

# Numerical Limits in Programming

## Project #4

### Computational Physics Lab

*Prof. Volker Crede*

*January 29, 2009 [due by February 5th]*

#### Part 1

It is always good to know your limits.

a) Write a program find the largest integer  $N$  (computationally) such that  $(N+1)$  equals the integer which is larger than  $N$  by one.

b) Write a program to find the largest positive floating-point number  $x$ , to within a factor of 2, such that  $(1.0 + x) - 1.0 == 0$ .

#### Part 2

Consider the Taylor expansion for the exponential

$$e^x = 1 + x + x^2/2! + x^3/3! + \dots = \lim_{N \rightarrow \infty} S(x,N)$$

where  $S(x,N)$  is the partial sum with  $N+1$  terms. (a) Write a program that provides data points for the absolute fractional error of the sum,  $|S(x,N) - e^x|/e^x$ , versus  $N$  (up to  $N = 20$ ) for a given value of  $x$ . Test your program for  $x = 10, 2, -2, -10$ . Plot your data and, explain why  $S(x,N)$  fails to reasonably approximate the function  $e^x$  when  $x < 0$ . Now modify your program so that it uses the identity  $e^x = 1/e^{-x} = 1/S(-x,N)$  to evaluate the exponential when  $x$  is negative. Explain why this approach works. For  $e^x$  use the math library `exp(x)` function which requires including the `math.h` header (`#include <math.h>`).

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## Part 3

Post exercise to your computational physics website on comphy. Create a html page for Project 4. Create a link from your main project web page to this html page. This html page should include the following heading information: exercise title, exercise number, your name, & today's date. The main content of this page should include the following:

### Part 1

- a short description of each tasks
- links to your final source code files
- text regions which contains the actual source code text
- a text region with the program outputs

### Part 2

- a short description of the exercise
- a link to the source code
- the text region which contains the actual source code text
- plots and a summary of discussions about the different approaches failures and sucesses

For your web images, make sure that the image is sized so that it reasonably fits on your html page. This can be done using the height and width options of the image html tag; example:

```
<IMG SRC="ex1_part1.jpg" WIDTH=165 HEIGHT=213 >
```

For text regions use the html object tag; example:

```
<object width="600" height="400" type="text/plain" data="ex1_part2.cc" border="0" ></object>
```