

# Determining the full meson photo-production amplitude from “complete” experiments

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- *spin-observables in  $J^{\pi}=0^{-}$  production and “complete” experiments*
- *Q: can such data provide total amplitudes ?*
- *the mechanics of fitting out the amplitudes*
  - *potentials and limitations*
  - *constraining the phase*
- *tests with mock data – a work in progress*

# Idealized path to search for $N^*$ , $\Delta^*$ states via $J^\pi=0^-$ photo-production

## (1) determine the production amplitude from experiment

- search for resonant structure:  
Analytic Continuation, Argand circles,  
phase motion, Speed plots, etc.

## (2) separate resonance and background components

- determine resonant  $\gamma N^*$  and decay couplings;  
contact with LQCD, DSE, Hadron models

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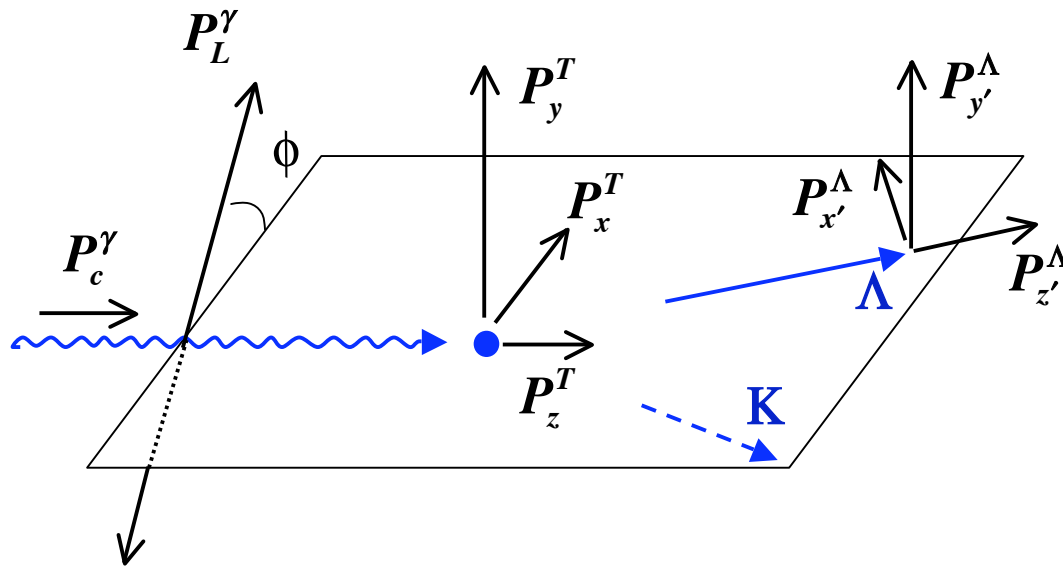
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## (2) separate resonance and background components

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-without exp Amplitudes  
models have conjectured resonances  
and adjusted couplings to compare  
with limited data

# ***Polarized Pseudoscalar meson photo-production:***



**Polarization observables in  $J^\pi = 0^-$  meson photo-production :**

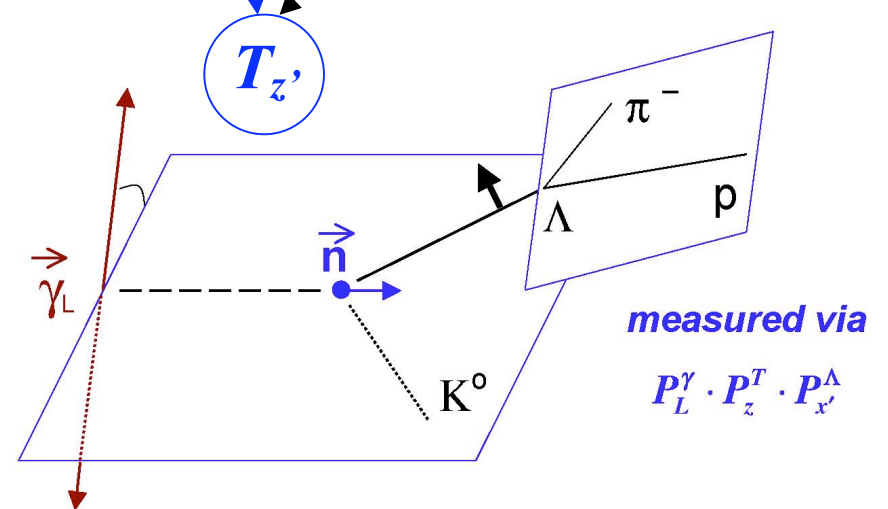
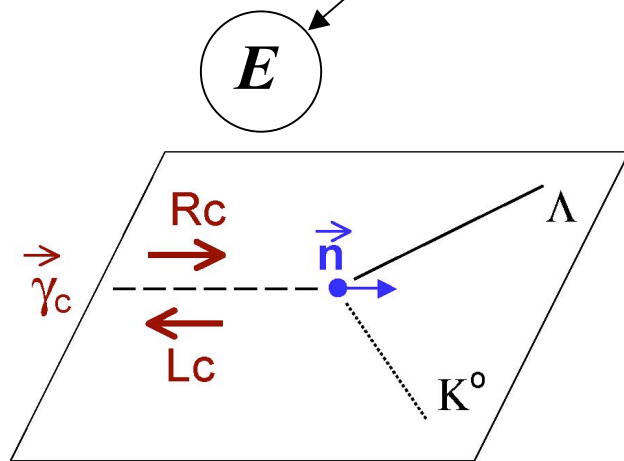
- *single-pol observables measured from double-pol asy*
- *double-pol observables measured from triple-pol asy*

<i>Photon beam</i>		<i>Target</i>			<i>Recoil</i>			<i>Target - Recoil</i>								
					$x'$	$y'$	$z'$	$x'$	$x'$	$x'$	$y'$	$y'$	$y'$	$z'$	$z'$	$z'$
		$x$	$y$	$z$				$x$	$y$	$z$	$x$	$y$	$z$	$x$	$y$	$z$
<i>unpolarized</i>	$\sigma_0$		<b><math>T</math></b>			<b><math>P</math></b>		$T_{x'}$		$L_{x'}$		<b><math>\Sigma</math></b>		$T_{z'}$		$L_{z'}$
<i>linearly <math>P_\gamma</math></i>	$\Sigma$	<b><math>H</math></b>	<b><math>P</math></b>	<b><math>G</math></b>	$O_{x'}$	<b><math>T</math></b>	$O_{z'}$	$L_{z'}$	$C_{z'}$	$T_{z'}$	<b><math>E</math></b>		<b><math>F</math></b>	$L_{x'}$	$C_{x'}$	$T_{x'}$
<i>circular <math>P_\gamma</math></i>		<b><math>F</math></b>		<b><math>E</math></b>	$C_{x'}$		$C_{z'}$		$O_{z'}$		<b><math>G</math></b>		<b><math>H</math></b>		$O_{x'}$	

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		$x$	$y$	$z$				$x$	$y$	$z$	$x$	$y$	$z$	$x$	$y$	$z$
unpolarized	$\sigma_0$		$T$			$P$		$T_{x'}$		$L_{x'}$		$\Sigma$		$T_{z'}$		$L_{z'}$
linearly $P_\gamma$	$\Sigma$	$H$	$P$	$G$	$O_{x'}$	$T$	$O_{z'}$	$L_{z'}$	$C_{z'}$	$T_{z'}$	$E$		$F$	$L_{x'}$	$C_{x'}$	$T_{x'}$
circular $P_\gamma$		$F$		$E$	$C_{x'}$		$C_{z'}$		$O_{z'}$		$G$		$H$		$O_{x'}$	



# Requirements for a complete determination of the amplitude, unique to within a phase and free of ambiguities:

⇔ **8 carefully chosen observables** - *Barker, Donnachie, Storrow, NP B95(75)*  
- *Chiang & Tabakin, PRC55,2054(97)*

**(I) cross section and the three single-polarization observables:**

$\{\sigma, \Sigma, P, T\}$

**(II) four double-polarization observables:**

- **2 + 2 select cases from:** *Beam-Target, Beam-Recoil, Target-Recoil*
- **2 + 1 + 1 select cases from:** *Beam-Target, Beam-Recoil, Target-Recoil*



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- in practice, one needs linear and circular beam polarization, at least one orientation of target polarization and recoil polarization

⇔ *by the time you accumulate that in a  $4\pi$  detector, you have all 16*

⇒ *∃ huge redundancy, at least in principle!*

$\gamma + p \rightarrow K^+ \Lambda$  series of JLab experiments:

Photon beam		Target			Recoil			Target - Recoil								
					$x'$	$y'$	$z'$	$x'$	$x'$	$x'$	$y'$	$y'$	$y'$	$z'$	$z'$	$z'$
		$x$	$y$	$z$				$x$	$y$	$z$	$x$	$y$	$z$	$x$	$y$	$z$
unpolarized	$\sigma_0$		$T$			$P$		$T_{x'}$		$L_{x'}$		$\Sigma$		$T_{z'}$		$L_{z'}$
linearly $P_\gamma$	$\Sigma$	$H$	$P$	$G$	$O_{x'}$	$T$	$O_{z'}$	$L_{z'}$	$C_{z'}$	$T_{z'}$	$E$		$F$	$L_{x'}$	$C_{x'}$	$T_{x'}$
circular $P_\gamma$		$F$		$E$	$C_{x'}$		$C_{z'}$		$O_{z'}$		$G$		$H$		$O_{x'}$	

