Kinematic Fitting in Hadron Physics

Prof. Sean Dobbs[1] & Daniel Lersch[2]

April 8, 2020

[1] (sdobbs@fsu.edu)
[2] (dlersch@jlab.org)

Scope of this (2 Parts) Lecture

- Application of kinematic fitting techniques to (hadron) physics data analysis
  - Example analysis of $pp \to ppX$ reactions
    ($X = \pi^+\pi^-$, $X = \pi^+\pi^-\pi^0$, $X = \eta$,..)
  - **Goal:** Reconstruct the reaction: $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$
  - Get familiar with kinematic fitting

Scope of this (2 Parts) Lecture

- Application of kinematic fitting techniques to (hadron) physics data analysis
  - Example analysis of $pp \to ppX$ reactions
    ($X = \pi^+\pi^-$, $X = \pi^+\pi^-\pi^0$, $X = \eta$,..)
  - **Goal:** Reconstruct the reaction: $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$
  - Get familiar with kinematic fitting
- Homework (will be hand out by Wednesday)
  - Repeat analysis techniques discussed within this lecture on a given data set
  - Homework data with be similar (but not identical!) to the data presented in this lecture
  - This presentation and the homework will (soon) be available at:
    http://hadron.physics.fsu.edu/~dlersch/
  - Reference: Paul Avery: "Fitting Theory Writeups and References"
    https://www.phys.ufl.edu/~avery/fitting.html
    $\to$ Nice summary lectures about kinematic fitting

Scope of this (2 Parts) Lecture

- Application of kinematic fitting techniques to (hadron) physics data analysis
  - Example analysis of $pp \to ppX$ reactions ($X = \pi^+\pi^-$, $X = \pi^+\pi^-\pi^0$, $X = \eta$,..)
  - **Goal:** Reconstruct the reaction: $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$
  - Get familiar with kinematic fitting
- Homework (will be hand out by Wednesday)
  - Repeat analysis techniques discussed within this lecture on a given data set
  - Homework data with be similar (but not identical!) to the data presented in this lecture
  - This presentation and the homework will (soon) be available at: `http://hadron.physics.fsu.edu/~dlersch/`
  - Reference: Paul Avery: "Fitting Theory Writeups and References" `https://www.phys.ufl.edu/~avery/fitting.html` $\to$ Nice summary lectures about kinematic fitting
- Have fun!

Part I

- Data set and experimental setup
- Monitoring spectra
- Reference analysis
- Introduction to kinematic fitting

The $pp \to ppX$ Data Set

- The data set analyzed in this lecture is composed of three generated[3] reactions:
  i) $pp \to pp\pi^+\pi^-$
  ii) $pp \to pp\pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]$
  iii) $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$

[3] Just phase-space, nothing too exciting

4

5

The $pp \to ppX$ Data Set

- The data set analyzed in this lecture is composed of three generated[3] reactions:
  i) $pp \to pp\pi^+\pi^-$
  ii) $pp \to pp\pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]$
  iii) $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$
- The relative abundance increase:[4]
  $N(pp \to pp\pi^+\pi^-) > N(pp \to pp\pi^+\pi^-\pi^0) > N(pp \to pp\eta[\eta \to ..])$

[3] Just phase-space, nothing too exciting
[4] Did not use realistic ratios,
[5]

The $pp \to ppX$ Data Set

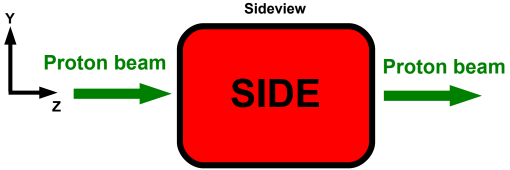
- The data set analyzed in this lecture is composed of three generated[3] reactions:
    i) $pp \to pp\pi^+\pi^-$
    ii) $pp \to pp\pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]$
    iii) $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$
- The relative abundance increase:[4]
  $N(pp \to pp\pi^+\pi^-) > N(pp \to pp\pi^+\pi^-\pi^0) > N(pp \to pp\eta[\eta \to ..])$
- Each reaction is reconstructed [5] within a hypothetical detector, including:
    - Resolution effects
    - Reconstruction inefficiencies

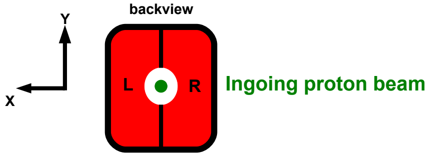
[3] Just phase-space, nothing too exciting

[4] Did not use realistic ratios,

[5] No GEANT4, just computed different response functions to mimic a detector,

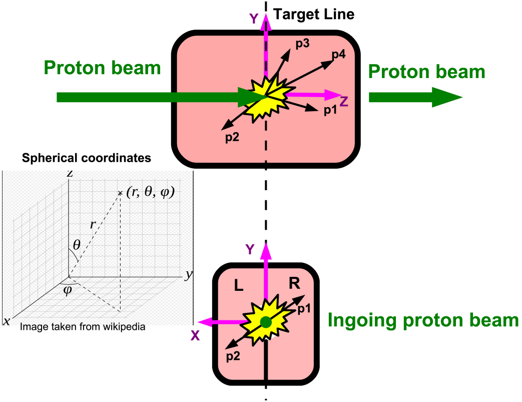
The SImple DEtector - SIDE Project





- Hypothetical experimental setup, inspired by **WASA-at-COSY** and **GlueX**
- Proton beam on proton target inside centre of SIDE
- Right-handed spherical coordinate system with origin at target location
- "Detector" mainly consists of two halves (**L**eft and **R**ight)

The SImple DEtector - SIDE Project



- Hypothetical experimental setup, inspired by **WASA-at-COSY** and **GlueX**
- Proton beam on proton target inside centre of SIDE
- Right-handed spherical coordinate system with origin at target location
- "Detector" mainly consists of two halves (**L**eft and **R**ight)

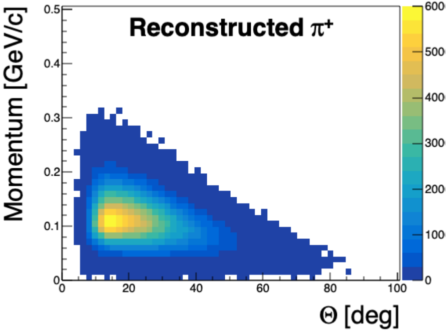
SIDE: Overview and Performance

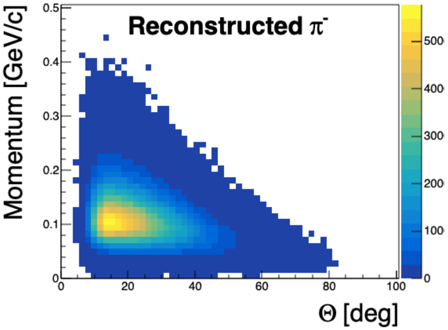
Experiment / Detector Property	Setting
Target	point-like
beam energy	1.5 GeV
Vertex information	No $\rightarrow$ point-like interaction region
Angular Coverage	$\sim 4\pi$
Particle Reconstruction	Charged and Neutral
Acceptance Loss	Beam line at $\theta \leq 10^\circ$
	Target line insertion at $\phi = \pm 90^\circ$
p / π^+ misidentification	$\sim$ 15%

- Remember, this detector does not exist in reality
- Some settings are not realistic

Charged Particle Reconstruction

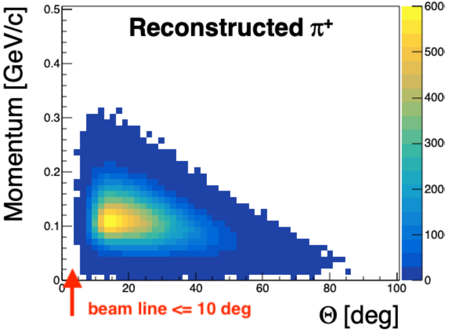
- Momentum resolution σ_p depends on charge: $\sigma_p(+q) < \sigma_p(-p)$
- Beam line at $\theta \leq 10^\circ$
- Detection inefficiency ϵ, depending on ϕ: $\epsilon(L) > \epsilon(R)$

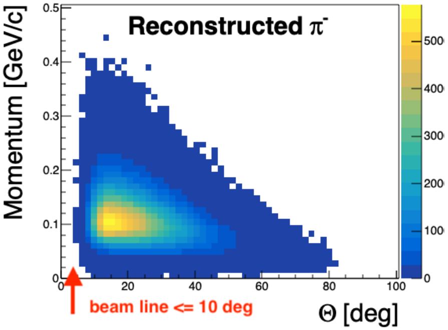




Charged Particle Reconstruction

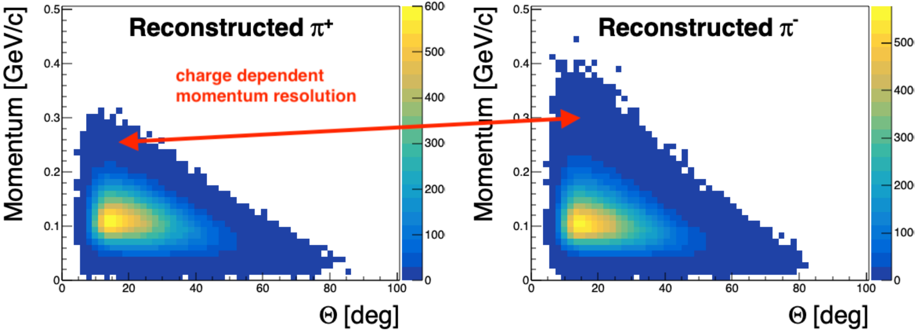
- Momentum resolution σ_p depends on charge: $\sigma_p(+q) < \sigma_p(-p)$
- Beam line at $\theta \leq 10^\circ$
- Detection inefficiency ϵ, depending on ϕ: $\epsilon(L) > \epsilon(R)$





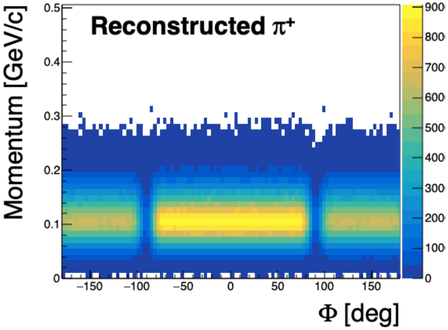
Charged Particle Reconstruction

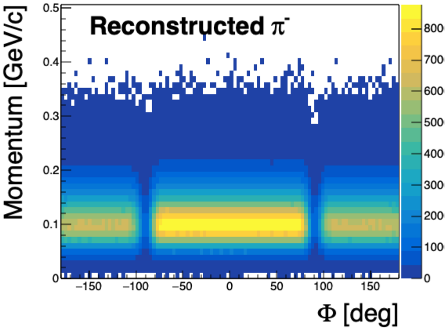
- Momentum resolution σ_p depends on charge: $\sigma_p(+q) < \sigma_p(-p)$
- Beam line at $\theta \leq 10^\circ$
- Detection inefficiency ϵ, depending on ϕ: $\epsilon(L) > \epsilon(R)$



Charged Particle Reconstruction

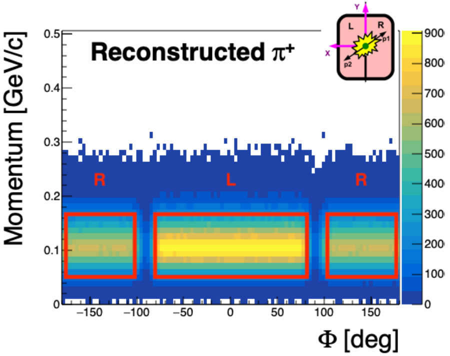
- Momentum resolution σ_p depends on charge: $\sigma_p(+q) < \sigma_p(-p)$
- Beam line at $\theta \leq 10^\circ$
- Detection inefficiency ϵ, depending on ϕ: $\epsilon(L) > \epsilon(R)$

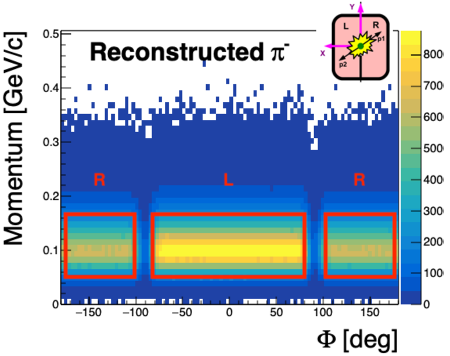




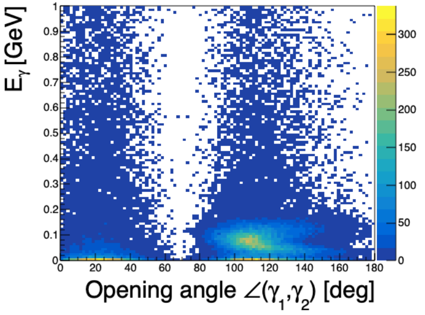
Charged Particle Reconstruction

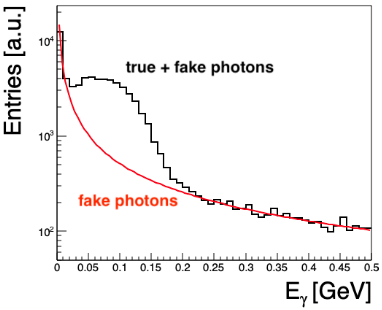
- Momentum resolution σ_p depends on charge: $\sigma_p(+q) < \sigma_p(-p)$
- Beam line at $\theta \leq 10^\circ$
- Detection inefficiency ϵ, depending on ϕ: $\epsilon(L) > \epsilon(R)$

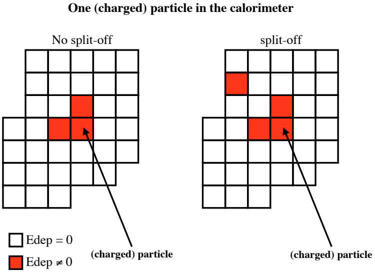




Photon Reconstruction and Fake Photons







- Most hadron physics experiments use a calorimeter for photon reconstruction
  $\rightarrow$ So does SIDE
- Energy spread pattern of charged / neutral particles hitting the calorimeter might cause falsely reconstructed photons
- These photons are referred as (hadronic) split-offs
  $\rightarrow$ Not discussed in detail here
- For simplicity, we call these photons: "fake photons"

Intermediate Summary and next Steps

- Simulated measurements of $pp \to ppX$ reactions
- Reconstructed particles with SIDE:
  - Not a perfect detector
  - Reconstruction inefficiencies
  - Resolution effects
  - Particle misidentification
- Next: select events with: $\eta \to \pi^+\pi^-\pi^0$ decays, but we need:
  - Selection criteria
  - Monitoring plots $\to$ to have control over the selection
- The following slides will present a small analysis walk through

Helpful Definitions and Monitoring Spectra

- No matter how you perform your analysis $\Rightarrow$ **monitor every single step** (future-you will be thankful!)

