Introduction to Numerical Differential Equations

Project #8

Computational Physics Lab

[Due Friday, February 29, 2008]

Radioactive Decays

This problem is a straightforward application of a forward difference. Given N(t) radioactive nuclei, they will decay randomly according to the following equation:

$$\frac{dN}{dt} = \frac{-N(t)}{\tau}$$

This equation can be solved analytically as $N(t) = N(0) \exp(-t/\tau)$, where N(0) is the initial number (or fraction) of radioactive nuclei. This solution allows us to compare our numerical results with the exact solution.

1. Numerically solve for the time dependence N(t) for the interval $0.0 \text{ s} \leq t \leq 15.0 \text{ s}$ assuming N(0) = 100% and $\tau = 2$ s. Do this for the following values of Δt: 1.0 s, 0.1 s, and 0.01 s. Graphically compare your results to the exact solution to this equation by plotting the fractional error vs. t. Overlay on one graph the plots for each Δt.

2. Using $\Delta t = 0.01$ s, plot the time dependence of N(t) for the following values of τ: 1.0, 3.0, and 5.0 s, and 0.01 s.

    - Briefly discuss the accuracy of your results. If there are any problems, explain.

3. Consider a system of a parent, ***P***, and a daughter nucleus, ***D*** both radioactive. The equations which describe their decays are as follows:

$$\frac{dN_P}{dt} = \frac{-N_P(t)}{\tau_P}$$

$$\frac{dN_D}{dt} = \frac{N_P(t)}{\tau_P} - \frac{N_D(t)}{\tau_D}$$

Numerically solve the above equations and plot the time dependence of N_P and N_D for $\tau_P = 2.0$ s and $\tau_D = 1.0$, 2.0 and 4.0 s.

Qualitatively explain the behavior of N_D for the situations in which $\tau_P \gg \tau_D$, $\tau_P \approx \tau_D$, and $\tau_P \ll \tau_D$.

4. Record your work and report on your results on your computational physics website. Create a html page for Project 8. Create a link from your main project web page to this html page. This html page should include the following heading information: exercise title, exercise number, your name, & today's date. The main content of this page should include the following:

- a short description of the exercise
- a short write up of the discussions from parts 2 &3
- a link to the source code
- a text region which contains the actual source code text
- a link to your program data file
- images (not links) of your plots

** For text regions use the html object tag; example:
<object width="600" height="400" type="text/plain" data="yourProgram.cc" border="0" ></object>