<i>status</i>	<i>CLAS run period</i>	<i>beam</i>	<i>target</i>	
<i>complete</i>	<i>g1</i>	$\gamma, \vec{\gamma}_c$	$LH_2$	<i>Miskimen/Schumacher</i>
<i>complete</i>	<i>g8</i>	$\vec{\gamma}_L$	$LH_2$	<i>Cole</i>
<i>complete</i>	<i>g9a - <math>P_z^T</math></i>	$\vec{\gamma}_L, \vec{\gamma}_c$	$FROST - C_4\vec{H}_9O\vec{H}$	<i>Klein, Pasyuk</i>
<i>2010</i>	<i>g9b - <math>P_x^T</math></i>	$\vec{\gamma}_L, \vec{\gamma}_c$	$FROST - C_4\vec{H}_9O\vec{H}$	<i>Klein, Pasyuk</i>

**Full set of 16**

$\gamma + n(p) \rightarrow K^0 \Lambda$  experiments at JLab with HDice:

- *single-pol observables measured from double-pol asy*
- *double-pol observables measured from triple-pol asy*

Photon beam	Target			Recoil			Target - Recoil									
				$x'$	$y'$	$z'$	$x'$	$x'$	$x'$	$y'$	$y'$	$y'$	$z'$	$z'$	$z'$	
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circular $P_\gamma$		$F$		$E$	$C_{x'}$		$C_{z'}$		$O_{z'}$		$G$		$H$		$O_{x'}$	



*simultaneous B-R with HD*



**Full set of 16**

schedule	CLAS run period	beam	target	spokesmen
2006-2007	g13	$\vec{\gamma}_L$	$LD_2$	Nadel-Turonski
2010-2011	g14	$\vec{\gamma}_L, \vec{\gamma}_c$	$H \cdot \vec{D}$ ice	Sandorfi/Klein

# The mechanics of inferring amplitudes from data

(I) *theory*  $\Rightarrow$  *experiment*: eg. generating mock data

$$\frac{d\sigma}{d\Omega} = \frac{|P_{\pi,\eta,K}^{CM}|}{E_\gamma^{cm}} \cdot \left| \langle N(\pi,\eta,K) | \sum F_i | \gamma N \rangle \right|^2, \quad \text{CGLN, PR106(1957)}$$

$\uparrow$  (Cartesian)  $\Leftrightarrow$  (Spherical)  $H_i$  helicity amplitudes

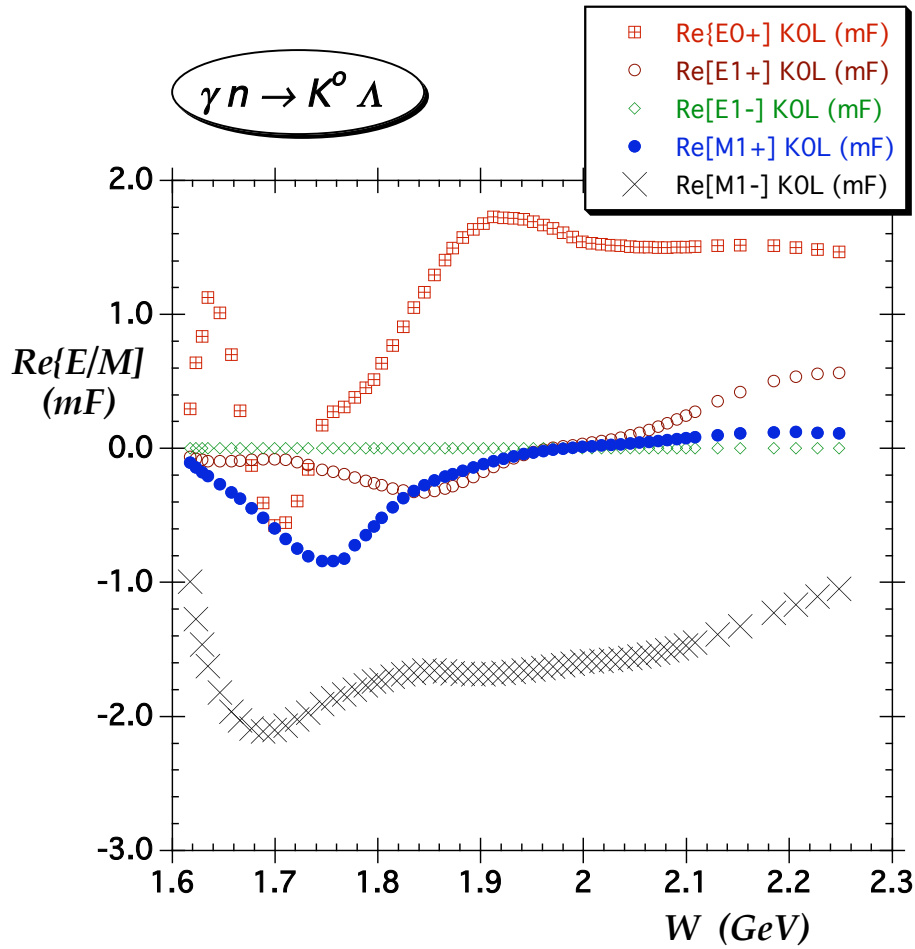
*multipoles*  $\Rightarrow$  *observables*

- $\{E_{\ell\pm}, M_{\ell\pm}\} \otimes \{P_\ell, P'_\ell, P''_\ell\} \Leftrightarrow \{\text{functions of } W / E_\gamma\} \otimes \{\text{functions of } \theta_{CM}\}$

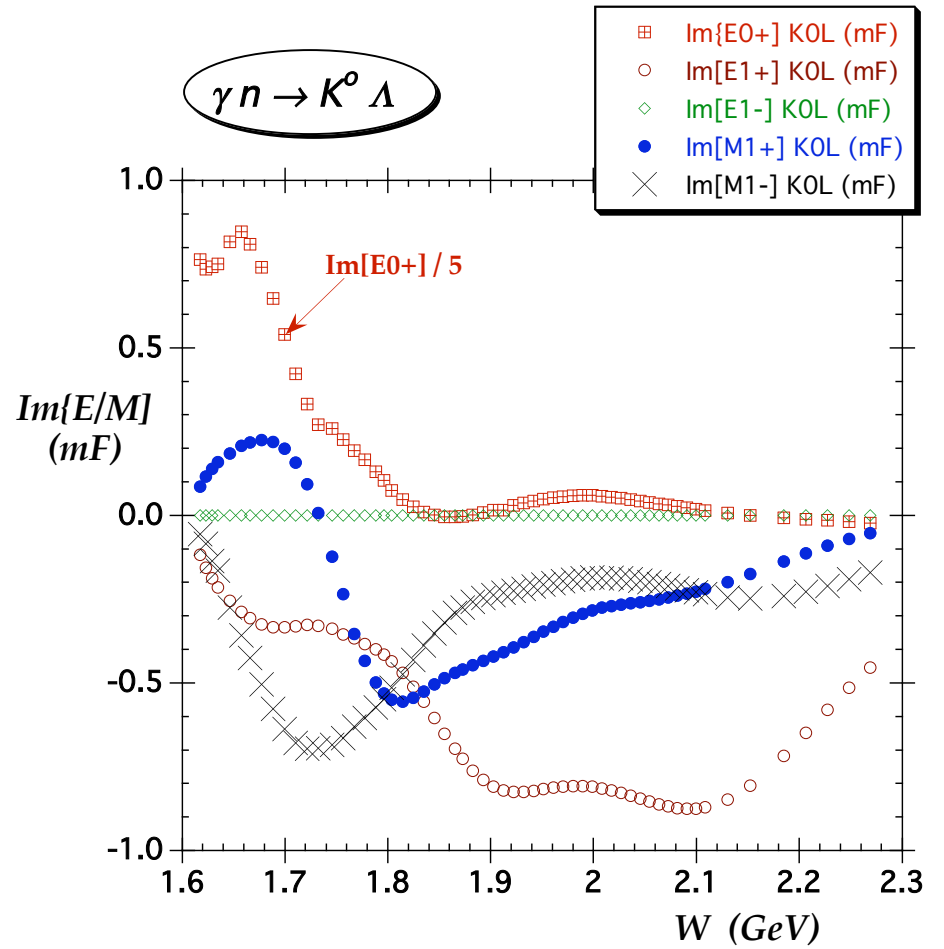
$$\Rightarrow \{F_1, F_2, F_3, F_4\}_{CGLN} \Leftrightarrow \{\text{functions of } (W / E_\gamma, \theta_{CM})\}$$

$$\Rightarrow \{\sigma_0, \Sigma, T, P, E, \dots L_{z'}\} \Leftrightarrow \{\text{functions of } (W / E_\gamma, \theta_{CM}, \phi)\}$$

