Review of the Paper "Completeness Rules for Spin Observables in Pseudoscalar Meson Photoproduction"

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FSU Weekly Group Meeting

Weekly Group Meeting

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Outline

1 Introduction

2 Amplitudes and Ambiguities

3 Fun Facts

Observables in pseudoscalar meson photoproduction

Paper reference - Chiang and Tabakin, PRC 55, 2054 (1997)

Observables in pseudoscalar meson photoproduction -

- Total 4 amplitudes corresponding to such reactions.
- 16 observables, denoted as Ω^α here. In general N^2 observables for N complex amplitudes. These observables can be measured experimentally.
- The 16 observables are -

- 4 S type - cross section I(θ)+ 3 single spin observables (Σ, T, P). Σ is related to beam polarization, T is related to target pol., and P is related to recoil pol.
- 4 beam-target (BT) type observables (G, H, E, F)
- 4 beam-recoil (BR) type observables (O_x, O_z, C_x, C_z)
- 4 target-recoil (TR) type observables (T_x, T_z, L_x, L_z)

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Question 1 - Unique determination of the 4 amplitudes ?

Can the 4 amplitudes be uniquely determined from the observables ? How many out of the 16 do we need ?

- It depends on whether these observables depend on each other. If they are completely independent from each other, then to uniquely determine N amplitudes we need $2N - 1$ observables (to find out the 4 magnitudes and 3 relative phases).

- These 16 observables are linearly independent.

- However they are non-linearly dependent on each other. E.g. -
  Linear-quadratic equations : $\Sigma = -TP - T_x L_z + L_x T_z$
  Quadratic equations : $C_z O_z + O_x C_x - GE - HF = 0$
  Square relations : $G^2 + H^2 + E^2 + F^2 = \sigma^2 - \Sigma^2 - T^2 + P^2$

- Due to the non-linear dependence, many solutions for the amplitudes can simultaneously satisfy the polarization observable measurements.

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So, how many measurements do we need ? And are they any particular set of measurements ?

- The classic Barker, Donnachie and Storrow **(BDS) paper** states "5 double spin observables (no four of them should belong to the same set of BT, BR or TR) + the four S type measurements are needed to determine all amplitudes w/o discrete ambiguities" - total 9 carefully chosen observables.
- **This paper** says that "The four S types + four appropriately chosen double spin observables suffice to resolve all ambiguities" - total 8 carefully chosen observables.

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Question 2 - What are discrete ambiguities and the relations between amplitudes and observables ?

Relation between the amplitudes and the observables -

- In 4X4 dimension, Gamma (Γ) matrices are used since they are linearly independent.
- We can either use helicity or transversity basis. The matrices are labelled Γ in the former and $\tilde{\Gamma}$ in the latter basis.
- All 16 observables can be expressed as -
  $\Omega^{\alpha} = \frac{1}{2} H_i{}^{*} \Gamma_{ij}{}^{\alpha} H_j = \frac{1}{2} b_i{}^{*} \Gamma_{ij}{}^{\alpha} b_j$, α goes from 1 to 16

  where, $H_i \rightarrow$ complex helicity amplitudes
  $b_i \rightarrow$ complex transversity amplitudes
  $\Omega^{\alpha} \rightarrow$ measured observable

  Because of its form, this formulation is called "Bilinear Helicity Product".

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Relation between the amplitudes and the observables - an example -

$$G = \frac{1}{2} b_i^* \tilde{\Gamma}^3_{ij} b_j$$

$$= \begin{pmatrix} b_1^* & b_2^* & b_3^* & b_4^* \end{pmatrix} \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

$$= Im(-b_1 b_3^* - b_2 b_4^*)$$

Question 2 - What are discrete ambiguities and the relations between amplitudes and observables ?

Discrete ambiguities -

- This paper assumes that we always measure the 4 S type observables, $(I(\theta), \Sigma, T, P)$. They are also labelled as $(\Omega^1, \Omega^4, \Omega^{10}, \Omega^{12})$. $\Omega^m = b_i^* \tilde{\Gamma}_{ij}^m b_j$, $m = 1, 4, 10, 12$
- Then, if there exists any linear or antilinear transformation such that these observables remain invariant under the transformation, then the amplitudes can't be uniquely determined.

- Linear transformation L : $b_i^{'} = L_{ij} b_j$. If L commutes with the Γ matrices that are related to the measured observables, i.e.,

$$\tilde{\Gamma}^\alpha L = L \tilde{\Gamma}^\alpha,$$

then the observables will remain unchanged under this transformation.

- Antilinear transformation A : $b_i^{'} = A_{ij} B_j^*$. If A satisfies

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Discrete ambiguities example -

- Assume that we measure (G, F) (BT type), (O_z, C_x) (BR type) and T_x (TR type) on top of the 4 S type measurements.
- In this case, there exists a transformation, $b_i^{'} = \tilde{\Gamma}^6_{ij} b_j^*$, $b_i^{'*} = b_j \tilde{\Gamma}^{6*}_{ij}$ such that the above mentioned 9 observables remain unchanged. For instance,

$$\begin{aligned} G &= \tfrac{1}{2} b_i^{'*} \tilde{\Gamma}^3_{ij} b_j^{'} \\ &= \tfrac{1}{2} b_m \tilde{\Gamma}^{6*}_{im} \tilde{\Gamma}^3_{ij} \tilde{\Gamma}^6_{jk} b_k^* \\ &= \tfrac{1}{2} b_m (\tilde{\Gamma}^{6\dagger} \tilde{\Gamma}^3 \tilde{\Gamma}^6)^T_{km} b_k^* \end{aligned}$$

Since $(\tilde{\Gamma}^{6\dagger} \tilde{\Gamma}^3 \tilde{\Gamma}^6)^T = \tilde{\Gamma}^3$, Antilinear transformation

therefore, $G = \frac{1}{2} b_i^* \tilde{\Gamma}^3_{ij} b_j$

i.e., both $b_i^{'}$ and b_i solutions for the transversity amplitudes give the same observable G. The same is true for F, O_z, C_x, T_x, leading to an ambiguity.

- Note : the BDS rule is proved wrong here !

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- Fierz relations - they are a convenient tool to give relations and bounds on observables. Some interesting facts that they show are -
  - If we know the 4 S-type observables and 3 double-spin observables from any one set (BT, BR or TR) then the 4th observable in that set is redundant and can be calculated from them.
  - The Fierz relations also show that the magnitude of the single spin observables (Σ, T, P) should be ≤ 1.0.

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