Helpful Definitions and Monitoring Spectra

- No matter how you perform your analysis $\Rightarrow$ **monitor every single step** (future-you will be thankful!)
- There are many monitoring variables / plots, some more helpful than others

Helpful Definitions and Monitoring Spectra

- No matter how you perform your analysis ⇒ **monitor every single step** (future-you will be thankful!)
- There are many monitoring variables / plots, some more helpful than others
- Always helpful: **Energy and momentum conservation**

Helpful Definitions and Monitoring Spectra

- No matter how you perform your analysis $\Rightarrow$ **monitor every single step** (future-you will be thankful!)
- There are many monitoring variables / plots, some more helpful than others
- Always helpful: **Energy and momentum conservation**
- Suppose reaction: particle1 + particle2 $\rightarrow$ particle3 + particle4 + ...

Helpful Definitions and Monitoring Spectra

- No matter how you perform your analysis $\Rightarrow$ **monitor every single step** (future-you will be thankful!)
- There are many monitoring variables / plots, some more helpful than others
- Always helpful: **Energy and momentum conservation**
- Suppose reaction: particle1 + particle2 $\rightarrow$ particle3 + particle4 + ...
- Each particle i is defined by 4-momentum vector (Lorentz-Vector) $\mathbf{P}_i$:

$$\mathbf{P}_i = \begin{pmatrix} P_x \\ P_y \\ P_z \\ E \end{pmatrix} \tag{1}$$

Helpful Definitions and Monitoring Spectra

- No matter how you perform your analysis $\Rightarrow$ **monitor every single step** (future-you will be thankful!)
- There are many monitoring variables / plots, some more helpful than others
- Always helpful: **Energy and momentum conservation**
- Suppose reaction: particle1 + particle2 $\rightarrow$ particle3 + particle4 + ...
- Each particle i is defined by 4-momentum vector (Lorentz-Vector) $\mathbf{P}_i$:

$$\mathbf{P}_i = \begin{pmatrix} P_x \\ P_y \\ P_z \\ E \end{pmatrix} \tag{1}$$

- **Energy and momentum conservation:** Incoming and outgoing 4-momenta should balance out

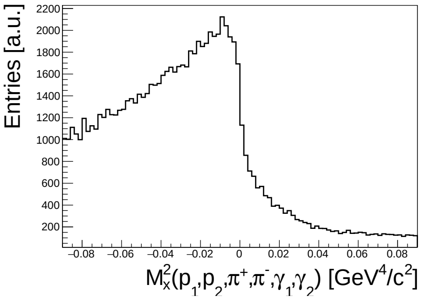
$$\mathbf{P}_{in} = \mathbf{P}_{out} \Leftrightarrow \mathbf{P}_1 + \mathbf{P}_2 - \mathbf{P}_3 - \mathbf{P}_4 - ... \stackrel{!}{=} 0 \tag{2}$$

(Overall) Missing Mass

- Using the definition from the previous slide:

$$M_x(P_3, P_4, ..) = \|\mathbf{P}_{in} - \mathbf{P}_{out}\| \Leftrightarrow \|\mathbf{P}_1 + \mathbf{P}_2 - \mathbf{P}_3 - \mathbf{P}_4 - ...\| \stackrel{!}{=} 0 \qquad (3)$$

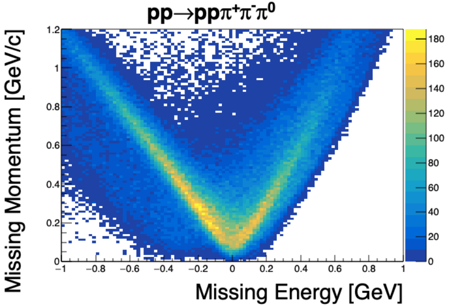
- Shown below: M_x^2 for the reaction hypothesis: $pp \rightarrow pp\pi^+\pi^-\gamma_1\gamma_2$

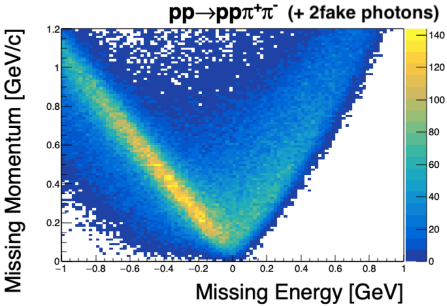


- Possible reasons for $M_x^2 \neq 0$: Resolution effects from experimental setup, background reactions which do not match the reaction topology, misidentified particles

Missing Momentum vs. Missing Energy

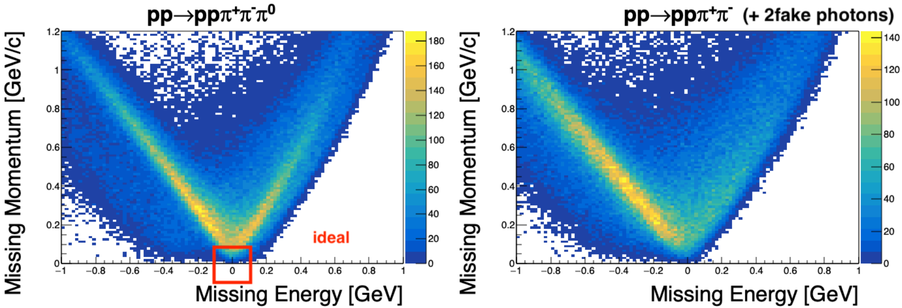
- Missing momentum $= |\vec{P}_1 + \vec{P}_2 - \vec{P}_3 - \vec{P}_4 - ...| \in [0, \mathbb{R})$
- Missing energy $= E_1 + E_2 - E_3 - E_4 - ... \in (-\mathbb{R}, \mathbb{R})$





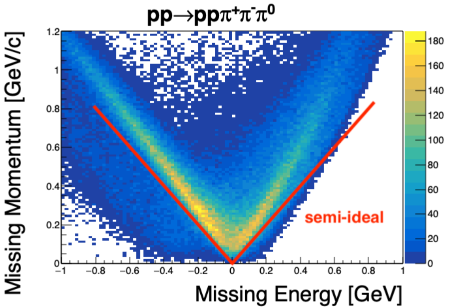
Missing Momentum vs. Missing Energy

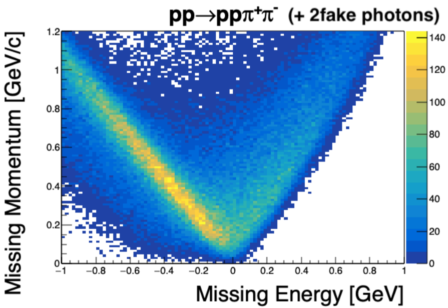
- Missing momentum $= |\vec{P}_1 + \vec{P}_2 - \vec{P}_3 - \vec{P}_4 - ...| \in [0, \mathbb{R})$
- Missing energy $= E_1 + E_2 - E_3 - E_4 - ... \in (-\mathbb{R}, \mathbb{R})$
- Ideal case: missing momentum = missing energy = 0



Missing Momentum vs. Missing Energy

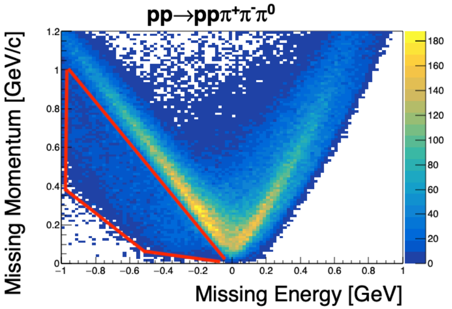
- Missing momentum $= |\vec{P}_1 + \vec{P}_2 - \vec{P}_3 - \vec{P}_4 - ...| \in [0, \mathbb{R})$
- Missing energy $= E_1 + E_2 - E_3 - E_4 - ... \in (-\mathbb{R}, \mathbb{R})$
- Ideal case: missing momentum $=$ missing energy $= 0$
- Semi-ideal case: missing momentum $\simeq \pm$ missing energy

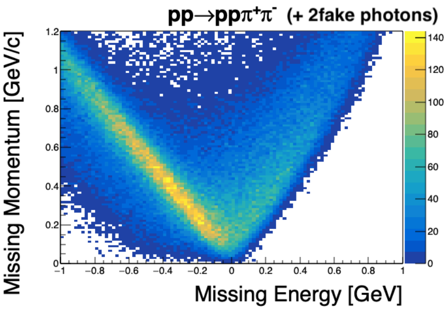




Missing Momentum vs. Missing Energy

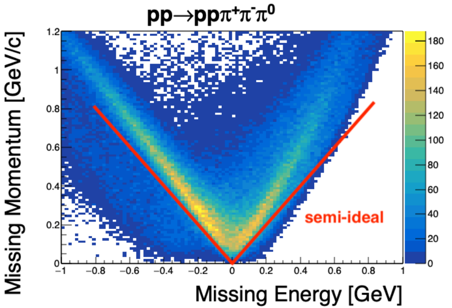
- Missing momentum $= |\vec{P}_1 + \vec{P}_2 - \vec{P}_3 - \vec{P}_4 - ...| \in [0, \mathbb{R})$
- Missing energy $= E_1 + E_2 - E_3 - E_4 - ... \in (-\mathbb{R}, \mathbb{R})$
- <u>Ideal case:</u> missing momentum $=$ missing energy $= 0$
- <u>Semi-ideal case:</u> missing momentum $\simeq \pm$ missing energy
- Shifts / offsets with respect to x-axis:
  - Particle misidentification, i.e. incorrect mass assignment
  - Background events which do not match the reaction hypothesis of interest

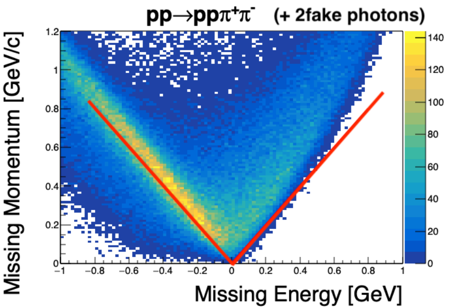




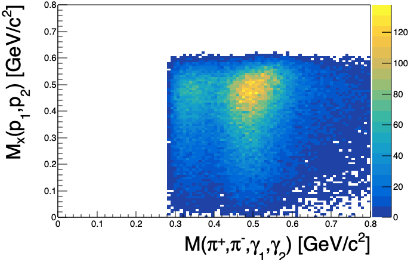
Missing Momentum vs. Missing Energy

- Missing momentum $= |\vec{P}_1 + \vec{P}_2 - \vec{P}_3 - \vec{P}_4 - ...| \in [0, \mathbb{R})$
- Missing energy $= E_1 + E_2 - E_3 - E_4 - ... \in (-\mathbb{R}, \mathbb{R})$
- <u>Ideal case:</u> missing momentum = missing energy = 0
- <u>Semi-ideal case:</u> missing momentum $\simeq \pm$ missing energy
- Shifts / offsets with respect to x-axis:
  - Particle misidentification, i.e. incorrect mass assignment
  - Background events which do not match the reaction hypothesis of interest
- **BUT:** Observing: missing momentum $\simeq \pm$ missing energy does NOT guarantee that your data sample is background free

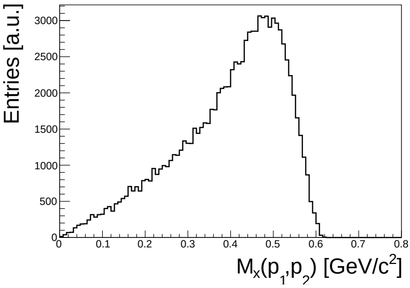


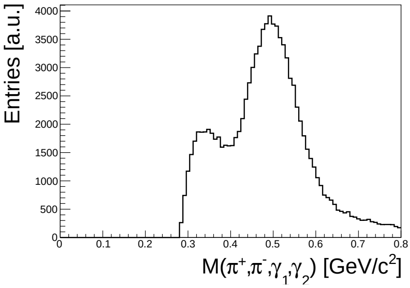


Intermediate States: Missing Mass vs. Invariant Mass

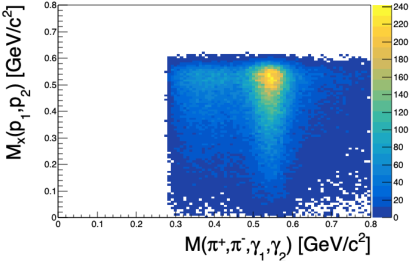


- Look at reaction:
  $pp \rightarrow pp\pi^+\pi^-\pi^0[\pi^0 \rightarrow \gamma_1\gamma_2]$
- Missing mass:
  $M_x(p_1, p_2) = \|\mathbf{P}_{beam} + \mathbf{P}_{target} - (\mathbf{P}_{p_1} + \mathbf{P}_{p_2})\|$
- Invariant mass:
  $M(\pi^+, \pi^-, \gamma_1, \gamma_2) = \|\mathbf{P}_{\pi^+} + \mathbf{P}_{\pi^-} + \mathbf{P}_{\gamma_1} + \mathbf{P}_{\gamma_1}\|$
- Ideally: $M_x(p_1, p_2) = M(\pi^+, \pi^-, \gamma_1, \gamma_2)$

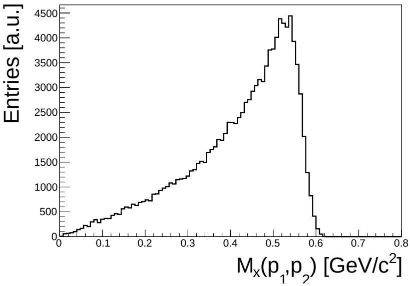


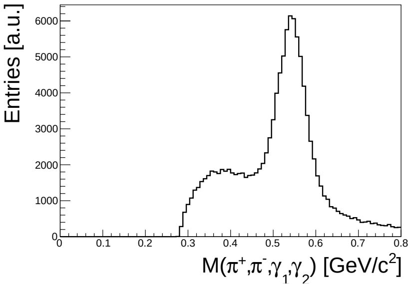


Intermediate States: Missing Mass vs. Invariant Mass

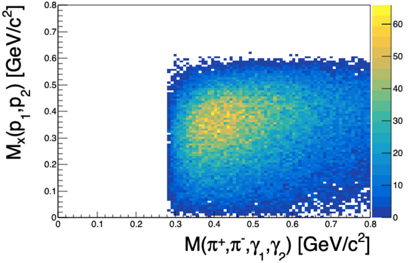


- Look at reaction:
  $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma_1\gamma_2]]$
- Missing mass:
  $M_x(p_1, p_2) = \|\mathbf{P}_{beam} + \mathbf{P}_{target} - (\mathbf{P}_{p_1} + \mathbf{P}_{p_2})\|$
- Invariant mass:
  $M(\pi^+, \pi^-, \gamma_1, \gamma_2) = \|\mathbf{P}_{\pi^+} + \mathbf{P}_{\pi^-} + \mathbf{P}_{\gamma_1} + \mathbf{P}_{\gamma_1}\|$
- Ideally: $M_x(p_1, p_2) = M(\pi^+, \pi^-, \gamma_1, \gamma_2)$