eg. multipoles from: B. Julia-Diaz, T-S. H. Lee  
 - M2 (full) solution -

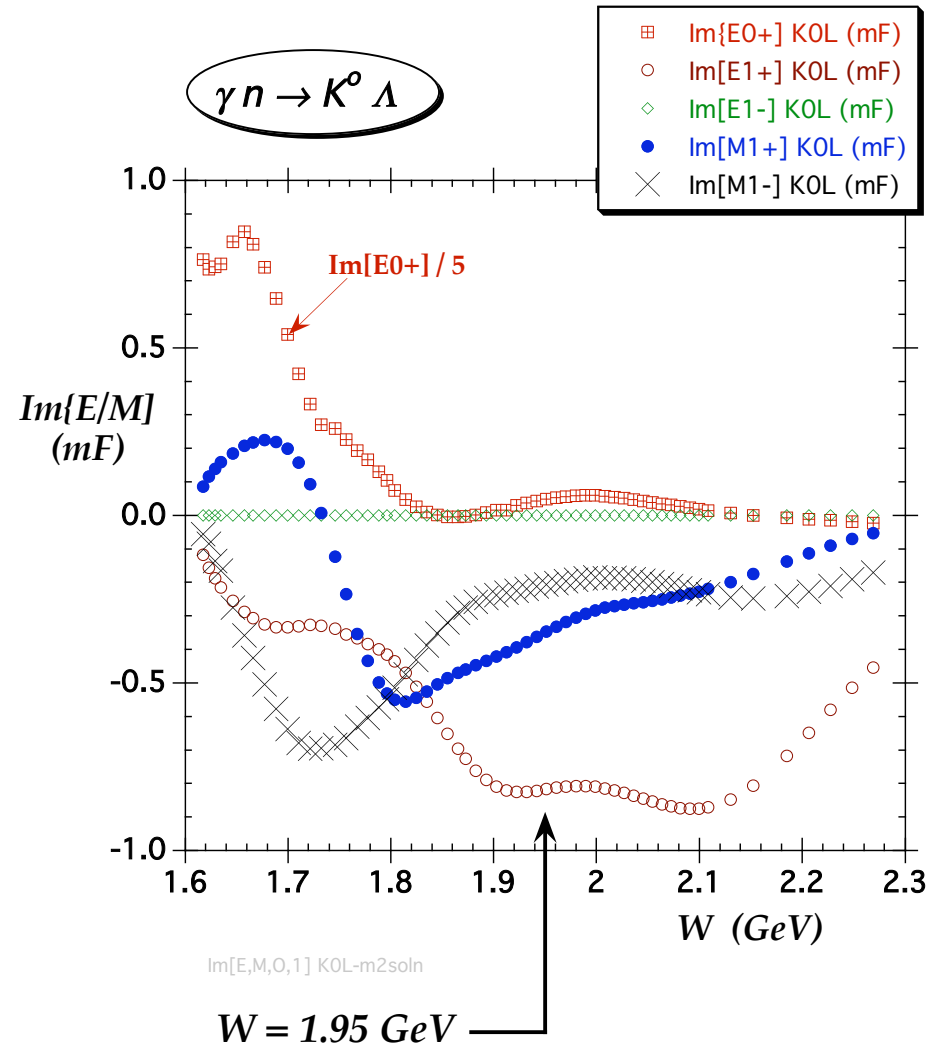
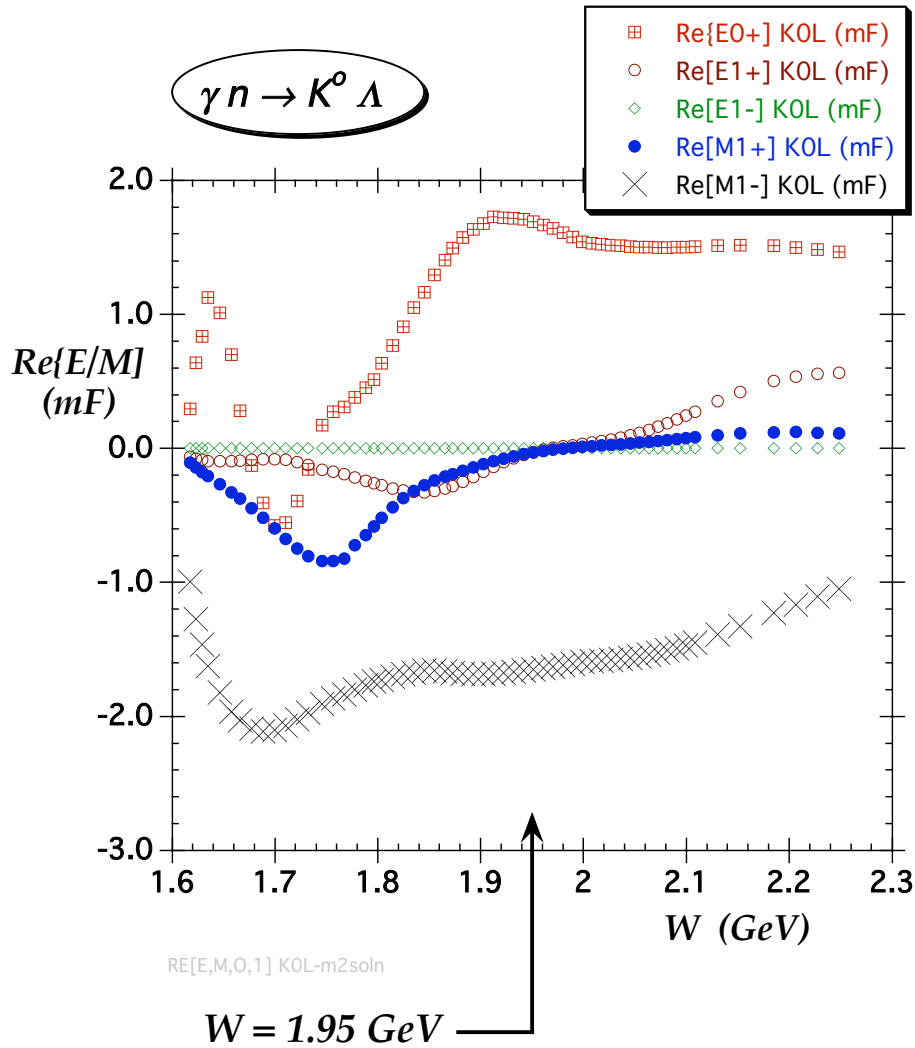


