PHZ4151C: Exercise 1

Computational Physics Lab
Due January 25

For each problem you will write one or more python programs. These programs should follow the Python 2.7.x coding and formatting conventions outlined for our course. <u>You must hand in copies of the programs and outputs as prescribed in each problem.</u>

In addition, you must submit all Python programs as an archive tgz file via email to phz4151c@hadron.physics.fsu.edu. Place copies of only your Python programs in a directory called <last_name>-exercise1/ where <last_name> is your last name. Below are the commands for creating, checking, and submitting archive files.

Creating archive file:

tar -zcvf last_name-exercise1.tgz last_name-exercise1/

Checking archive contents:

tar -ztvf last_name-exercise1.tgz

Submitting via email:

mail -s "exercise 1" -c <your-email> -a last_name-exercise1.tgz phz4151c@hadron.physics.fsu.edu

> *Note: the mail command expects a message to be entered before sending the email. After entering the full "mail" command line one must end the message with either a "." on a new line then the [return] key or use the key combination [control-d] on a new line.*

1. **Altitude of a satellite:** A satellite is to be launched into a circular orbit around the Earth so that it orbits the planet once every T seconds.

(a) Treating the Earth as a perfect sphere with its center of mass in the middle (which is only approximately correct), show that the altitude h above the Earth's surface that the satellite must have is

$$h=\left(\frac{GMT^2}{4\pi^2}\right)^{1/3}-R,$$

where $G = 6.67 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ is Newton's gravitational constant, $M = 5.97 \times 10^{24}$ kg is the mass of the Earth, and $R = 6371$ km is its radius.

(b) Write a program that asks the user to enter the desired value of T and then calculates and prints out the corresponding altitude in meters. Note that floating point real numbers can be represented in scientific notation with E or e indicating the power of 10. For example, 6.67e-11 = 6.67×10^{-11}.

(c) Use your program to calculate the altitudes of satellites that orbit the Earth once a day (so-called "geosynchronous" orbit), once every 90 minutes, and once every 45 minutes. What do you conclude from the last of these calculations?

(d) Technically a geosynchronous satellite is one that orbits the Earth once per *sidereal day* (which is 23.93 hours not 24 hours). Why is this? How much difference will it make to the altitude of the satellite?

For full credit turn in your derivation for part (a), a printout of your finished program, a printout of the three runs of the program showing the three answers it produces, and your answers to the questions in parts (c) and (d).

2. **Special relativity:** A spaceship travels from Earth in a straight line at a (relativistic) speed v to another planet x light years away. Write a program to ask the user for the value of x and the speed v as a fraction of the speed of light, then print out the time in years that the spaceship takes to reach its destination (a) in the rest frame of an observer on Earth and (b) as perceived by a passenger on board the ship. Use your program to calculate the answers for a planet 10 light years away with v = 0.99c.

For full credit turn in a printout of your program plus a printout of the program in action showing the answers it produces.

3. **The semi-empirical mass formula:** In nuclear physics, the semi-empirical mass formula is a formula for calculating the approximate nuclear binding energy B of an atomic nucleus with atomic number Z and mass number A. The formula looks like this:

$$B = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \frac{a_5}{A^{1/2}},$$

where, in units of millions of electron volts, the constants are a_1 = 15.67, a_2 = 17.23, a_3 = 0.75, a_4 = 93.2, and

$$a_5 = \begin{cases} 12.0 & \text{if } Z \text{ and } A-Z \text{ are both even} \\ -12.0 & \text{if } Z \text{ and } A-Z \text{ are both odd} \\ 0 & \text{otherwise.} \end{cases}$$

Write a program that takes as its input the values of A and Z, and prints out (a) the binding energy B for the corresponding atom and (b) the binding energy per nucleon, which is B/A. Use your program to find the binding energy of an atom with A = 58 and Z = 28. (Hint: The correct answer is around 490 MeV.) In your program make use of the *if* statement (described in chapter 2 section 2.3.1).

For full credit turn in a printout of your program and a printout of the program in action showing the output it produces.

4. **Planetary orbits:** The orbit in space of one body around another, such as a planet around the Sun, need not be circular. In general it takes the form of an ellipse, with the body sometimes closer in and sometimes further out. If you are given the distance ℓ_1 of closest approach that a planet makes to the Sun, also called its *perihelion*, and its linear velocity v_1 at perihelion, then any other property of the orbit can be calculated from these two as follows.

(a) Kepler's second law tells us that the distance ℓ_2 and velocity v_2 of the planet at its most distant point, or *aphelion*, satisfy $\ell_2 v_2 = \ell_1 v_1$. At the same time the total energy, kinetic plus gravitational, of a planet with velocity v and distance r from the Sun is given by

$$E = \frac{1}{2}mv^2 - G\frac{mM}{r},$$

where m is the planet's mass, $M = 1.9891 \times 10^{30}$ kg is the mass of the Sun, and $G = 6.6738 \times 10^{-11}$ m^3 kg^{-1} s^{-2} is Newton's gravitational constant. Given that energy must be conserved, show that v_2 is the smaller root of the quadratic equation

$$v_2^2 - \frac{2GM}{v_1 l_1} v_2 - \left[v_1^2 - \frac{2GM}{l_1}\right] = 0.$$

Once we have v_2 we can calculate ℓ_2 using the relation $\ell_2 = \ell_1 v_1 / v_2$.

(b) Given the values of v_1, ℓ_1, and ℓ_2, other parameters of the orbit are given by simple formulas can that be derived from Kepler's laws and the fact that the orbit is an ellipse:

Semi-major axis: $$a = \frac{1}{2}(l_1 + l_2),$$

Semi-minor axis: $$b = \sqrt{l_1 l_2},$$

Orbital period: $$T = \frac{2\pi ab}{l_1 v_1},$$

Orbital eccentricity: $$e = \frac{l_2 - l_1}{l_2 + l_1}$$

Write a program that asks the user to enter the distance to the Sun and the velocity at perihelion, then calculates and prints the quantities ℓ_2, v_2, T, and e.

(c) Test your program by having it calculate the properties of the orbits of the Earth (for which $\ell_1 = 1.4710 \times 10^{11}$ m and $v_1 = 3.0287 \times 10^4$ m s^{-1}) and Halley's comet ($\ell_1 = 8.7830 \times 10^{10}$ m and $v_1 = 5.4529 \times 10^4$ m s^{-1}). Among other things, you should find that the orbital period of the Earth is one year and that of Halley's comet is about 76 years.

For full credit turn in a printout of your program and a printout of the program in action showing the output it produces for the Earth and Halley's comet examples.