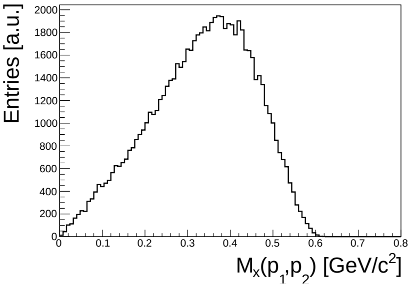


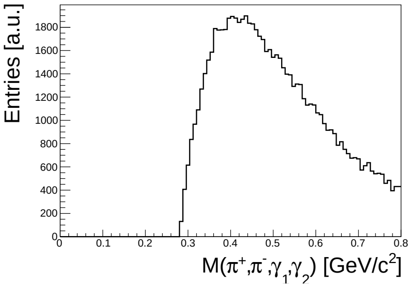


Intermediate States: Missing Mass vs. Invariant Mass

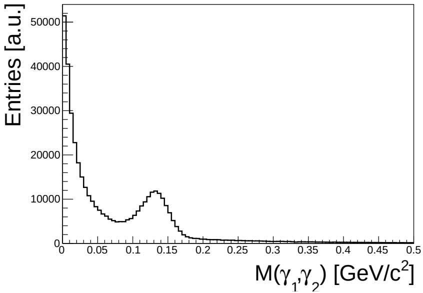


- Look at reaction: $pp \to pp\pi^+\pi^- + 2$ fake photons
- Missing mass:
  $M_x(p_1, p_2) = \|\mathbf{P}_{beam} + \mathbf{P}_{target} - (\mathbf{P}_{p_1} + \mathbf{P}_{p_2})\|$
- Invariant mass:
  $M(\pi^+, \pi^-, \gamma_1, \gamma_2) = \|\mathbf{P}_{\pi^+} + \mathbf{P}_{\pi^-} + \mathbf{P}_{\gamma_1} + \mathbf{P}_{\gamma_1}\|$
- Ideally: $M_x(p_1, p_2) = M(\pi^+, \pi^-, \gamma_1, \gamma_2)$

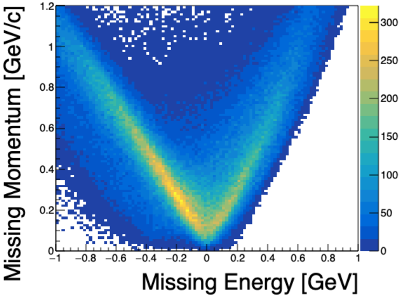


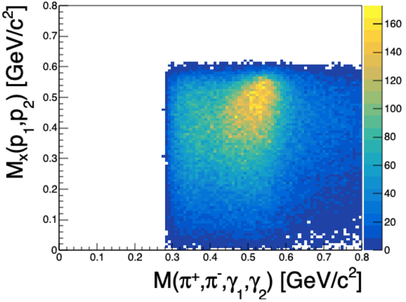


Analysis

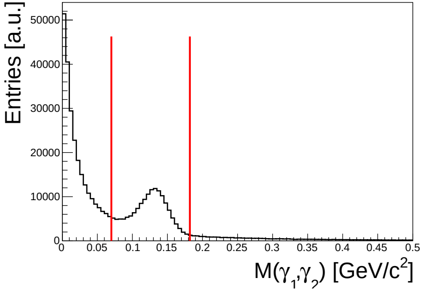


- **NOTE:** All cuts shown in the following are "first-guess" cuts and chosen for demonstration purposes only
- Use the missing momentum vs. missing energy plot to monitor effects of cuts



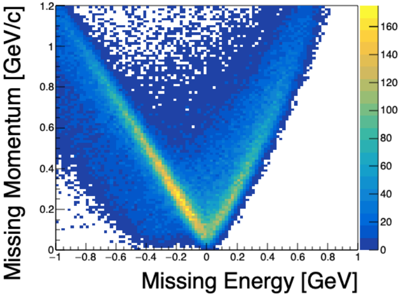


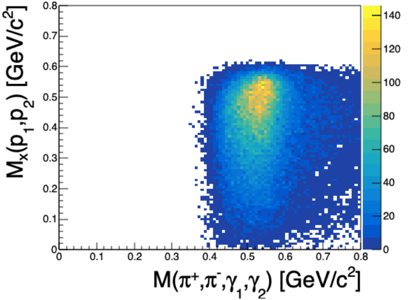
Analysis



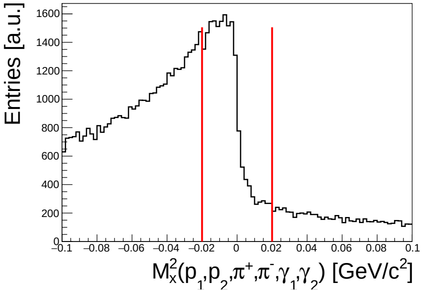
- **NOTE:** All cuts shown in the following are "first-guess" cuts and chosen for demonstration purposes only
- Use the missing momentum vs. missing energy plot to monitor effects of cuts

1. Request: $M(\gamma_1, \gamma_2) \approx m_{\pi^0}$
   $\rightarrow$ suppress all $pp \rightarrow pp\pi^+\pi^-$ events, by rejection low energetic photons



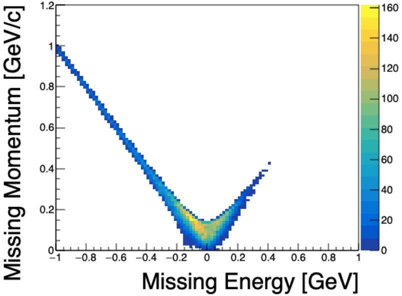


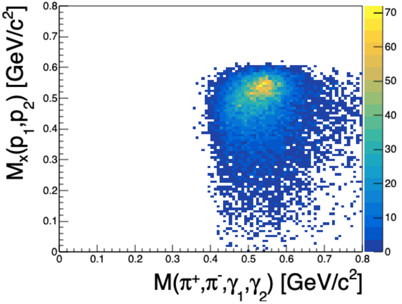
Analysis



- **NOTE:** All cuts shown in the following are "first-guess" cuts and chosen for demonstration purposes only
- Use the missing momentum vs. missing energy plot to monitor effects of cuts

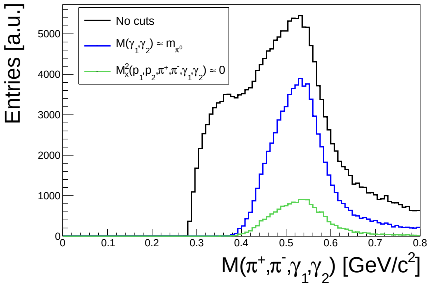
1. Request: $M(\gamma_1, \gamma_2) \approx m_{\pi^0}$
   $\rightarrow$ suppress all $pp \rightarrow pp\pi^+\pi^-$ events, by rejection low energetic photons
2. Select: $M_x^2(p_1, p_2, \pi^+, \pi^- \gamma_1, \gamma_2) \approx 0$
   $\rightarrow$ ensure energy and momentum conservation





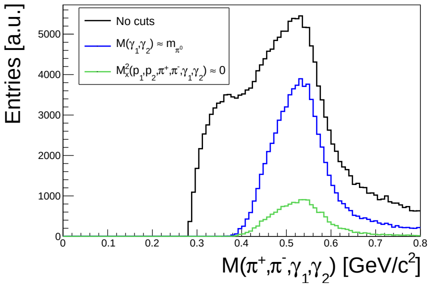
Recap

- Successfully selected events with: π^+, π^- and two photons in the final state
- But, which events stem from $\eta \to \pi^+\pi^-\pi^0$?
- Might / should have performed analysis differently



Recap

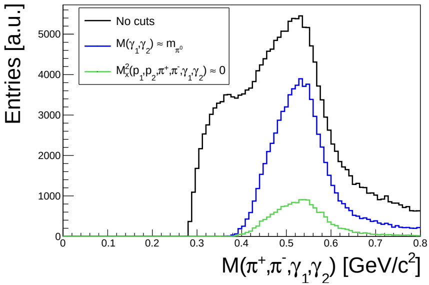
- Successfully selected events with: π^+, π^- and two photons in the final state
- But, which events stem from $\eta \to \pi^+\pi^-\pi^0$?
- Might / should have performed analysis differently



- The selection criteria shown on the previous slides are NOT the ultimate truth
- Someone else might have performed this analysis differently
- Exploring different analysis chains / techniques helps you to better understand your data
- **Physics Data Analysis is always an iterative process**

Recap

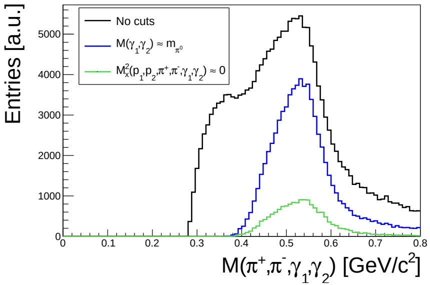
- Successfully selected events with: π^+, π^- and two photons in the final state
- But, which events stem from $\eta \to \pi^+\pi^-\pi^0$?
- Might / should have performed analysis differently



- The selection criteria shown on the previous slides are NOT the ultimate truth
- Someone else might have performed this analysis differently
- Exploring different analysis chains / techniques helps you to better understand your data
- **Physics Data Analysis is always an iterative process**
- No matter which cut(s) you end up choosing, two basic requirements need to be fulfilled:
  - i) Take detector resolution into account
    $\to$ A 0.00003 GeV energy cut window is pointless if the detector resolution is in the order of $\sim$ 0.1 GeV
  - ii) Consider reaction kinematics
    $\to$ All previously shown cuts did this to some extend

Recap

- Successfully selected events with: π^+, π^- and two photons in the final state
- But, which events stem from $\eta \to \pi^+\pi^-\pi^0$?
- Might / should have performed analysis differently



- The selection criteria shown on the previous slides are NOT the ultimate truth
- Someone else might have performed this analysis differently
- Exploring different analysis chains / techniques helps you to better understand your data
- **Physics Data Analysis is always an iterative process**
- No matter which cut(s) you end up choosing, two basic requirements need to be fulfilled:
  i) Take detector resolution into account
  $\rightarrow$ A $0.00003\,\mathrm{GeV}$ energy cut window is pointless if the detector resolution is in the order of $\sim 0.1\,\mathrm{GeV}$
  ii) Consider reaction kinematics
  $\rightarrow$ All previously shown cuts did this to some extend

$\Rightarrow$ **Kinematic Fitting**

Introduction to Kinematic Fitting

- Suppose (again) the reaction: $\mathbf{P}_1 + \mathbf{P}_2 \rightarrow \mathbf{P}_3 + \mathbf{P}_4 + ...$, where $\mathbf{P}_i$ is the Lorentz-Vector of particle i and all particle momenta are known

Introduction to Kinematic Fitting

- Suppose (again) the reaction: $\mathbf{P}_1 + \mathbf{P}_2 \to \mathbf{P}_3 + \mathbf{P}_4 + ...$, where $\mathbf{P}_i$ is the Lorentz-Vector of particle i and all particle momenta are known
- Each vector is determined by measured quantities (angles, momenta,..), i.e. your detector

Introduction to Kinematic Fitting

- Suppose (again) the reaction: $\mathbf{P}_1 + \mathbf{P}_2 \rightarrow \mathbf{P}_3 + \mathbf{P}_4 + ...$, where $\mathbf{P}_i$ is the Lorentz-Vector of particle i and all particle momenta are known
- Each vector is determined by <u>measured</u> quantities (angles, momenta,..), i.e. your detector
- The detector is (unfortunately) not perfect $\rightarrow$ Each quantity bears an uncertainty σ

Introduction to Kinematic Fitting

- Suppose (again) the reaction: $\mathbf{P}_1 + \mathbf{P}_2 \rightarrow \mathbf{P}_3 + \mathbf{P}_4 + ...$, where $\mathbf{P}_i$ is the Lorentz-Vector of particle i and all particle momenta are known
- Each vector is determined by <u>measured</u> quantities (angles, momenta,..), i.e. your detector
- The detector is (unfortunately) not perfect $\rightarrow$ Each quantity bears an uncertainty σ
- Given spherical coordinates, one might assign for each particle i:
    - $\sigma_{p,i} \leftrightarrow$ Momentum uncertainty
    - $\sigma_{\theta,i} \leftrightarrow$ Angular uncertainty for θ
    - $\sigma_{\phi,i} \leftrightarrow$ Angular uncertainty for ϕ

Introduction to Kinematic Fitting

- Suppose (again) the reaction: $\mathbf{P}_1 + \mathbf{P}_2 \rightarrow \mathbf{P}_3 + \mathbf{P}_4 + ...$, where $\mathbf{P}_i$ is the Lorentz-Vector of particle i and all particle momenta are known
- Each vector is determined by measured quantities (angles, momenta,..), i.e. your detector
- The detector is (unfortunately) not perfect $\rightarrow$ Each quantity bears an uncertainty σ
- Given spherical coordinates, one might assign for each particle i:
  - $\sigma_{p,i} \leftrightarrow$ Momentum uncertainty
  - $\sigma_{\theta,i} \leftrightarrow$ Angular uncertainty for θ
  - $\sigma_{\phi,i} \leftrightarrow$ Angular uncertainty for ϕ
- This means, that each particle stems from a true[6]-Vector with: $p_{i,true}$, $\theta_{i,true}$ and $\phi_{i,true}$

[6] The vector which describes the particle in reality

Introduction to Kinematic Fitting

- Suppose (again) the reaction: $\mathbf{P}_1 + \mathbf{P}_2 \rightarrow \mathbf{P}_3 + \mathbf{P}_4 + ...$, where $\mathbf{P}_i$ is the Lorentz-Vector of particle i and all particle momenta are known
- Each vector is determined by <u>measured</u> quantities (angles, momenta,..), i.e. your detector
- The detector is (unfortunately) not perfect $\rightarrow$ Each quantity bears an uncertainty σ
- Given spherical coordinates, one might assign for each particle i:
  - $\sigma_{p,i} \leftrightarrow$ Momentum uncertainty
  - $\sigma_{\theta,i} \leftrightarrow$ Angular uncertainty for θ
  - $\sigma_{\phi,i} \leftrightarrow$ Angular uncertainty for ϕ
- This means, that each particle stems from a true[6]-Vector with: $p_{i,true}$, $\theta_{i,true}$ and $\phi_{i,true}$
- Assuming that each measured quantity follows a gaussian distribution, one might write:
  - $p_{i,measured} = p_{i,true} \pm \sigma_{p,i}$
  - $\theta_{i,measured} = \theta_{i,true} \pm \sigma_{\theta,i}$
  - $\phi_{i,measured} = \phi_{i,true} \pm \sigma_{\phi,i}$