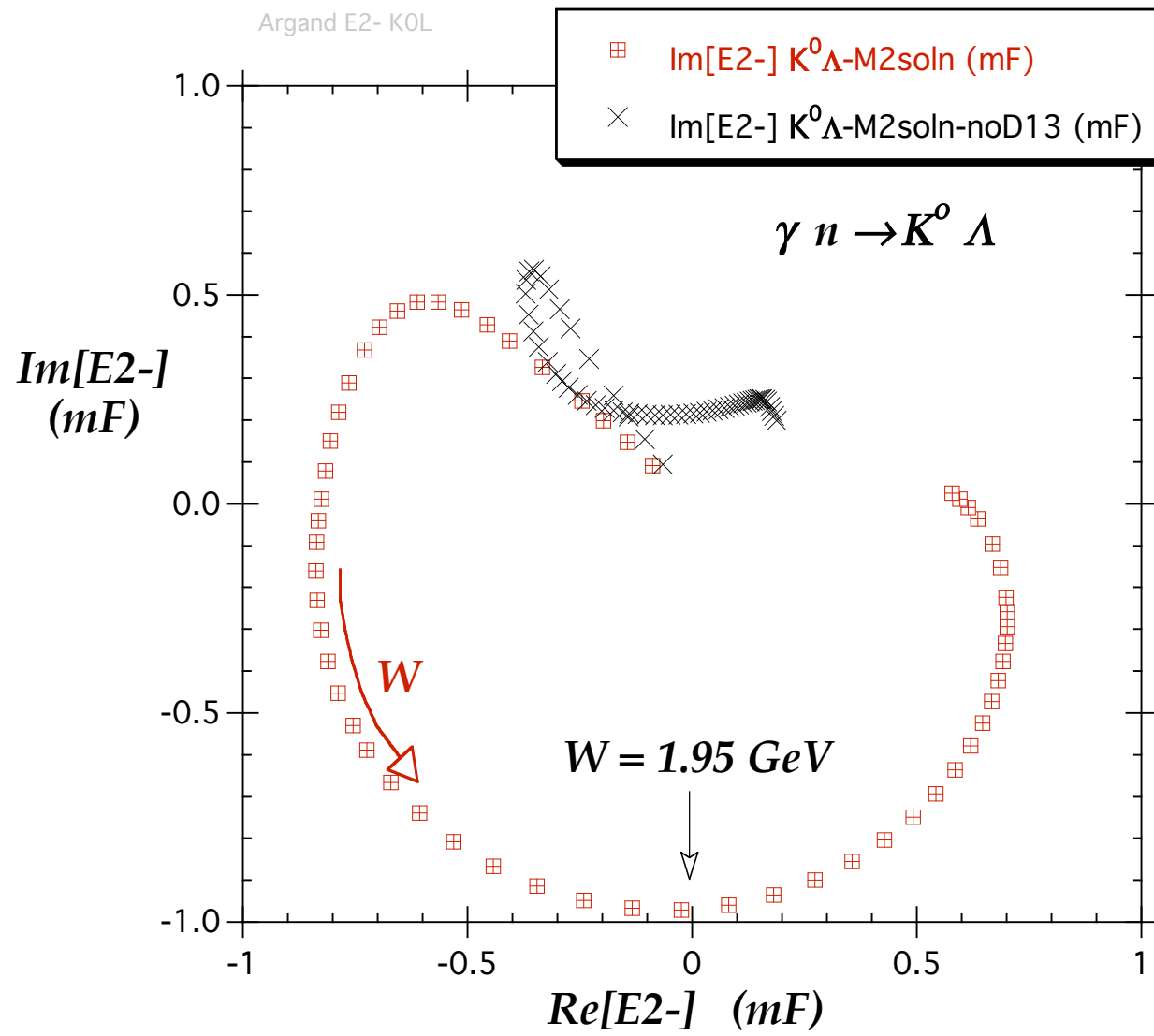
RE[E,M,O,1] KOL-m2soln



Im[E,M,O,1] KOL-m2soln

eg. multipoles from: B. Julia-Diaz, T-S. H. Lee  
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## Constructing the CGLN $F_i$ amplitudes:

$$\begin{aligned}
 F_1(x) = & E_{0+} + (M_{1+} + E_{1+}) \cdot P_2'(x) \\
 & + (2M_{2+} + E_{2+}) \cdot P_3'(x) + (3M_{2-} + E_{2-}) \\
 & + (3M_{3+} + E_{3+}) \cdot P_4'(x) + (4M_{3-} + E_{3-}) \cdot P_2'(x) \\
 & + (4M_{4+} + E_{4+}) \cdot P_5'(x) + (5M_{4-} + E_{4-}) \cdot P_3'(x)
 \end{aligned}$$

$$\begin{aligned}
 F_2(x) = & (2M_{1+} + M_{1-}) + (3M_{2+} + 2M_{2-}) \cdot P_2'(x) \\
 & + (4M_{3+} + 3M_{3-}) \cdot P_3'(x) + (5M_{4+} + 4M_{4-}) \cdot P_4'(x)
 \end{aligned}$$

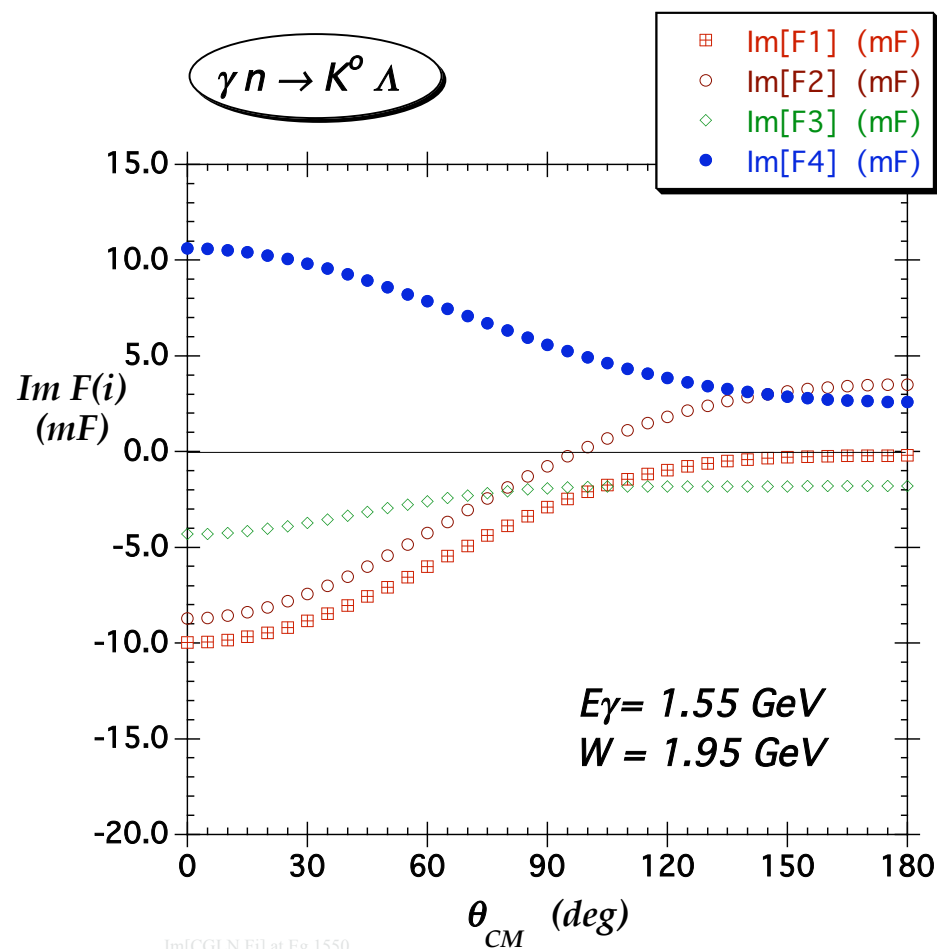
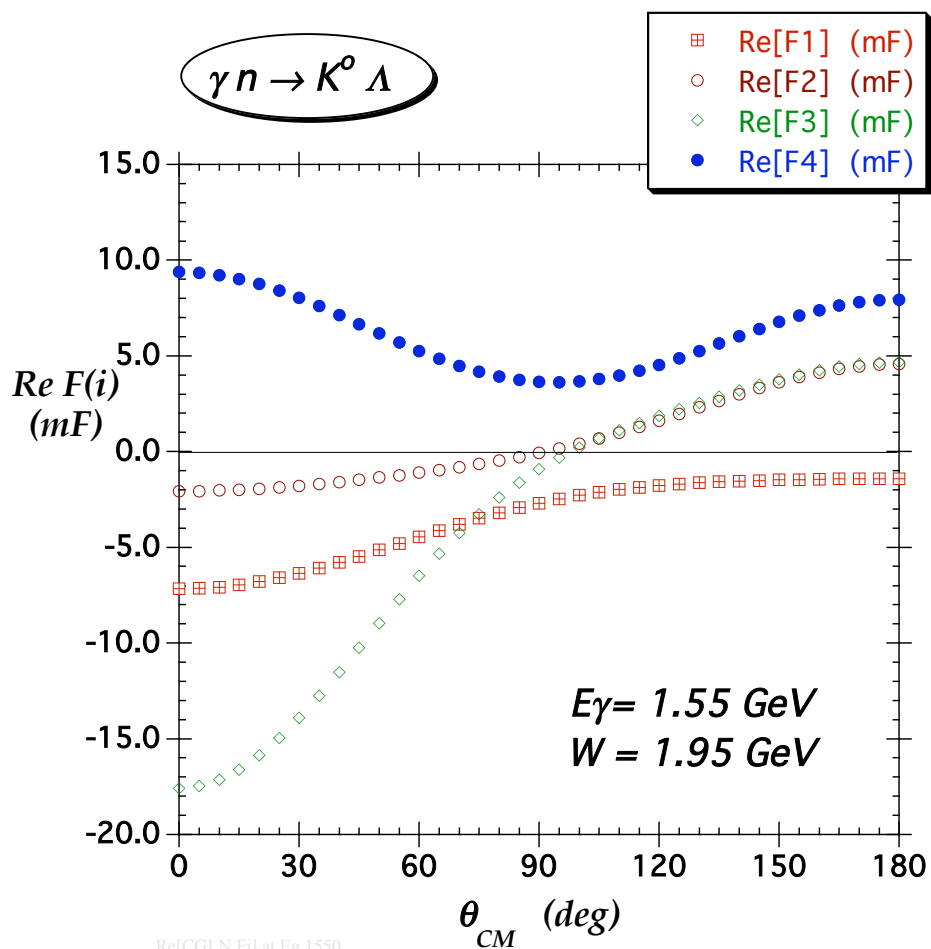
$P_\ell, P_\ell', P_\ell'' \equiv$  Legendre functions

$$x = \cos(\theta_{CM})$$

$$\begin{aligned}
 F_3(x) = & + (E_{1+} - M_{1+}) \cdot P_2''(x) \\
 & + (E_{2+} - M_{2+}) \cdot P_3''(x) \\
 & + (E_{3+} - M_{3+}) \cdot P_4''(x) + (E_{3-} + M_{3-}) \cdot P_2''(x) \\
 & + (E_{4+} - M_{4+}) \cdot P_5''(x) + (E_{4-} + M_{4-}) \cdot P_3''(x)
 \end{aligned}$$

$$\begin{aligned}
 F_4(x) = & (M_{2+} - E_{2+} - E_{2-} - M_{2-}) \cdot P_2''(x) \\
 & + (M_{3+} - E_{3+} - E_{3-} - M_{3-}) \cdot P_3''(x) \\
 & + (M_{4+} - E_{4+} - E_{4-} - M_{4-}) \cdot P_4''(x)
 \end{aligned}$$

