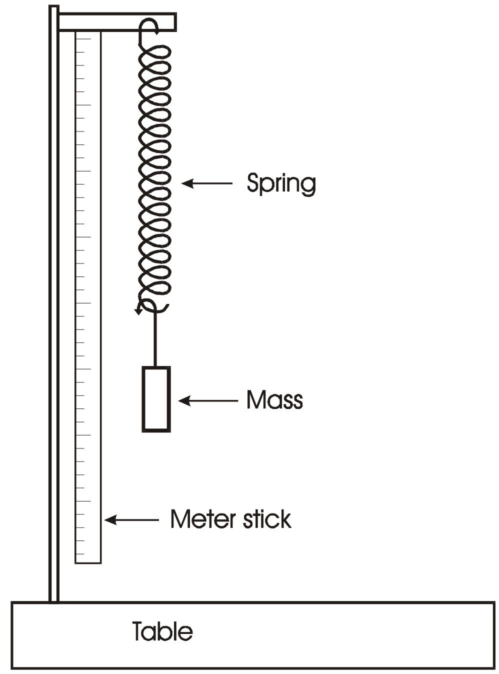

Figure 9.2 – Simple harmonic oscillation.

Experimental Procedure and Data Analysis

A. Hooke's Law

1. Hang the spring with the larger end down as shown in Figure 9-2. Clamp down a meter stick vertically to monitor the position of the mass. Place a total of 100 gram mass at the end of the spring. Record the equilibrium position of the mass. This point will be your *reference point* (x = 0) from which you will measure how much the spring stretch. The reason you need to start with this much mass is to ensure that all of the spring coils are separated and not squeezing each other.

2. Add a series of *additional* masses to the hanger in 50 gram increment up to 500 grams. Record these *additional* forces (not counting the original 100 g) and the corresponding rest position of the hanger.

3. Use the Excel template named "SHM" to plot the added additional force versus the displacement of the hanger from the *reference point*. Perform a Linear Regression Fit of Weight versus Displacement. Make a plot with your measured values and a best-fit line.

   - Obtain the spring constant k and the uncertainty in k from the linear regression fit.
   - Does your spring appear to obey Hooke's law?

***B. Simple Harmonic Motion*:**

1. Determine the period of oscillation for at least five different masses added to the spring. Since it is the entire mass that is oscillating, it is now necessary to include the entire mass

added to the spring. Start with 150 grams (total) on the spring and work up to the largest mass practicable.
Do not give the oscillations such a large amplitude that the coils of the spring bunched up together at the top of the motion.
Time at least 20 oscillations for each mass. Do this twice and take the average. (Remember the count is 0, not 1, when the timer is started).

2. Enter the data into the second part of the template "SHM". Calculate the period T and then the period squared T^2. Plot T^2 as a function of the added (total) mass, m_a. Perform a Linear Regression Fit of T^2 versus m_a.

   - What is the slope from the linear regression fit?
   - From Equation 9-3, what should this slope yield?
   - Calculate the spring constant from this slope. By what percentage doe it differ from the value you obtained in Part A?

3. From Equation 9-3, what should be the intercept for a plot of T^2 versus m? What is the actual value you obtained from the Linear Regression Fit?

   The reason for the discrepancy between the theoretical value and the measured value is that the theoretical relationship is based a massless spring, while a real spring has mass and it contributes to the total oscillating mass. On the other hand, you cannot simple add the entire mass of the spring to the oscillating mass because each part of the spring undergoes less and less motion as the top end of the spring is approached. Instead, the spring will contribute an effective mass, m_e, that is somewhat less than the total spring mass to the oscillating mass:

   $$m = m_a + m_e \tag{9-4}$$

   and

   $$T^2 = \frac{4\pi^2}{k}(m_a + m_e) = \frac{4\pi^2}{k} m_a + \frac{4\pi^2}{k} m_e \tag{9-5}$$

   Now calculate the effective mass of the spring m_e from the intercept in the Linear Regression Fit and the value k you have obtained.

   Measure the entire mass of the spring with a balance. About what fraction is m_e of the total spring mass?

4. For simple harmonic motion the period should not depend on the oscillation period. Perform a brief experimental verification of this fact. Do this by timing 20 oscillations for one fixed mass at three different amplitudes.

Conclusions

Briefly discuss whether you have accomplished the goals listed at the beginning.