[6] The vector which describes the particle in reality

Introduction to Kinematic Fitting

- Suppose (again) the reaction: $\mathbf{P}_1 + \mathbf{P}_2 \rightarrow \mathbf{P}_3 + \mathbf{P}_4 + ...$, where $\mathbf{P}_i$ is the Lorentz-Vector of particle i and all particle momenta are known
- Each vector is determined by measured quantities (angles, momenta,..), i.e. your detector
- The detector is (unfortunately) not perfect $\rightarrow$ Each quantity bears an uncertainty σ
- Given spherical coordinates, one might assign for each particle i:
  - $\sigma_{p,i} \leftrightarrow$ Momentum uncertainty
  - $\sigma_{\theta,i} \leftrightarrow$ Angular uncertainty for θ
  - $\sigma_{\phi,i} \leftrightarrow$ Angular uncertainty for ϕ
- This means, that each particle stems from a true[6]-Vector with: $p_{i,true}$, $\theta_{i,true}$ and $\phi_{i,true}$
- Assuming that each measured quantity follows a gaussian distribution, one might write:
  - $p_{i,measured} = p_{i,true} \pm \sigma_{p,i}$
  - $\theta_{i,measured} = \theta_{i,true} \pm \sigma_{\theta,i}$
  - $\phi_{i,measured} = \phi_{i,true} \pm \sigma_{\phi,i}$
- Since the measured quantities and the uncertainties are known, one might find the true information by fitting

[6] The vector which describes the particle in reality

Introduction to Kinematic Fitting: Least Squares

- Presuming that all variables (and the corresponding uncertainties) are measured independently[7] and gaussian distributed, one might formulate:

$$\chi^2 \equiv \sum_i \left[\left(\frac{p_{meas,i} - p_{fit,i}}{\sigma_{p,i}} \right)^2 + \left(\frac{\theta_{meas,i} - \theta_{fit,i}}{\sigma_{\theta,i}} \right)^2 + \left(\frac{\phi_{meas,i} - \phi_{fit,i}}{\sigma_{\phi,i}} \right)^2 \right] \stackrel{!}{=} 0 \quad (4)$$

[7] This is already a dangerous assumption!

Introduction to Kinematic Fitting: Least Squares

- Presuming that all variables (and the corresponding uncertainties) are measured independently[7] and gaussian distributed, one might formulate:

$$\chi^2 \equiv \sum_i \left[\left(\frac{p_{meas,i} - p_{fit,i}}{\sigma_{p,i}} \right)^2 + \left(\frac{\theta_{meas,i} - \theta_{fit,i}}{\sigma_{\theta,i}} \right)^2 + \left(\frac{\phi_{meas,i} - \phi_{fit,i}}{\sigma_{\phi,i}} \right)^2 \right] \stackrel{!}{=} 0 \quad (4)$$

- This does not help much, because one could simply set $p_{meas,i} = p_{fit,i}$, $\theta_{meas,i} = \theta_{fit,i}, \ldots$ and obtain $\chi^2 = 0$

[7] This is already a dangerous assumption!

Introduction to Kinematic Fitting: Least Squares

- Presuming that all variables (and the corresponding uncertainties) are measured independently[7] and gaussian distributed, one might formulate:

$$\chi^2 \equiv \sum_i \left[\left(\frac{p_{meas,i} - p_{fit,i}}{\sigma_{p,i}} \right)^2 + \left(\frac{\theta_{meas,i} - \theta_{fit,i}}{\sigma_{\theta,i}} \right)^2 + \left(\frac{\phi_{meas,i} - \phi_{fit,i}}{\sigma_{\phi,i}} \right)^2 \right] \stackrel{!}{=} 0 \quad (4)$$

- This does not help much, because one could simply set $p_{meas,i} = p_{fit,i}$, $\theta_{meas,i} = \theta_{fit,i}, \ldots$ and obtain $\chi^2 = 0$
- BUT: The true particle 4-vectors are not random, as they are attached to the underlying reaction which produced the particles $\rightarrow$ Energy and momentum conservation

[7] This is already a dangerous assumption!

Introduction to Kinematic Fitting: Least Squares

- Presuming that all variables (and the corresponding uncertainties) are measured independently[7] and gaussian distributed, one might formulate:

$$\chi^2 \equiv \sum_i \left[\left(\frac{p_{meas,i} - p_{fit,i}}{\sigma_{p,i}} \right)^2 + \left(\frac{\theta_{meas,i} - \theta_{fit,i}}{\sigma_{\theta,i}} \right)^2 + \left(\frac{\phi_{meas,i} - \phi_{fit,i}}{\sigma_{\phi,i}} \right)^2 \right] \stackrel{!}{=} 0 \quad (4)$$

- This does not help much, because one could simply set $p_{meas,i} = p_{fit,i}$, $\theta_{meas,i} = \theta_{fit,i}, \ldots$ and obtain $\chi^2 = 0$
- BUT: The true particle 4-vectors are not random, as they are attached to the underlying reaction which produced the particles $\rightarrow$ Energy and momentum conservation
- Therefore, we request that the fitted momenta not only minimize χ^2, but also obey:
  - i) $0 = \mathbf{P}_1(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + \mathbf{P}_2(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - \mathbf{P}_3(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - \mathbf{P}_4(p_{fit,4}, \theta_{fit,4}, \phi_{fit,4}) - \ldots$
  - ii) $0 = E_1(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + E_2(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - E_3(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - E_4(p_{fit,4}, \theta_{fit,4}, \phi_{fit,4}) - \ldots$

[7] This is already a dangerous assumption!

Introduction to Kinematic Fitting: Constraints

- The request for energy and momentum conservation might be summarized into a set of **constraint** functions:
  - $F_x \equiv p_{x,1}(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + p_{x,2}(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - p_{x,3}(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
  - $F_y \equiv p_{y,1}(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + p_{y,2}(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - p_{y,3}(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
  - $F_z \equiv p_{z,1}(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + p_{z,2}(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - p_{z,3}(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
  - $F_E \equiv E_1(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + E_2(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - E_3(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$

Introduction to Kinematic Fitting: Constraints

- The request for energy and momentum conservation might be summarized into a set of **constraint** functions:
  - $F_x \equiv p_{x,1}(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + p_{x,2}(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - p_{x,3}(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
  - $F_y \equiv p_{y,1}(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + p_{y,2}(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - p_{y,3}(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
  - $F_z \equiv p_{z,1}(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + p_{z,2}(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - p_{z,3}(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
  - $F_E \equiv E_1(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + E_2(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - E_3(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
- Which finally leads to:

$$\chi^2 = \sum_i \left[\sum_j \frac{v_{meas,ij} - v_{fit,ij}}{\sigma_{ij}} \right] + 2 \sum_\mu \lambda_\mu F_\mu \stackrel{!}{=} 0 \tag{5}$$

Introduction to Kinematic Fitting: Constraints

- The request for energy and momentum conservation might be summarized into a set of **constraint** functions:
  - $F_x \equiv p_{x,1}(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + p_{x,2}(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - p_{x,3}(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
  - $F_y \equiv p_{y,1}(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + p_{y,2}(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - p_{y,3}(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
  - $F_z \equiv p_{z,1}(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + p_{z,2}(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - p_{z,3}(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
  - $F_E \equiv E_1(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + E_2(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - E_3(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
- Which finally leads to:

$$\chi^2 = \sum_i \left[\sum_j \frac{v_{meas,ij} - v_{fit,ij}}{\sigma_{ij}} \right] + 2 \sum_\mu \lambda_\mu F_\mu \stackrel{!}{=} 0 \tag{5}$$

- With: i=1,2,3,4,.., $v_{ij} = p_i, \theta_i, \phi_i$, $\sigma_{ij} = \sigma_{p,i}, \sigma_{\theta,i}, \sigma_{\phi,i}$ and: $\mu = p_x, p_y, p_z$

Introduction to Kinematic Fitting: Constraints

- The request for energy and momentum conservation might be summarized into a set of **constraint** functions:
  - $F_x \equiv p_{x,1}(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + p_{x,2}(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - p_{x,3}(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
  - $F_y \equiv p_{y,1}(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + p_{y,2}(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - p_{y,3}(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
  - $F_z \equiv p_{z,1}(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + p_{z,2}(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - p_{z,3}(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
  - $F_E \equiv E_1(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + E_2(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - E_3(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
- Which finally leads to:

$$\chi^2 = \sum_i \left[\sum_j \frac{v_{meas,ij} - v_{fit,ij}}{\sigma_{ij}} \right] + 2 \sum_\mu \lambda_\mu F_\mu \stackrel{!}{=} 0 \qquad (5)$$

- With: i=1,2,3,4,.., $v_{ij} = p_i, \theta_i, \phi_i$, $\sigma_{ij} = \sigma_{p,i}, \sigma_{\theta,i}, \sigma_{\phi,i}$ and: $\mu = p_x, p_y, p_z$
- λ_μ are the Lagrange-multipliers

Introduction to Kinematic Fitting: Constraints

- The request for energy and momentum conservation might be summarized into a set of **constraint** functions:
  - $F_x \equiv p_{x,1}(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + p_{x,2}(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - p_{x,3}(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
  - $F_y \equiv p_{y,1}(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + p_{y,2}(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - p_{y,3}(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
  - $F_z \equiv p_{z,1}(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + p_{z,2}(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - p_{z,3}(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
  - $F_E \equiv E_1(p_{fit,1}, \theta_{fit,1}, \phi_{fit,1}) + E_2(p_{fit,2}, \theta_{fit,2}, \phi_{fit,2}) - E_3(p_{fit,3}, \theta_{fit,3}, \phi_{fit,3}) - ... = 0$
- Which finally leads to:

$$\chi^2 = \sum_i \left[\sum_j \frac{v_{meas,ij} - v_{fit,ij}}{\sigma_{ij}} \right] + 2 \sum_\mu \lambda_\mu F_\mu \stackrel{!}{=} 0 \tag{5}$$

- With: i=1,2,3,4,.., $v_{ij} = p_i, \theta_i, \phi_i$, $\sigma_{ij} = \sigma_{p,i}, \sigma_{\theta,i}, \sigma_{\phi,i}$ and: $\mu = p_x, p_y, p_z$
- λ_μ are the Lagrange-multipliers
- Minimizing the equation above will (hopefully) lead to the true 4-vectors of each particle!

Introduction to Kinematic Fitting: Matrix Notation

- Using: $\epsilon_{ij} = v_{meas,ij} - v_{fit,ij}$, one might rewrite the previous equation to:

$$\chi^2 = \boldsymbol{\epsilon}^T \hat{V}^{-1} \boldsymbol{\epsilon} + 2\boldsymbol{\lambda}^T \boldsymbol{F} \stackrel{!}{=} 0 \tag{6}$$

- This is the same equation as shown before, but using vectors and matrices
- $\hat{V}^{-1}$ is the inverse **covariance matrix**[8] and summarizes all measurement uncertainties
- **Remember:** The equation above is only valid, if the measured quantities are (somewhat) uncorrelated and follow a gaussian distribution

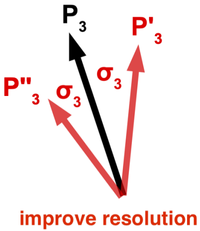
[8] Sometimes referred to as: "the error-matrix"

In a Nutshell

$$\chi^2 = \boldsymbol{\epsilon}^T \hat{V}^{-1} \boldsymbol{\epsilon} + 2\boldsymbol{\lambda}^T \boldsymbol{F} \stackrel{!}{=} 0$$

In a Nutshell

$$\chi^2 = \boxed{\epsilon^T \hat{V}^{-1} \epsilon} + 2\lambda^T \boldsymbol{F} \stackrel{!}{=} 0$$

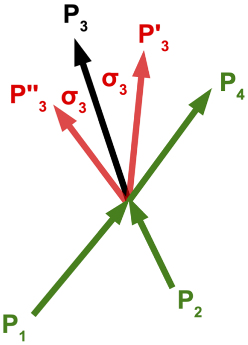


In a Nutshell

$$\chi^2 = \boxed{\epsilon^T \hat{V}^{-1} \epsilon} + \boxed{2\lambda^T F} \stackrel{!}{=} 0$$

improve resolution

with respect to
reaction hypothesis



In a Nutshell

$$\chi^2 = \boxed{\epsilon^T \hat{V}^{-1} \epsilon} + \boxed{2\lambda^T \boldsymbol{F}} \stackrel{!}{=} 0$$

Kinematic Fit provides:

1. Fitted particle 4-momenta

2. Possibility to reject events which do not match the reaction hypothesis

More on Constraints and Covariance Matrix

- The constraints are not limited to energy and momentum conservation only, one might also include (if reasonable):
  - Vertex (not discussed here $\rightarrow$ SIDE does not provide vertex information)
  - Mass constraint between particles
  - Missing particles (not discussed here)

More on Constraints and Covariance Matrix

- The constraints are not limited to energy and momentum conservation only, one might also include (if reasonable):
  - Vertex (not discussed here $\rightarrow$ SIDE does not provide vertex information)
  - Mass constraint between particles
  - Missing particles (not discussed here)
- Knowing the uncertainties σ_{ij} for each particle and each variable, the covariance matrix $\hat{V}$ might be written:

$$\hat{V} = \begin{pmatrix} \sigma_1^2 & \rho_{1,2}\sigma_1\sigma_2 & \cdots & \rho_{1,3N}\sigma_1\sigma_{3N} \\ \rho_{2,1}\sigma_2\sigma_1 & \sigma_2^2 & \cdots & \rho_{2,3N}\sigma_2\sigma_{3N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{3N,1}\sigma_{3N}\sigma_1 & \rho_{3N,2}\sigma_{3N}\sigma_2 & \cdots & \sigma_{3N}^2 \end{pmatrix} \tag{7}$$