*eg. CGLN amplitudes at  $W = 1.95$  GeV  
 - calculated from the multipoles of B. Julia-Diaz, T-S. H. Lee*

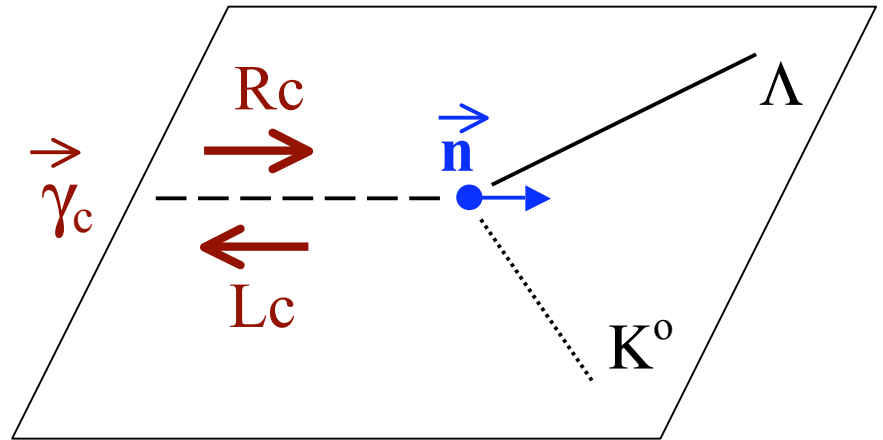


notation:

$$\hat{A}_{asy} = \sigma_o \cdot A_{asy}$$

eg:

$$\left. \begin{aligned} E &= \frac{\sigma_{Anti} - \sigma_{Par}}{\sigma_{Anti} + \sigma_{Par}} \\ \sigma_o &= \frac{1}{2}(\sigma_{Anti} + \sigma_{Par}) \\ \Rightarrow \hat{E} &= \frac{1}{2}(\sigma_{Anti} - \sigma_{Par}) \end{aligned} \right\}$$



$$\{F_1, F_2, F_3, F_4\}_{CGLN} \Leftrightarrow \{ \text{functions of } (W / E_\gamma, \theta_{CM}) \}$$

$$\Rightarrow \{ \sigma_o, \hat{\Sigma}, \hat{T}, \hat{P}, \hat{E}, \dots \hat{L}_{z'} \} \Leftrightarrow \{ \text{functions of } (W / E_\gamma, \theta_{CM}, \phi) \}$$

Observables in terms of the CGLN  $F_i$  amplitudes,  $i = 1-4$  :

$$\rho = \left| \mathbf{P}_{\pi,\eta,K}^{CM} \right| / \mathbf{E}_\gamma^{cm}$$

- Andy & Harry's signs – confirmed Aug 22'09.

$$\hat{\sigma}_0 = \left\{ \begin{aligned} &|F_1|^2 + |F_2|^2 + \frac{1}{2} \sin^2 \theta \cdot (|F_3|^2 + |F_4|^2) \\ &+ \Re \left[ \sin^2 \theta \cdot (F_2^* F_3 + F_1^* F_4 + \cos \theta \cdot F_3^* F_4) - 2 \cos \theta \cdot F_1^* F_2 \right] \end{aligned} \right\} \cdot \rho$$

$$\hat{\Sigma} = - \left[ \frac{1}{2} \sin^2 \theta \cdot (|F_3|^2 + |F_4|^2) + \sin^2 \theta \cdot \Re \left\{ F_2^* F_3 + F_1^* F_4 + \cos \theta \cdot (F_3^* F_4) \right\} \right] \cdot \rho$$

$$\hat{T} = \Im \left\{ \sin \theta \left[ F_1^* F_3 - F_2^* F_4 + \cos \theta \cdot (F_1^* F_4 - F_2^* F_3) - \sin^2 \theta \cdot F_3^* F_4 \right] \right\} \cdot \rho$$

$$\hat{P} = \Im \left\{ \sin \theta \left[ -2F_1^* F_2 - F_1^* F_3 + F_2^* F_4 + \cos \theta \cdot (F_2^* F_3 - F_1^* F_4) + \sin^2 \theta \cdot F_3^* F_4 \right] \right\} \cdot \rho$$

$$\hat{T}_{x'} = \Re \left\{ \sin^2 \theta \left[ -F_1^* F_3 - F_2^* F_4 - F_3^* F_4 - \frac{1}{2} \cos \theta \cdot (|F_3|^2 + |F_4|^2) \right] \right\} \cdot \rho$$

$$\hat{L}_{x'} = + \Re \left\{ \sin \theta \left[ |F_1|^2 - |F_2|^2 + \frac{1}{2} \sin^2 \theta \cdot (|F_4|^2 - |F_3|^2) - F_2^* F_3 + F_1^* F_4 + \cos \theta (F_1^* F_3 - F_2^* F_4) \right] \right\} \cdot \rho$$

$$\hat{T}_{z'} = \Re \left\{ \sin \theta \left[ -F_2^* F_3 + F_1^* F_4 + \cos \theta (F_1^* F_3 - F_2^* F_4) + \frac{1}{2} \sin^2 \theta \cdot (|F_4|^2 - |F_3|^2) \right] \right\} \cdot \rho$$

$$\hat{L}_{z'} = \Re \left\{ \begin{aligned} &2F_1^* F_2 - \cos \theta (|F_1|^2 + |F_2|^2) + \sin^2 \theta \cdot (F_1^* F_3 + F_2^* F_4 + F_3^* F_4) \\ &+ \frac{1}{2} \cos \theta \sin^2 \theta \cdot (|F_3|^2 + |F_4|^2) \end{aligned} \right\} \cdot \rho$$

$$\hat{E} = - \left[ -|F_1|^2 - |F_2|^2 + \Re \left\{ 2 \cos \theta \cdot (F_1^* F_2) - \sin^2 \theta \cdot (F_2^* F_3 + F_1^* F_4) \right\} \right] \cdot \rho$$

$$\hat{G} = + \sin^2 \theta \cdot \Im \left\{ F_2^* F_3 + F_1^* F_4 \right\} \cdot \rho$$

$$\hat{F} = \sin \theta \cdot \Re \left[ F_1^* F_3 - F_2^* F_4 - \cos \theta \cdot (F_2^* F_3 - F_1^* F_4) \right] \cdot \rho$$

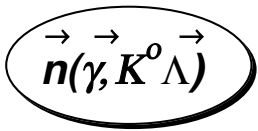
$$\hat{H} = - \sin \theta \cdot \Im \left[ 2F_1^* F_2 + F_1^* F_3 - F_2^* F_4 + \cos \theta \cdot (F_1^* F_4 - F_2^* F_3) \right] \cdot \rho$$

## Simulating the extraction of multipoles (a work in progress):

- *create mock data:*
  - using multipoles from EBAC-Juliá-Días/Lee/Sato “M2” amplitude, generate observables with appropriate energy and angular coverage
  - Gaussian smear the predictions with the expected statistical widths

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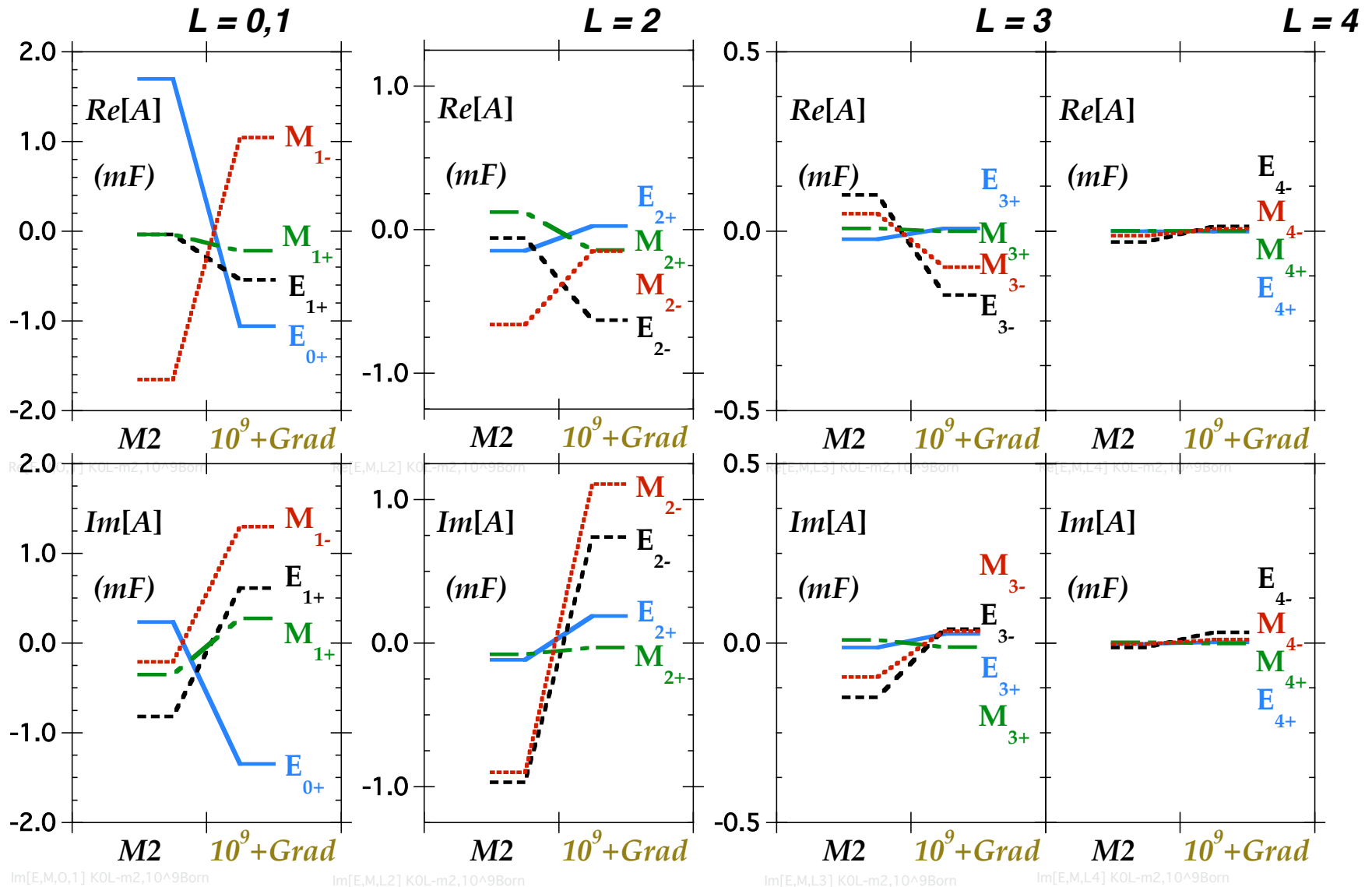
- **create mock data:**
  - using multipoles from EBAC-Juliá-Días/Lee/Sato "M2" amplitude, generate observables with appropriate energy and angular coverage
  - Gaussian smear the predictions with the expected statistical widths
- **fitting mock data, varying multipoles:**
  - (1) **limiting case:**
    - generate observables in 10 deg steps from  $10^\circ$  to  $170^\circ$  CM; assign each an error of 0.1% and Gaussian smear the predictions (Note: no data set will ever be that good !)
    - a) search from a starting point within  $\sim 25\%$  of the generating soln  
 $\Rightarrow$  converges rapidly to EBAC "M2" multipoles ✓
    - b) as a starting point for the search, use Born values;  
Monte Carlo sampling of parameter space, then Gradient search



**EBAC M2 vs Born +  $10^9$  MC + gradient search**

$\chi^2/df = 0.996$  - better than M2 to mock data

**$W = 1.95$  GeV**



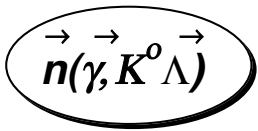
Im[E,M,0,1] KOL-m2,10^9Born

Im[E,M,L2] KOL-m2,10^9Born

Im[E,M,L3] KOL-m2,10^9Born

Im[E,M,L4] KOL-m2,10^9Born

M2 vs Born + 10^9 MC + Grad freefit

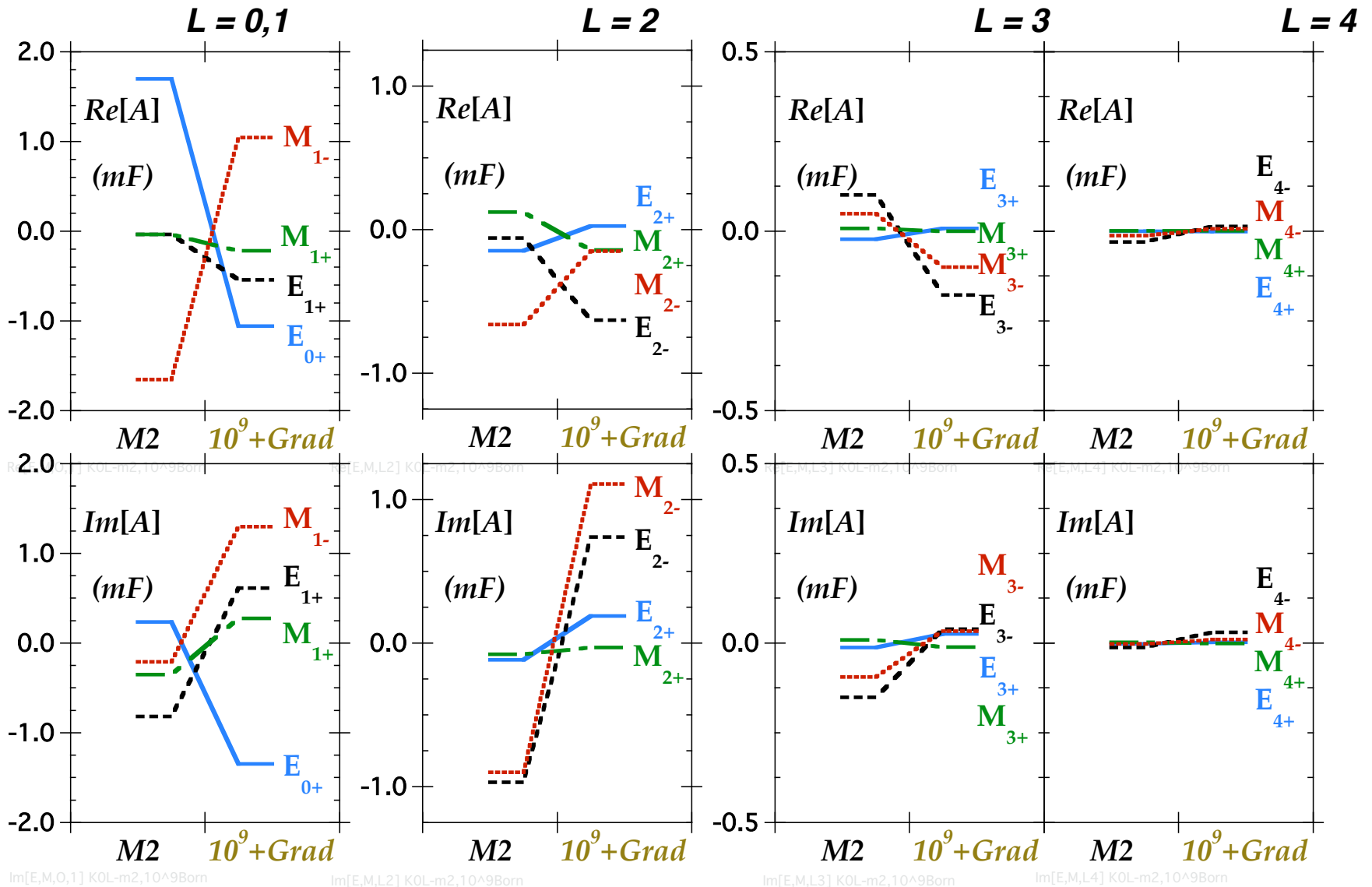


EBAC M2 vs Born +  $10^9$  MC + gradient search

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$W = 1.95$  GeV

new soln = M2 x  $e^{i(136.04 \text{ deg})}$



Im[E,M,0,1] KOL-m2,10^9Born

Im[E,M,L2] KOL-m2,10^9Born

Im[E,M,L3] KOL-m2,10^9Born

Im[E,M,L4] KOL-m2,10^9Born



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- in general, fitted soln will be rotated from generating amplitude by some angle dependent on the statistical fluctuations in data

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c) fixing the common phase with high L Born :

- $S_{\beta\alpha} = 1 + 2i T_{\beta\alpha}$  , for reactions  $\alpha \rightarrow \beta$

- $$T_{\beta\alpha} = \sum_L T_{\beta\alpha}^L P_L \rightarrow T'_{\beta\alpha} = \sum_L e^{i\phi_{\beta\alpha}(w)} T_{\beta\alpha}^L P_L$$

- $$O_{exp} = \left| \sum c T_{\beta\alpha} \right|^2 \Leftrightarrow \left| \sum c e^{i\phi_{\beta\alpha}} T_{\beta\alpha} \right|^2$$