More on Constraints and Covariance Matrix

- The constraints are not limited to energy and momentum conservation only, one might also include (if reasonable):
  - Vertex (not discussed here $\rightarrow$ SIDE does not provide vertex information)
  - Mass constraint between particles
  - Missing particles (not discussed here)
- Knowing the uncertainties σ_{ij} for each particle and each variable, the covariance matrix $\hat{V}$ might be written:

$$\hat{V} = \begin{pmatrix} \sigma_1^2 & \rho_{1,2}\sigma_1\sigma_2 & \cdots & \rho_{1,3N}\sigma_1\sigma_{3N} \\ \rho_{2,1}\sigma_2\sigma_1 & \sigma_2^2 & \cdots & \rho_{2,3N}\sigma_2\sigma_{3N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{3N,1}\sigma_{3N}\sigma_1 & \rho_{3N,2}\sigma_{3N}\sigma_2 & \cdots & \sigma_{3N}^2 \end{pmatrix} \tag{7}$$

- ρ_{ij} are the correlation factors between the variables and symmetric: $\rho_{ij} = \rho_{ji}$
  $\rightarrow$ $\hat{V}$ is a symmetric $3N \times 3N$ matrix (N particles with p, θ, ϕ each)

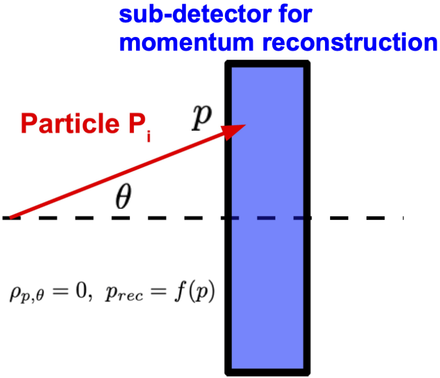
More on Constraints and Covariance Matrix

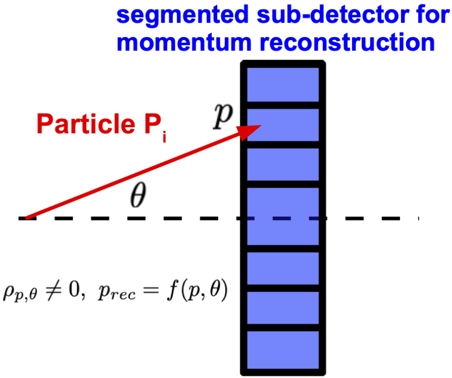
- The constraints are not limited to energy and momentum conservation only, one might also include (if reasonable):
  - Vertex (not discussed here $\rightarrow$ SIDE does not provide vertex information)
  - Mass constraint between particles
  - Missing particles (not discussed here)
- Knowing the uncertainties σ_{ij} for each particle and each variable, the covariance matrix $\hat{V}$ might be written:

$$\hat{V} = \begin{pmatrix} \sigma_1^2 & \rho_{1,2}\sigma_1\sigma_2 & \cdots & \rho_{1,3N}\sigma_1\sigma_{3N} \\ \rho_{2,1}\sigma_2\sigma_1 & \sigma_2^2 & \cdots & \rho_{2,3N}\sigma_2\sigma_{3N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{3N,1}\sigma_{3N}\sigma_1 & \rho_{3N,2}\sigma_{3N}\sigma_2 & \cdots & \sigma_{3N}^2 \end{pmatrix} \tag{7}$$

- ρ_{ij} are the correlation factors between the variables and symmetric: $\rho_{ij} = \rho_{ji}$ $\rightarrow$ $\hat{V}$ is a symmetric $3N \times 3N$ matrix (N particles with p, θ, ϕ each)
- In order for the kinematic fit to work, it is extremely important to know the elements of $\hat{V}$

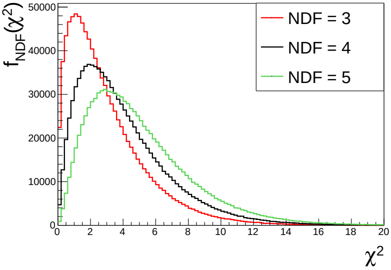
Un-Correlated Measurements





- **Left:** Detector response is uniform with respect to $\theta \rightarrow$ Reconstructed momentum p_{rec} does not depend on θ
- **Right:** Detector response is not uniform with respect to $\theta \rightarrow$ Reconstructed momentum p_{rec} depends on θ
- Knowing your detector $\Leftrightarrow$ Knowing $\hat{V}$

The χ^2-Distribution



- For uncorrelated variables with gaussian uncertainties**, the (kinematic) fit χ^2 follows the distribution shown on the left

$$f_{NDF}(x) = \frac{x^{(NDF/2-1)}}{2^{(NDF/2)}\Gamma(NDF/2)} \cdot e^{-x/2} \quad (8)$$

- Where $\Gamma()$ is the Gamma function and NDF the number of degrees of freedom

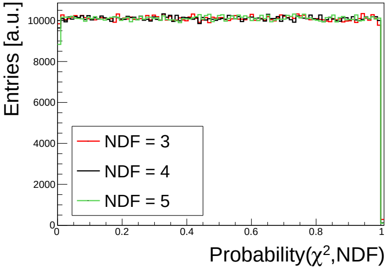
- This distribution is the first to check, after performing a kinematic fit
- Any deviation from this distribution might be related to:
  - Background reactions that do not fulfill the reaction hypothesis
  - Wrong setting of $\hat{V}$
  - Wrong reaction hypothesis
  - Violation of $\star\star$

The Probability-Distribution

- The χ^2 discussed on the previous slide might be translated to a fit probability via:

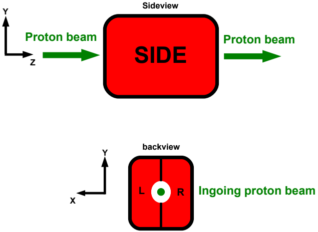
$$Prob(\chi^2, NDF) = \frac{\int_{\chi^2}^{\infty} t^{NDF/2-1} e^{-t/2} dt}{\sqrt{2^{NDF}\Gamma(NDF/2)}} \quad (9)$$

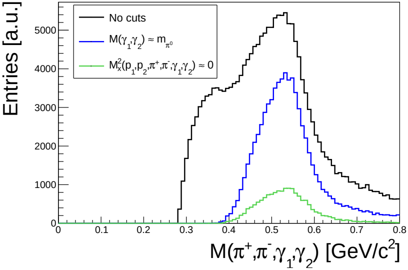
- Where $\Gamma()$ is the Gamma function and NDF the number of degrees of freedom



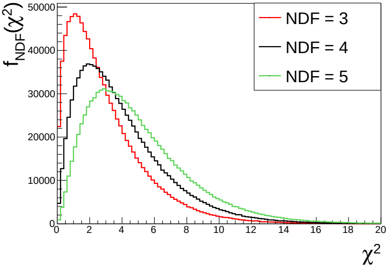
- Any deviation from a non-flat probability distribution might be related to:
  - Background reactions that do not fulfill the reaction hypothesis
  - Wrong setting of $\hat{V}$
  - Wrong reaction hypothesis
  - Violation of ⋆⋆ (see previous slide)

Part I Summary





- Hypothetical measurement of $pp \to ppX$ reactions
- Introduced monitoring plots based on energy and momentum conservation
- Performed simple analysis to reconstruct $\pi^+\pi^-\pi^0$ final states
- Introduction of kinematic fitting



Part II

- Evaluation of kinematic fit performance
- Application of kinematic fit in physics data analysis
- Practical aspects

Addendum: Non-Diagonal Weight Matrix

- Stated earlier in this lecture, that the weight matrix $\hat{V}$ has to be diagonal in order for the least squared method to work
  $\rightarrow$ **Gauss-Markov-Theorem**
- However, the method still holds if the matrix $\hat{V}$ is equal to the covariance matrix which was introduced earlier, i.e. $\hat{V}_{ij} = \hat{V}_{ji} = \rho_{ij}\sigma_i\sigma_j$
  $\rightarrow$ **Extension of the Gauss-Markov-Theorem by Aitken**
- I am sorry for the confusion and being that imprecise!

Setting up $\hat{V}$

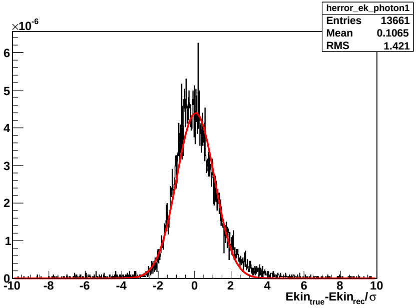
- As pointed out many times, $\hat{V}$ is crucial for the performance of the kinematic fitter

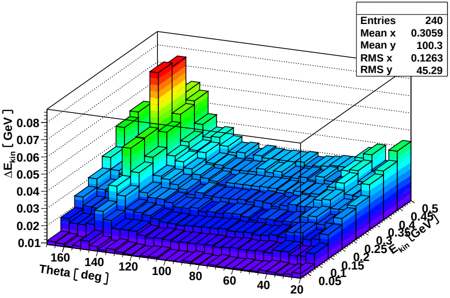
Setting up $\hat{V}$

- As pointed out many times, $\hat{V}$ is crucial for the performance of the kinematic fitter
- There are different approaches for determining $\hat{V}$:
  - Know your experiment and calculate the elements individually (e.g. include tracking parameters)
  - Look at residuals $\rightarrow$ Need MC for this
  - ...

Setting up $\hat{V}$

- As pointed out many times, $\hat{V}$ is crucial for the performance of the kinematic fitter
- There are different approaches for determining $\hat{V}$:
  - Know your experiment and calculate the elements individually (e.g. include tracking parameters)
  - Look at residuals $\rightarrow$ Need MC for this
  - ...
- Shown below: Determination of photon related parts in $\hat{V}$ in WASA-at-COSY





NDF - Number of Degrees of Freedom

- Suppose N particles (which are subject to the fit)[9], each defined by a momentum vector $\rightarrow$ 3N variables

[9] Beam and target proton are not part of the fit, in our case.

NDF - Number of Degrees of Freedom

- Suppose N particles (which are subject to the fit)[9], each defined by a momentum vector $\rightarrow$ 3N variables
- In your experiment, you measured / reconstructed $N_m \leq 3N$ particle properties (e.g. the momentum of one particle could not be determined)

[9] Beam and target proton are not part of the fit, in our case.

NDF - Number of Degrees of Freedom

- Suppose N particles (which are subject to the fit)[9], each defined by a momentum vector $\rightarrow$ 3N variables
- In your experiment, you measured / reconstructed $N_m \leq 3N$ particle properties (e.g. the momentum of one particle could not be determined)
- The kinematic fit is performed with $4 + N_c$ constraints (4: energy and momentum conservation, N_c: any additional constraint)

[9] Beam and target proton are not part of the fit, in our case.

NDF - Number of Degrees of Freedom

- Suppose N particles (which are subject to the fit)[9], each defined by a momentum vector $\rightarrow$ 3N variables
- In your experiment, you measured / reconstructed $N_m \leq 3N$ particle properties (e.g. the momentum of one particle could not be determined)
- The kinematic fit is performed with $4 + N_c$ constraints (4: energy and momentum conservation, N_c: any additional constraint)
- This leads to: $NDF = 3N - N_m + 4 + N_c$

[9] Beam and target proton are not part of the fit, in our case.

NDF - Number of Degrees of Freedom

- Suppose N particles (which are subject to the fit)[9], each defined by a momentum vector $\rightarrow$ 3N variables
- In your experiment, you measured / reconstructed $N_m \leq 3N$ particle properties (e.g. the momentum of one particle could not be determined)
- The kinematic fit is performed with $4 + N_c$ constraints (4: energy and momentum conservation, N_c: any additional constraint)
- This leads to: $NDF = 3N - N_m + 4 + N_c$
- Introducing the number of unknowns: $N_u = 3N - N_m$

[9] Beam and target proton are not part of the fit, in our case.

NDF - Number of Degrees of Freedom

- Suppose N particles (which are subject to the fit)[9], each defined by a momentum vector $\rightarrow$ 3N variables
- In your experiment, you measured / reconstructed $N_m \leq 3N$ particle properties (e.g. the momentum of one particle could not be determined)
- The kinematic fit is performed with $4 + N_c$ constraints (4: energy and momentum conservation, N_c: any additional constraint)
- This leads to: $NDF = 3N - N_m + 4 + N_c$
- Introducing the number of unknowns: $N_u = 3N - N_m$
- One obtains: $NDF = 4 - N_u + N_c$

[9] Beam and target proton are not part of the fit, in our case.

NDF - Number of Degrees of Freedom

- Suppose N particles (which are subject to the fit)[9], each defined by a momentum vector $\rightarrow$ 3N variables
- In your experiment, you measured / reconstructed $N_m \leq 3N$ particle properties (e.g. the momentum of one particle could not be determined)
- The kinematic fit is performed with $4 + N_c$ constraints (4: energy and momentum conservation, N_c: any additional constraint)
- This leads to: $NDF = 3N - N_m + 4 + N_c$
- Introducing the number of unknowns: $N_u = 3N - N_m$
- One obtains: $NDF = 4 - N_u + N_c$
- All particle momenta are known in our case $\rightarrow NDF = 4 + N_c$

[9] Beam and target proton are not part of the fit, in our case.