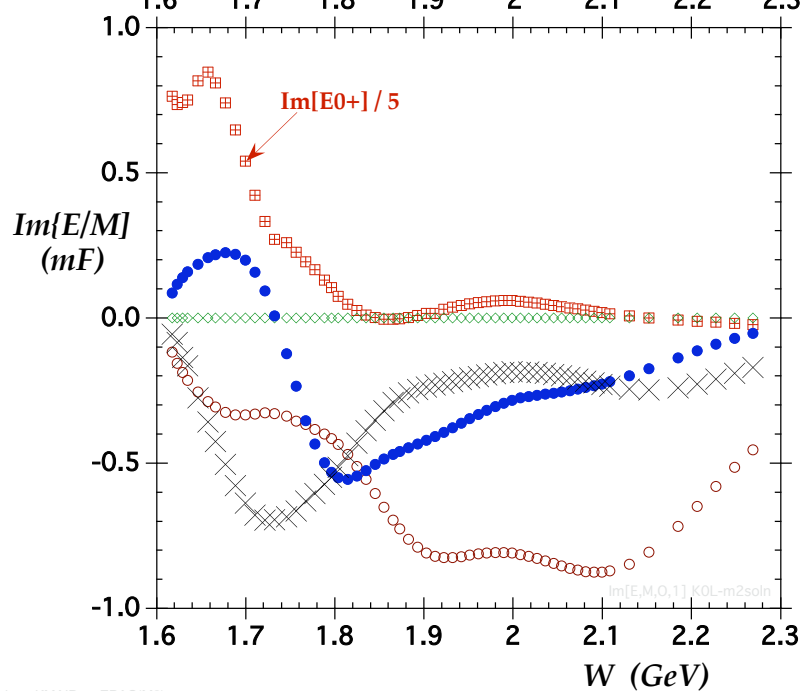
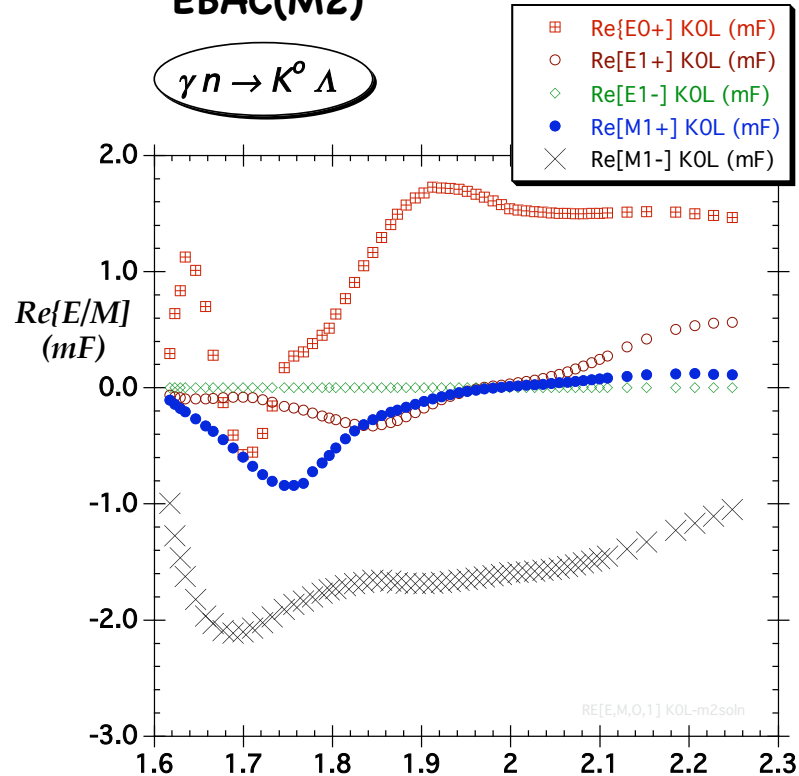
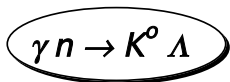
- vary multipoles for all L that are expected to differ from Born (eg. L=0-3); include higher L multipoles fixed to their Born values

⇒ uniquely finds the generating amplitude !

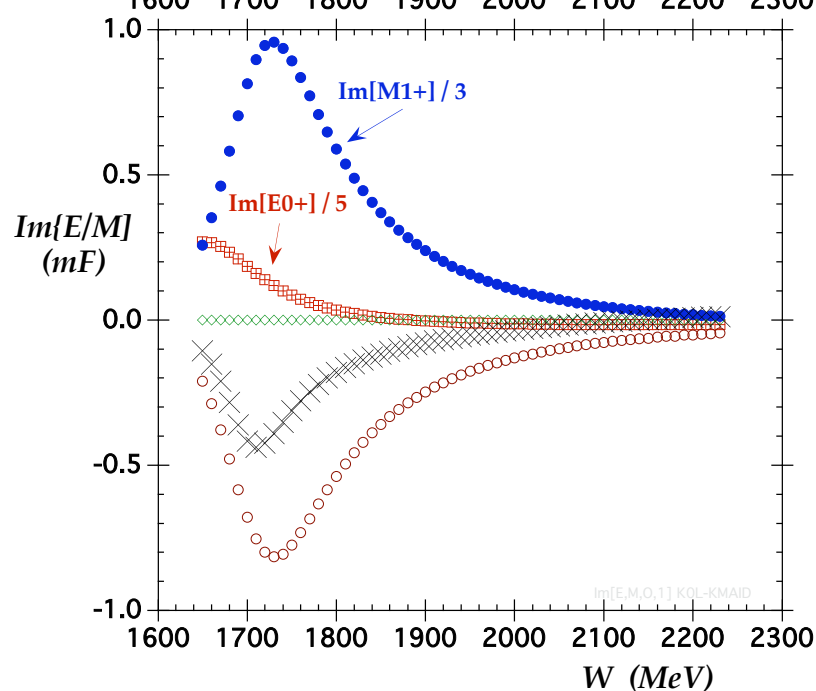
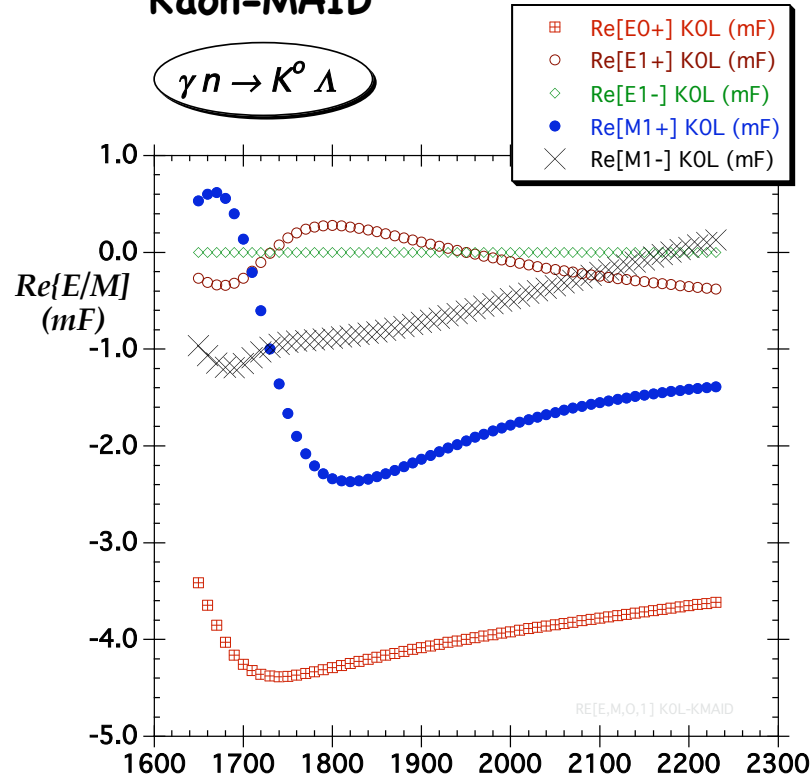
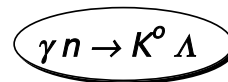
(but requires sufficient statistics for all non-Born multipoles)