Pull-Distributions

- Another powerful tool to evaluate the kinematic fitter performance are pulls
- For are particle i with variables v_{ij} and uncertainties σ_{ij}, the corresponding pull is defined:

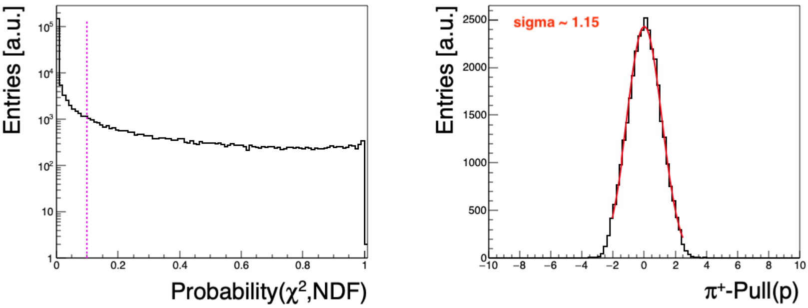
$$\text{Pull}(v_{ij}) = \frac{v_{meas,ij} - v_{fit,ij}}{\sqrt{\sigma^2_{meas,ij} - \sigma^2_{fit,ij}}} \tag{10}$$

- Where *meas* / *fit* denote the measured / fitted quantities
- Since we assume the measured variables to be gaussian, the fitted quantities are expected to be gaussian too

mean[Pull(v_{ij})]	σ[Pull(v_{ij})]	Scenario
0	1.0	everything is fine
0	< 1.0	$\sigma_{meas,ij}$ is overestimated
0	> 1.0	$\sigma_{meas,ij}$ is underestimated
$\neq 0$	$\in [0, 1]$	introduced bias
n.a.	n.a.	non-gaussian pulls $\rightarrow$ you are in trouble

Evaluating the Fit Performance: Example

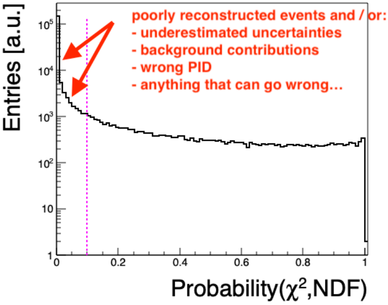
- Applied kinematic fit on data set with $pp \rightarrow pp\pi^+\pi^-\pi^0[\pi^0 \rightarrow \gamma\gamma]$ events only
- No additional constraints, beside energy and momentum conservation
- Set the covariance matrix $\hat{V}$ to be diagonal and loosely match the detector resolution (i.e. assumed constant uncertainties for each particle variable)
- Look at pull-distributions for probabilities $\geq 10\%$(horizontal, dashed line)[10]

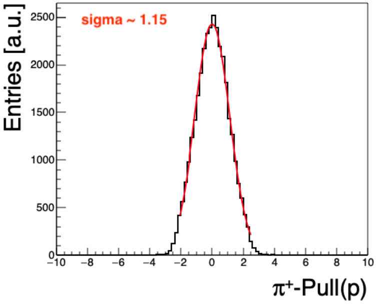


[10]More or less arbitrarily chosen

Evaluating the Fit Performance: Example

- Applied kinematic fit on data set with $pp \to pp\pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]$ events only
- No additional constraints, beside energy and momentum conservation
- Set the covariance matrix $\hat{V}$ to be diagonal and loosely match the detector resolution (i.e. assumed constant uncertainties for each particle variable)
- Look at pull-distributions for probabilities $\geq 10\%$(horizontal, dashed line)[10]

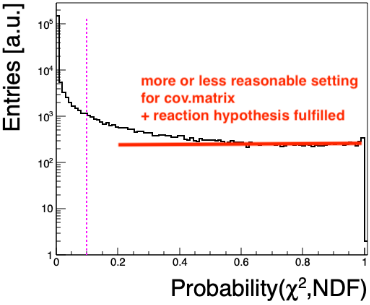


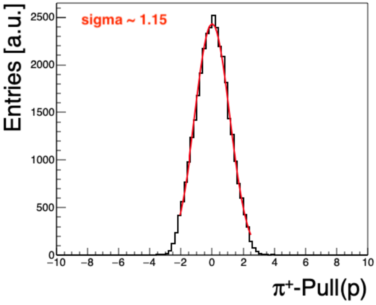


[10] More or less arbitrarily chosen

Evaluating the Fit Performance: Example

- Applied kinematic fit on data set with $pp \to pp\pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]$ events only
- No additional constraints, beside energy and momentum conservation
- Set the covariance matrix $\hat{V}$ to be diagonal and loosely match the detector resolution (i.e. assumed constant uncertainties for each particle variable)
- Look at pull-distributions for probabilities $\geq 10\%$(horizontal, dashed line)[10]

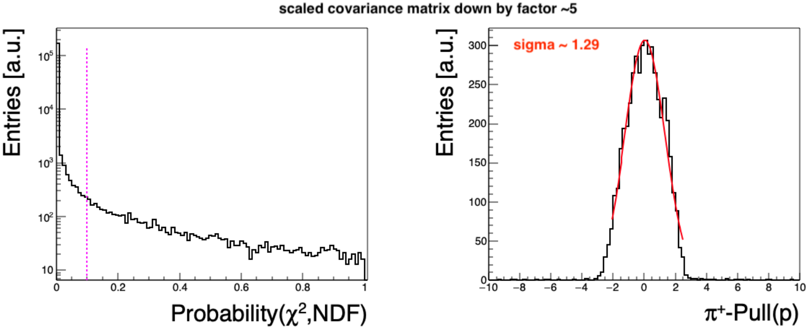




[10] More or less arbitrarily chosen

Evaluating the Fit Performance: Example

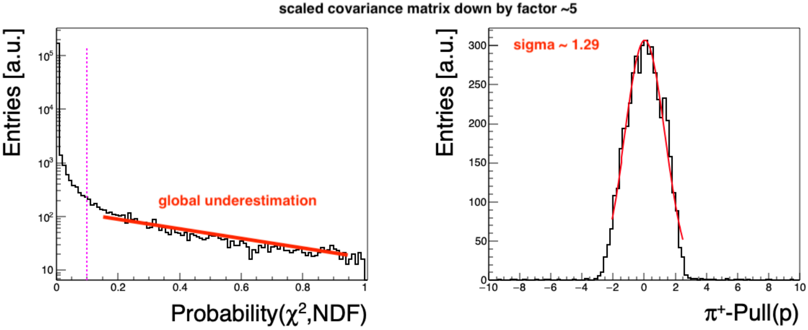
- Applied kinematic fit on data set with $pp \to pp\pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]$ events only
- No additional constraints, beside energy and momentum conservation
- Set the covariance matrix $\hat{V}$ to be diagonal and loosely match the detector resolution (i.e. assumed constant uncertainties for each particle variable)
- Look at pull-distributions for probabilities $\geq 10\%$(horizontal, dashed line)[10]



[10] More or less arbitrarily chosen

Evaluating the Fit Performance: Example

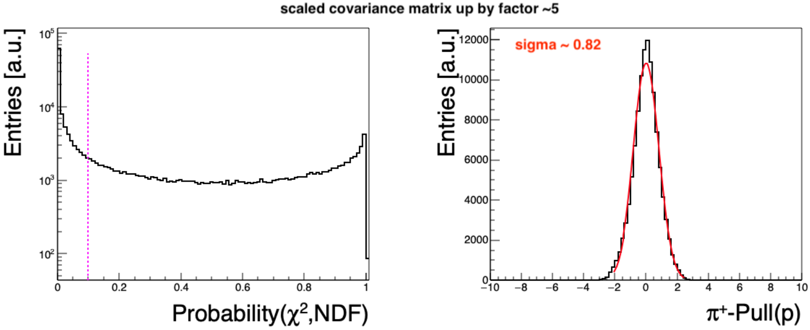
- Applied kinematic fit on data set with $pp \to pp\pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]$ events only
- No additional constraints, beside energy and momentum conservation
- Set the covariance matrix $\hat{V}$ to be diagonal and loosely match the detector resolution (i.e. assumed constant uncertainties for each particle variable)
- Look at pull-distributions for probabilities $\geq 10\%$(horizontal, dashed line)[10]



[10] More or less arbitrarily chosen

Evaluating the Fit Performance: Example

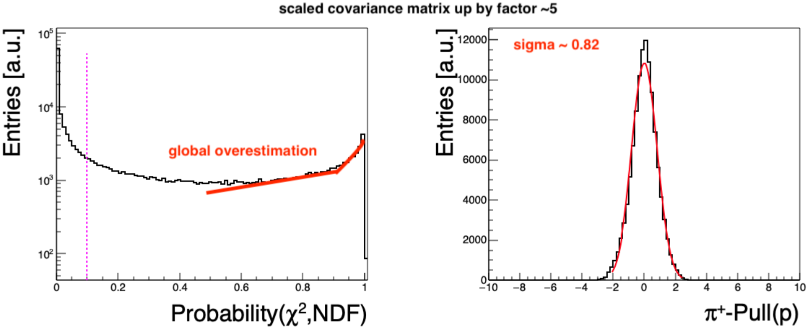
- Applied kinematic fit on data set with $pp \to pp\pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]$ events only
- No additional constraints, beside energy and momentum conservation
- Set the covariance matrix $\hat{V}$ to be diagonal and loosely match the detector resolution (i.e. assumed constant uncertainties for each particle variable)
- Look at pull-distributions for probabilities $\geq 10\%$(horizontal, dashed line)[10]



[10]More or less arbitrarily chosen

Evaluating the Fit Performance: Example

- Applied kinematic fit on data set with $pp \to pp\pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]$ events only
- No additional constraints, beside energy and momentum conservation
- Set the covariance matrix $\hat{V}$ to be diagonal and loosely match the detector resolution (i.e. assumed constant uncertainties for each particle variable)
- Look at pull-distributions for probabilities $\geq 10\%$(horizontal, dashed line)[10]



[10]More or less arbitrarily chosen

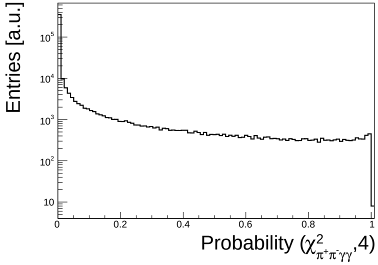
Preparation for Analysis

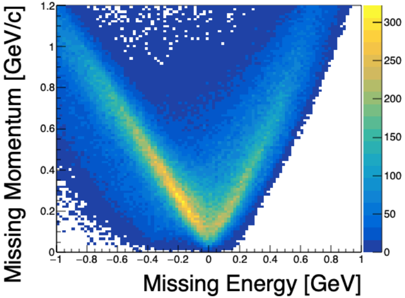
- **Before** using your kinematic fitter in your analysis, you should perform the checks discussed before:
  - Investigate the probability distribution $\leftrightarrow$ Any unexpected features
  - Look at pull-distributions: centered at zero and sigma close to one
  - Globally scaling the covariance matrix is sometimes helpful[11]
- **NEVER** use a fitter blindly in your analysis
- Always have a reference analysis at hand
- In many cases the fitter needs to be re-tuned after the first analysis-pass

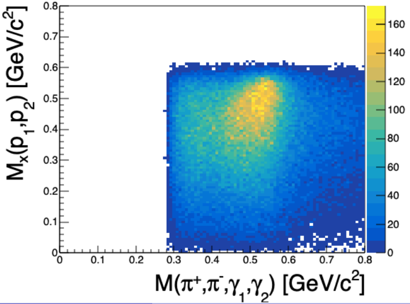
[11] As long as the relative adjustments of the matrix elements are correct

Include Kinematic Fit in $\eta \to \pi^+\pi^-\pi^0$ Analysis I

- Reaction hypothesis: $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$
- No constraints on η / π^0 mass (will do this later)
- Spectra shown below are deduced from reconstructed particle momenta

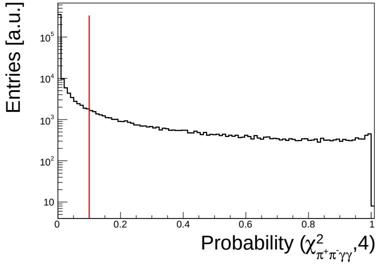


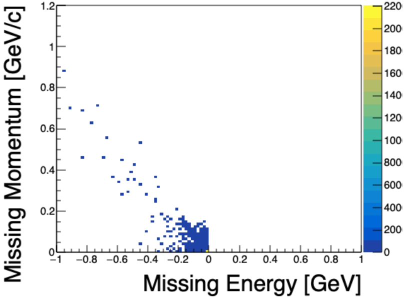


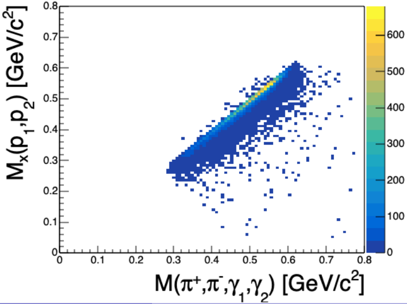


Include Kinematic Fit in $\eta \to \pi^+\pi^-\pi^0$ Analysis I

- Reaction hypothesis: $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$
- No constraints on η / π^0 mass (will do this later)
- Spectra shown below are deduced from fitted particle momenta
- Reject events with probability $< 10\%$ (Again, a first-guess cut)

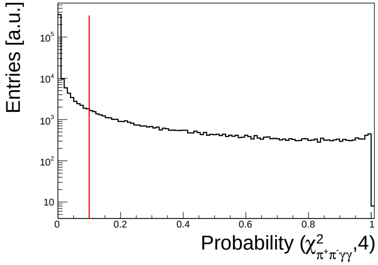


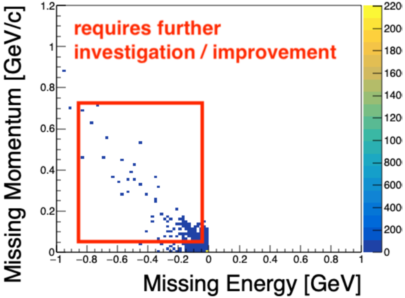


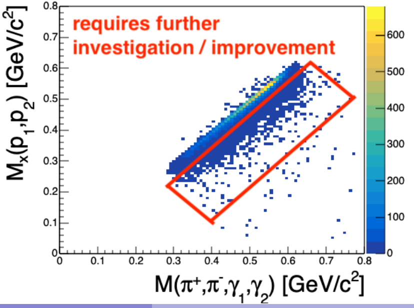


Include Kinematic Fit in $\eta \to \pi^+\pi^-\pi^0$ Analysis I

- Reaction hypothesis:
  $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$
- No constraints on η / π^0 mass
  (will do this later)
- Spectra shown below are deduced
  from fitted particle momenta
- Reject events with probability $< 10\%$
  (Again, a first-guess cut)

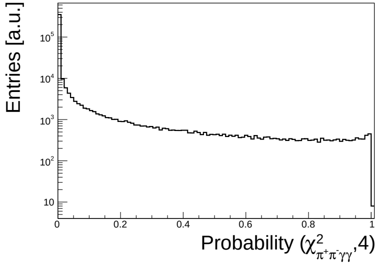


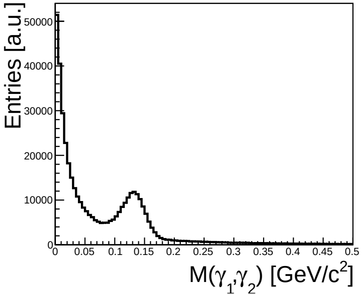


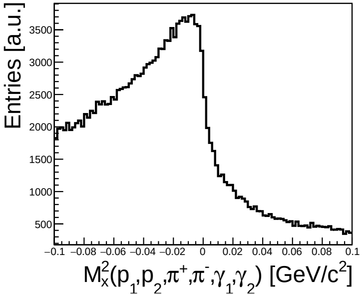


Include Kinematic Fit in $\eta \to \pi^+\pi^-\pi^0$ Analysis II

- Reaction hypothesis: $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$
- No constraints on η / π^0 mass (will do this later)
- Spectra shown below are deduced from reconstructed particle momenta

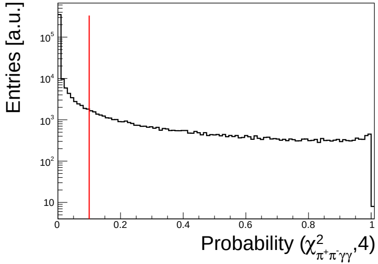


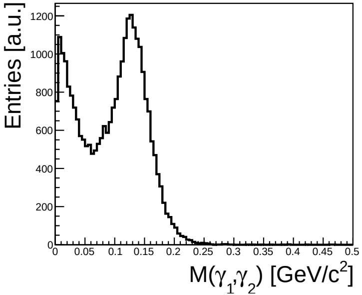


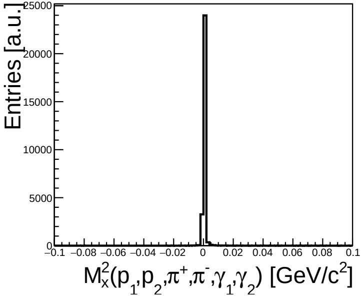


Include Kinematic Fit in $\eta \to \pi^+\pi^-\pi^0$ Analysis II

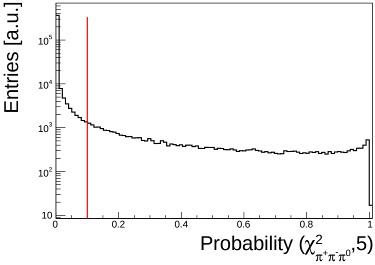
- Reaction hypothesis: $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$
- No constraints on η / π^0 mass (will do this later)
- Spectra shown below are deduced from fitted particle momenta
- Reject events with probability $< 10\%$ (Again, a first-guess cut)



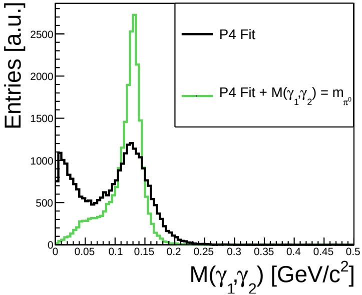


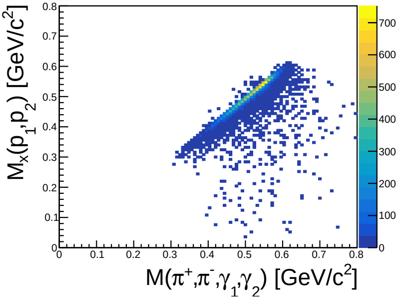


Include Kinematic Fit in $\eta \to \pi^+\pi^-\pi^0$ Analysis III

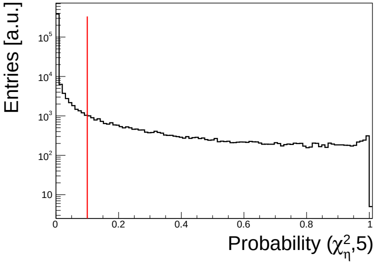


- Reaction hypothesis: $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$
- Include constraint: $M(\gamma_1, \gamma_2) = m_{\pi^0}$
- Spectra shown below are deduced from fitted particle momenta
- Reject events with probability $< 10\%$ (Again, a first-guess cut)

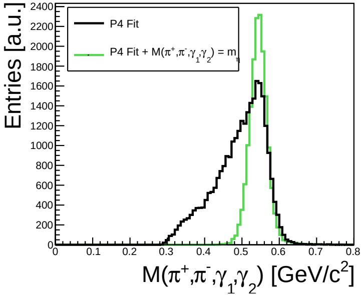


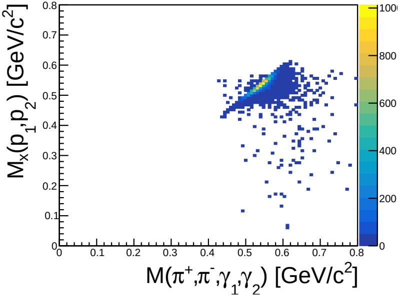


Include Kinematic Fit in $\eta \to \pi^+\pi^-\pi^0$ Analysis IV

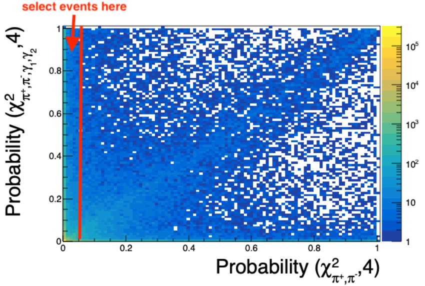


- Reaction hypothesis: $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$
- Include constraint: $M(\pi^+, \pi^-, \gamma_1, \gamma_2) = m_\eta$
- Spectra shown below are deduced from fitted particle momenta
- Reject events with probability $< 10\%$ (Again, a first-guess cut)

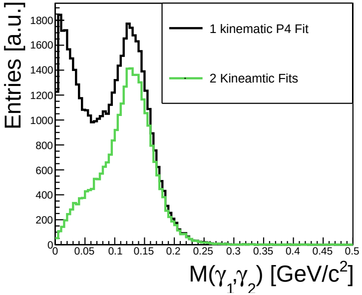


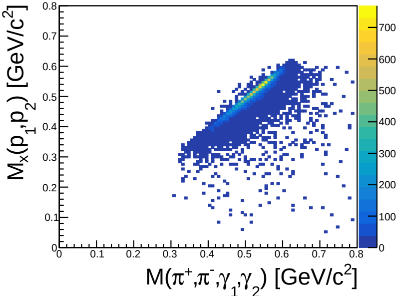


Include Kinematic Fit in $\eta \to \pi^+\pi^-\pi^0$ Analysis V

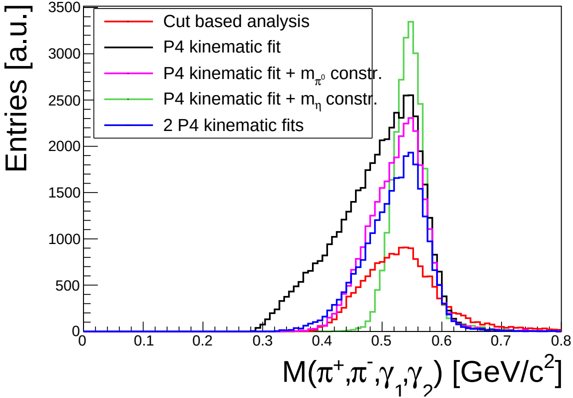


- Sometimes it is helpful to run 2fits with different hypothesis
  $\to$ Especially when reactions are somewhat similar
- Helps to monitor the fitter performance
- Spectra shown below are deduced from fitted particle momenta





Analysis Summary



- Re-analyzed the $pp \to pp$ data set, using a kinematic fitter
- Fitter seems to work so far, but needs to be re-adjusted
  $\rightarrow$ Constraint spectra should be delta-distributions
- Remaining problem: Individual contributions from $pp \to \pi^+\pi^-\pi^0$ and $\eta \to \pi^+\pi^-\pi^0$ are still hard to judge

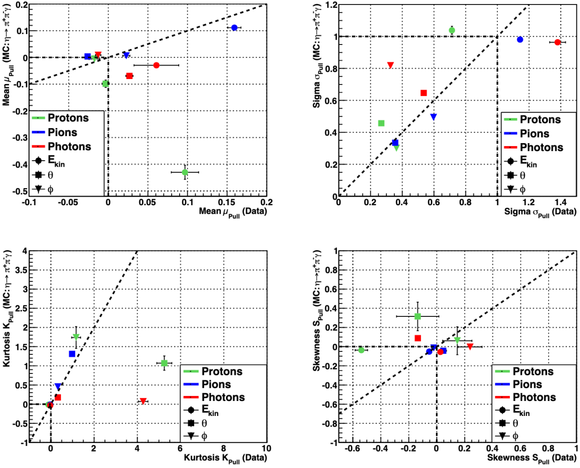
Where to go from here and practical Aspects

- First thing to do after this analysis run would be a comparison with MC simulated events

Observable	Scenario	Required Action
Probability	Flat in Data & MC	Ok, proceed with analysis
	not flat, but identical	Sometimes next best scenario, proceed with caution
	significantly different	re-tune fitter and/or MC, or re-calibrate Data
Pulls	$\mu = 0$, $\sigma = 1$ in Data & MC	Wonderful! Proceed
	$\mu = 0$, $\sigma \approx 1$ in Data & MC	ok, proceed
	$\mu \neq 0$, $\sigma \neq 1$ but identical	proceed, but try to re-tune
	Totally different	re-tune fitter and/or MC, or re-calibrate Data

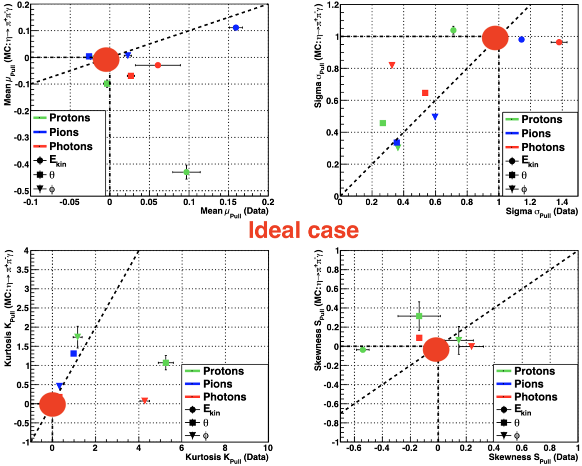
- If the comparisons above turn out well, you might want to compare different kinematic properties (i.e. missing mass, invariant mass,..) between data and MC
- Never trust your fitter blindly!

Comparing Pull-Features



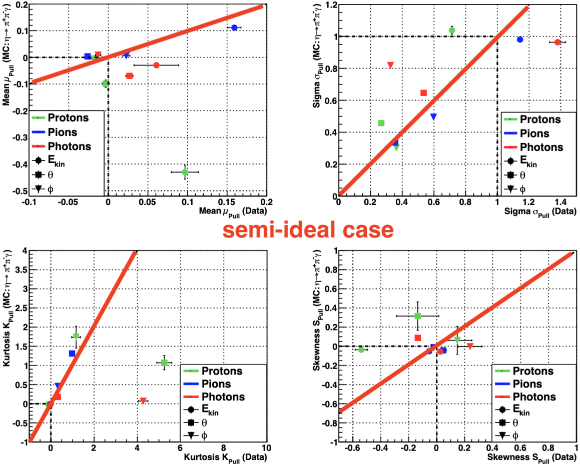
- Shown above: One way of comparing pull distributions in data / MC
- Included two shape variables to monitor deviation from Gaussian Distrubution
- Plot taken from here: `urn:nbn:de:hbz:468-20150204-112638-3`

Comparing Pull-Features



- Shown above: One way of comparing pull distributions in data / MC
- Included two shape variables to monitor deviation from Gaussian Distrubution
- Plot taken from here: `urn:nbn:de:hbz:468-20150204-112638-3`

Comparing Pull-Features



- Shown above: One way of comparing pull distributions in data / MC
- Included two shape variables to monitor deviation from Gaussian Distrubution
- Plot taken from here: `urn:nbn:de:hbz:468-20150204-112638-3`

Monitoring Physics Observables

- It is not helpful to keep stuck on tuning the fitter forever

Monitoring Physics Observables

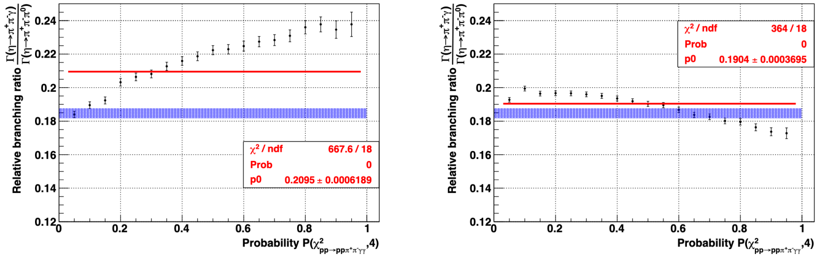
- It is not helpful to keep stuck on tuning the fitter forever
- Best way to see inefficiencies or oddities $\rightarrow$ Perform analysis until the end $\rightarrow$ See impact of fitter on final result

Monitoring Physics Observables

- It is not helpful to keep stuck on tuning the fitter forever
- Best way to see inefficiencies or oddities $\rightarrow$ Perform analysis until the end $\rightarrow$ See impact of fitter on final result
- This does NOT mean to tune the fitter until your results match the PDG value or your colleagues thesis!!!

Monitoring Physics Observables

- It is not helpful to keep stuck on tuning the fitter forever
- Best way to see inefficiencies or oddities $\rightarrow$ Perform analysis until the end $\rightarrow$ See impact of fitter on final result
- This does NOT mean to tune the fitter until your results match the PDG value or your colleagues thesis!!!
- Rather look at stability of whatever you calculate as a function of the fit probability



- Left: before MC tuning / Right: After MC tuning
- Did not care about the absolute value, but rather about the fluctuation
- Plot taken from here: `urn:nbn:de:hbz:468-20150204-112638-3`

Re-tuning the Kinematic Fit

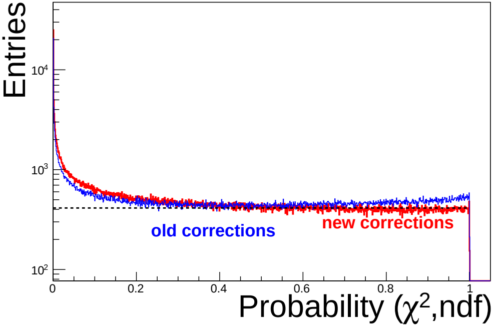
- When working with kinematic fits, you will most likely come to the point where you need to re-tune the fitter $\rightarrow$ Update $\hat{V}$

Re-tuning the Kinematic Fit

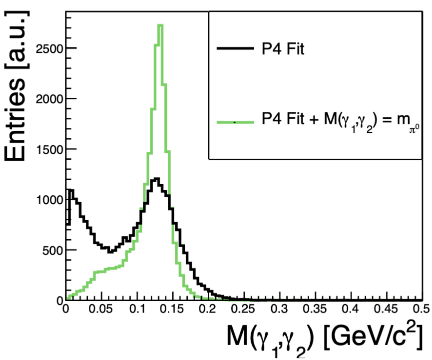
- When working with kinematic fits, you will most likely come to the point where you need to re-tune the fitter $\rightarrow$ Update $\hat{V}$
- As usual, there are many ways to do this:
  - Recalculate $\hat{V}$ $\rightarrow$ Painful, but sometimes unavoidable
  - Rescale certain fractions of $\hat{V}$
  - Hyper Parameter Optimization - HPO $\rightarrow$ Does the tuning for you, but might take very long

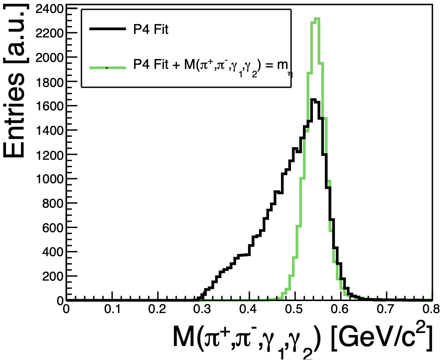
Re-tuning the Kinematic Fit

- When working with kinematic fits, you will most likely come to the point where you need to re-tune the fitter $\rightarrow$ Update $\hat{V}$
- As usual, there are many ways to do this:
  - Recalculate $\hat{V}$ $\rightarrow$ Painful, but sometimes unavoidable
  - Rescale certain fractions of $\hat{V}$
  - Hyper Parameter Optimization - HPO $\rightarrow$ Does the tuning for you, but might take very long
- Shown below the tuning of the kinematic fit for a CLAS g12 analysis
  $\rightarrow$ Used two scaling factors (one for momenta and one for angles)



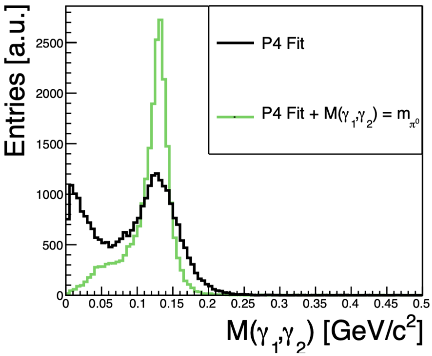
One Word on Mass Constraints

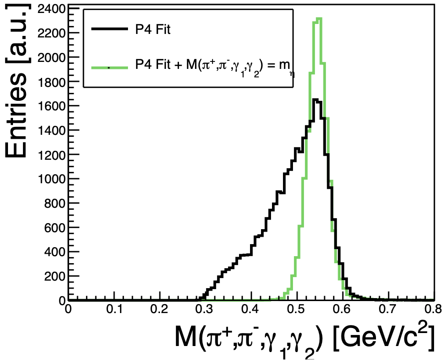




- Mass constraints can be very helpful, but also very dangerous

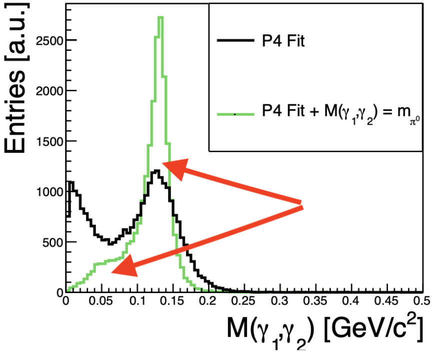
One Word on Mass Constraints

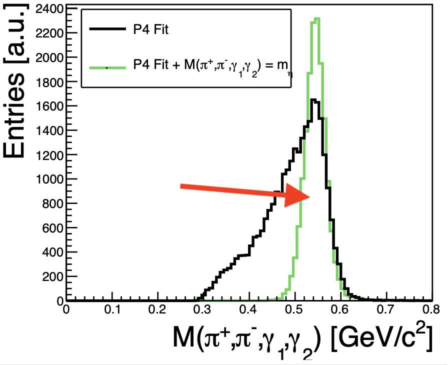




- Mass constraints can be very helpful, but also very dangerous
- Helpful: You are not particularly interested in the particle you are fitting to, but want to reject background

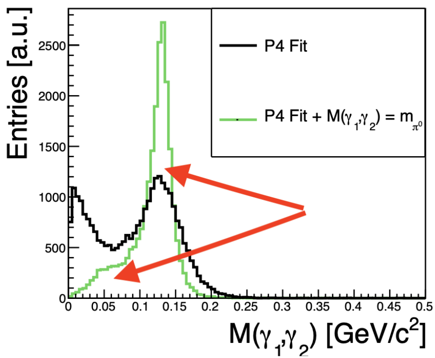
One Word on Mass Constraints

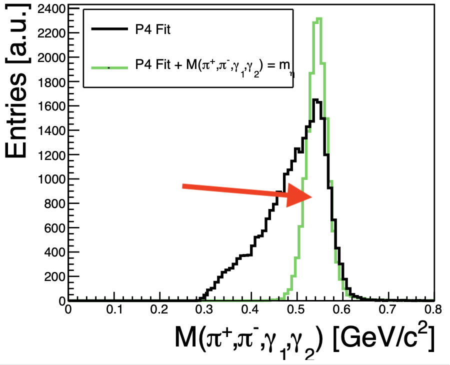




- Mass constraints can be very helpful, but also very dangerous
- Helpful: You are not particularly interested in the particle you are fitting to, but want to reject background
- Dangerous: Your are interested in this particle $\rightarrow$ Merge signal and background events under one distribution $\rightarrow$ No control over background contribution

One Word on Mass Constraints





- Mass constraints can be very helpful, but also very dangerous
- Helpful: You are not particularly interested in the particle you are fitting to, but want to reject background
- Dangerous: Your are interested in this particle → Merge signal and background events under one distribution → No control over background contribution
- When to use mass constraints? ↔ It depends on what you want to do

Building your own Fitter

- Why would you want to do that?

Building your own Fitter

- Why would you want to do that?
- Basic assumption: Variations of constraint functions are slow:

$$F_\mu(v_{fit,1}, \dots, v_{fit,2}) \simeq F_\mu(v_{meas,1}, \dots, v_{meas,2}) + \sum_j \epsilon_j \frac{\partial F_\mu}{\partial v_{fit,j}}\Big|_{v_{fit,j}=v_{meas,j}} = 0 \qquad (11)$$

Building your own Fitter

- Why would you want to do that?
- Basic assumption: Variations of constraint functions are slow:

$$F_\mu(v_{fit,1},...,v_{fit,2}) \simeq F_\mu(v_{meas,1},...,v_{meas,2}) + \sum_j \epsilon_j \frac{\partial F_\mu}{\partial v_{fit,j}}\bigg|_{v_{fit,j}=v_{meas,j}} = 0 \qquad (11)$$

- With: $B_{\mu j} = \frac{\partial F_\mu}{\partial v_{fit,j}}\Big|_{v_{fit,j}=v_{meas,j}}$ and $f_\mu = F_\mu(v_{meas,1},...,v_{meas,2})$, one can find:

$$\chi^2 = \boldsymbol{\epsilon}^T \hat{V}^{-1}\boldsymbol{\epsilon} + 2\boldsymbol{\lambda}^T(\hat{B}\boldsymbol{\epsilon} + \mathbf{f}) \qquad (12)$$

Building your own Fitter

- Why would you want to do that?
- Basic assumption: Variations of constraint functions are slow:

$$F_\mu(v_{fit,1}, ..., v_{fit,2}) \simeq F_\mu(v_{meas,1}, ..., v_{meas,2}) + \sum_j \epsilon_j \frac{\partial F_\mu}{\partial v_{fit,j}}\Big|_{v_{fit,j}=v_{meas,j}} = 0 \qquad (11)$$

- With: $B_{\mu j} = \frac{\partial F_\mu}{\partial v_{fit,j}}\Big|_{v_{fit,j}=v_{meas,j}}$ and $f_\mu = F_\mu(v_{meas,1}, ..., v_{meas,2})$, one can find:

$$\chi^2 = \boldsymbol{\epsilon}^T \hat{V}^{-1} \boldsymbol{\epsilon} + 2\boldsymbol{\lambda}^T(\hat{B}\boldsymbol{\epsilon} + \mathbf{f}) \qquad (12)$$

- Differentiating χ^2 with respect to ϵ (we want to minimize the χ^2) and using the equation on top, yields:

$$\boldsymbol{\epsilon} = -\hat{V}\hat{B}^T\hat{S}\mathbf{f} \qquad (13)$$

Building your own Fitter

- Why would you want to do that?
- Basic assumption: Variations of constraint functions are slow:

$$F_\mu(v_{fit,1}, ..., v_{fit,2}) \simeq F_\mu(v_{meas,1}, ..., v_{meas,2}) + \sum_j \epsilon_j \frac{\partial F_\mu}{\partial v_{fit,j}}\Big|_{v_{fit,j}=v_{meas,j}} = 0 \tag{11}$$

- With: $B_{\mu j} = \frac{\partial F_\mu}{\partial v_{fit,j}}\Big|_{v_{fit,j}=v_{meas,j}}$ and $f_\mu = F_\mu(v_{meas,1}, ..., v_{meas,2})$, one can find:

$$\chi^2 = \boldsymbol{\epsilon}^T \hat{V}^{-1} \boldsymbol{\epsilon} + 2\boldsymbol{\lambda}^T(\hat{B}\boldsymbol{\epsilon} + \mathbf{f}) \tag{12}$$

- Differentiating χ^2 with respect to ϵ (we want to minimize the χ^2) and using the equation on top, yields:

$$\boldsymbol{\epsilon} = -\hat{V}\hat{B}^T\hat{S}\mathbf{f} \tag{13}$$

- with: $\hat{S} = (\hat{B}\hat{V}\hat{B}^T)^{-1}$

Building your own Fitter

- Why would you want to do that?
- Basic assumption: Variations of constraint functions are slow:

$$F_\mu(v_{fit,1}, ..., v_{fit,2}) \simeq F_\mu(v_{meas,1}, ..., v_{meas,2}) + \sum_j \epsilon_j \frac{\partial F_\mu}{\partial v_{fit,j}}\Big|_{v_{fit,j}=v_{meas,j}} = 0 \tag{11}$$

- With: $B_{\mu j} = \frac{\partial F_\mu}{\partial v_{fit,j}}\Big|_{v_{fit,j}=v_{meas,j}}$ and $f_\mu = F_\mu(v_{meas,1}, ..., v_{meas,2})$, one can find:

$$\chi^2 = \boldsymbol{\epsilon}^T \hat{V}^{-1}\boldsymbol{\epsilon} + 2\boldsymbol{\lambda}^T(\hat{B}\boldsymbol{\epsilon} + \mathbf{f}) \tag{12}$$

- Differentiating χ^2 with respect to ϵ (we want to minimize the χ^2) and using the equation on top, yields:

$$\boldsymbol{\epsilon} = -\hat{V}\hat{B}^T\hat{S}\mathbf{f} \tag{13}$$

- with: $\hat{S} = (\hat{B}\hat{V}\hat{B}^T)^{-1}$
- This means, one only needs to calculate $\hat{B}$ and $\mathbf{f}$ in order to find the corrections ϵ

Building your own Fitter

- Why would you want to do that?
- Basic assumption: Variations of constraint functions are slow:

$$F_\mu(v_{fit,1}, ..., v_{fit,2}) \simeq F_\mu(v_{meas,1}, ..., v_{meas,2}) + \sum_j \epsilon_j \frac{\partial F_\mu}{\partial v_{fit,j}}\bigg|_{v_{fit,j}=v_{meas,j}} = 0 \qquad (11)$$

- With: $B_{\mu j} = \frac{\partial F_\mu}{\partial v_{fit,j}}\Big|_{v_{fit,j}=v_{meas,j}}$ and $f_\mu = F_\mu(v_{meas,1}, ..., v_{meas,2})$, one can find:

$$\chi^2 = \boldsymbol{\epsilon}^T \hat{V}^{-1}\boldsymbol{\epsilon} + 2\boldsymbol{\lambda}^T(\hat{B}\boldsymbol{\epsilon} + \mathbf{f}) \qquad (12)$$

- Differentiating χ^2 with respect to ϵ (we want to minimize the χ^2) and using the equation on top, yields:

$$\boldsymbol{\epsilon} = -\hat{V}\hat{B}^T\hat{S}\mathbf{f} \qquad (13)$$

- with: $\hat{S} = (\hat{B}\hat{V}\hat{B}^T)^{-1}$
- This means, one only needs to calculate $\hat{B}$ and $\mathbf{f}$ in order to find the corrections ϵ
- **NOTE:** In general, this is an iterative procedure, because convergence is not guaranteed after the first calculation

Summary and Outlook

- Introduced kinematic fitting techniques to physics data analysis

Summary and Outlook

- Introduced kinematic fitting techniques to physics data analysis
    i) Improve resolution

Summary and Outlook

- Introduced kinematic fitting techniques to physics data analysis
  - i) Improve resolution
  - ii) Suppress background

Summary and Outlook

- Introduced kinematic fitting techniques to physics data analysis
  i) Improve resolution
  ii) Suppress background
- Did not cover many interesting aspects:
  - Vertex constraints
  - Fitting with unmeasured particles / missing particle information

Summary and Outlook

- Introduced kinematic fitting techniques to physics data analysis
  i) Improve resolution
  ii) Suppress background
- Did not cover many interesting aspects:
  - Vertex constraints
  - Fitting with unmeasured particles / missing particle information
- The kinematic fit is a powerful tool, but also very dangerous
  $\rightarrow$ 3N variables, including the detector response, are folded into one single number

Summary and Outlook

- Introduced kinematic fitting techniques to physics data analysis
    i) Improve resolution
    ii) Suppress background
- Did not cover many interesting aspects:
    - Vertex constraints
    - Fitting with unmeasured particles / missing particle information
- The kinematic fit is a powerful tool, but also very dangerous
  $\rightarrow$ 3N variables, including the detector response, are folded into one single number
- Therefore, one has to pay close attention to the fitter performance / tuning

Summary and Outlook

- Introduced kinematic fitting techniques to physics data analysis
  - i) Improve resolution
  - ii) Suppress background
- Did not cover many interesting aspects:
  - Vertex constraints
  - Fitting with unmeasured particles / missing particle information
- The kinematic fit is a powerful tool, but also very dangerous
  $\rightarrow$ 3N variables, including the detector response, are folded into one single number
- Therefore, one has to pay close attention to the fitter performance / tuning
- Whether to use or not to use a fitter in the analysis is up to you, but I highly recommend to give it a try

Summary and Outlook

- Introduced kinematic fitting techniques to physics data analysis
  i) Improve resolution
  ii) Suppress background
- Did not cover many interesting aspects:
  - Vertex constraints
  - Fitting with unmeasured particles / missing particle information
- The kinematic fit is a powerful tool, but also very dangerous
  $\rightarrow$ 3N variables, including the detector response, are folded into one single number
- Therefore, one has to pay close attention to the fitter performance / tuning
- Whether to use or not to use a fitter in the analysis is up to you, but I highly recommend to give it a try
- Most topics discussed in this lecture are based on my own experience $\rightarrow$ Very likely, that I missed something

Summary and Outlook

- Introduced kinematic fitting techniques to physics data analysis
  i) Improve resolution
  ii) Suppress background
- Did not cover many interesting aspects:
  - Vertex constraints
  - Fitting with unmeasured particles / missing particle information
- The kinematic fit is a powerful tool, but also very dangerous
  $\rightarrow$ 3N variables, including the detector response, are folded into one single number
- Therefore, one has to pay close attention to the fitter performance / tuning
- Whether to use or not to use a fitter in the analysis is up to you, but I highly recommend to give it a try
- Most topics discussed in this lecture are based on my own experience $\rightarrow$ Very likely, that I missed something
- My favorite quote from A. Kupscz: "Analysis is a matter of taste"