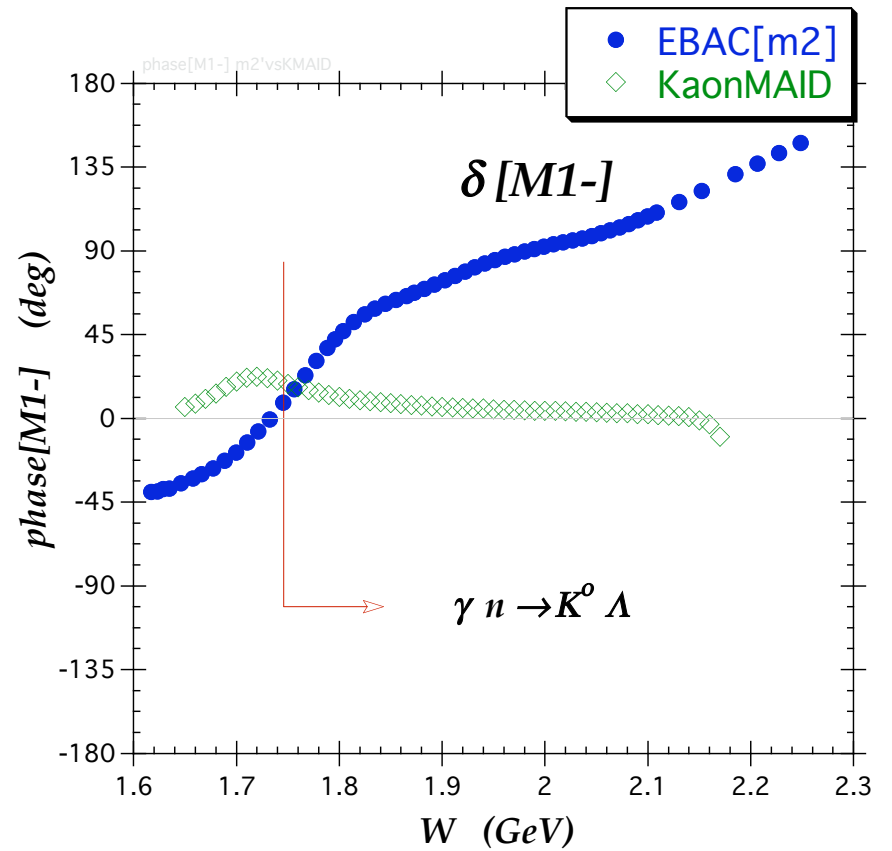
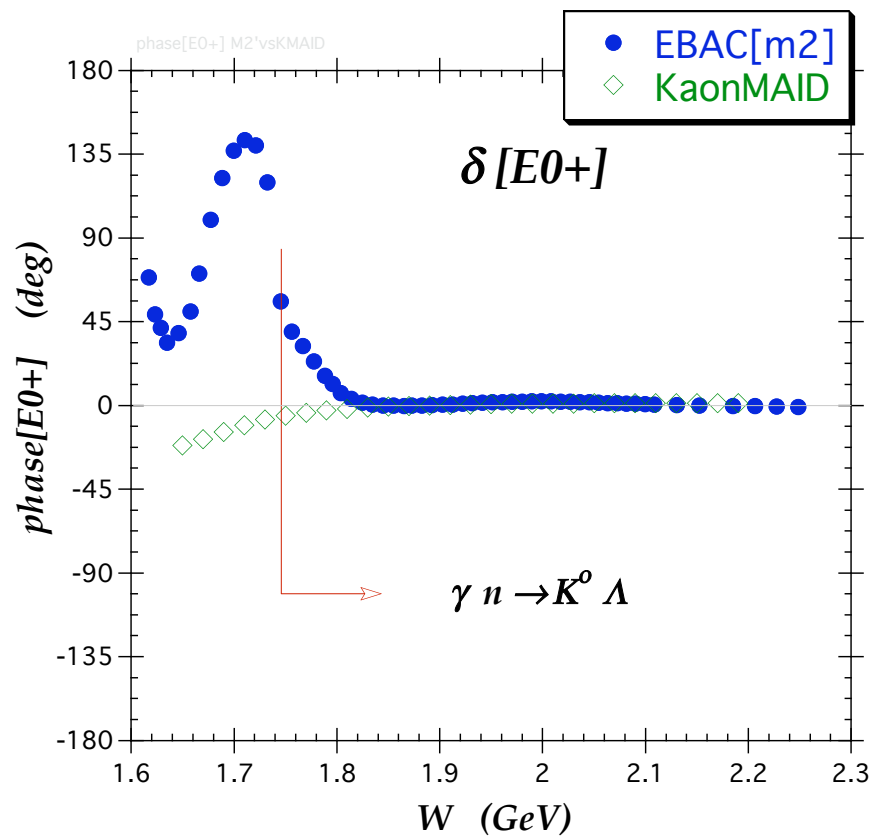
- realistic case: the data will run out of statistical accuracy before the multipoles run out of non-Born components;  
(eg. stat sufficient to fit  $L=0-2$ , but  $L=3-4$  may be non-Born; fixing  $L=5-8$  to Born DOES NOT determine the phase !)
- alternatively, lock phase of one multipole to something regarded as "known"  $\Leftrightarrow$  not a trivial choice for  $\gamma_N \rightarrow K\Lambda$

## EBAC(M2)



## Kaon-MAID



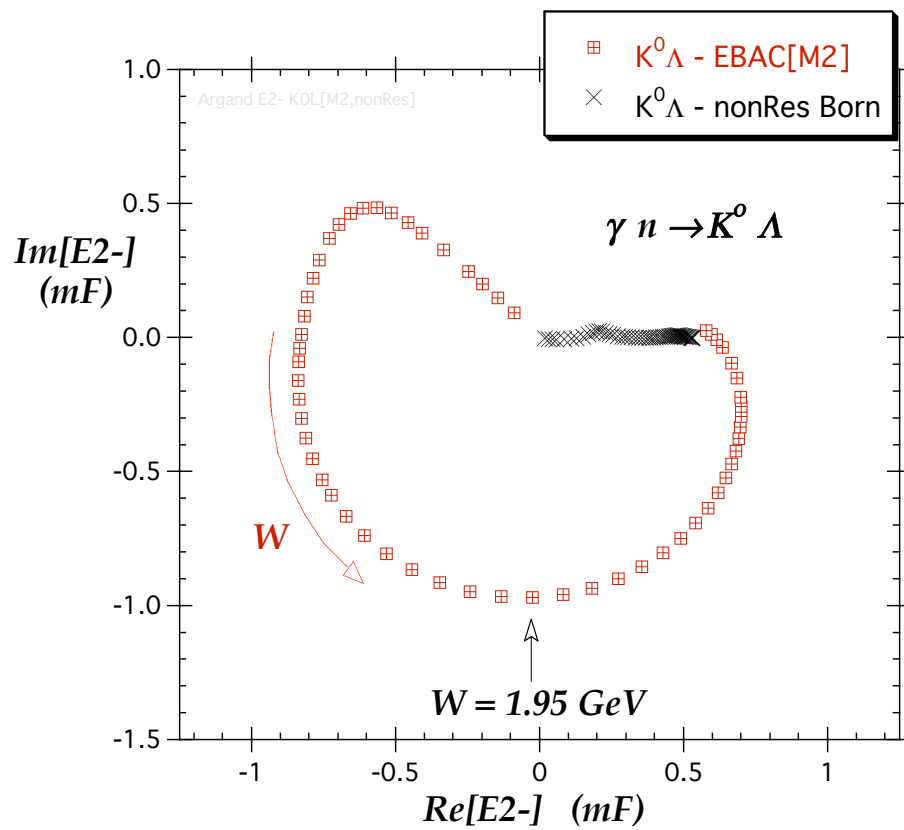


- realistic case: the data will run out of statistical accuracy before the multipoles run out of non-Born components;  
(eg. stat sufficient to fit  $L=0-2$ , but  $L=3-4$  may be non-Born; fixing  $L=5-8$  to Born DOES NOT determine the phase !)
- alternatively, lock phase of one multipole to something regarded as “known”  $\Leftrightarrow$  not a trivial choice for  $\gamma N \rightarrow K\Lambda$

## (2) Realistic case:

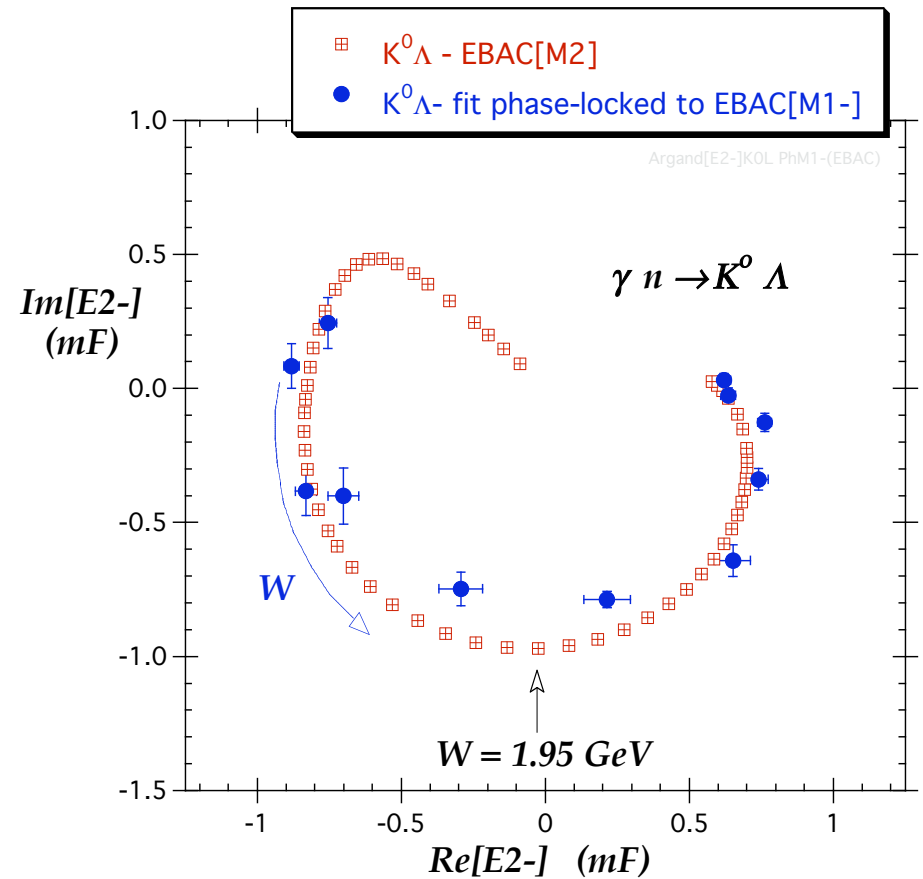
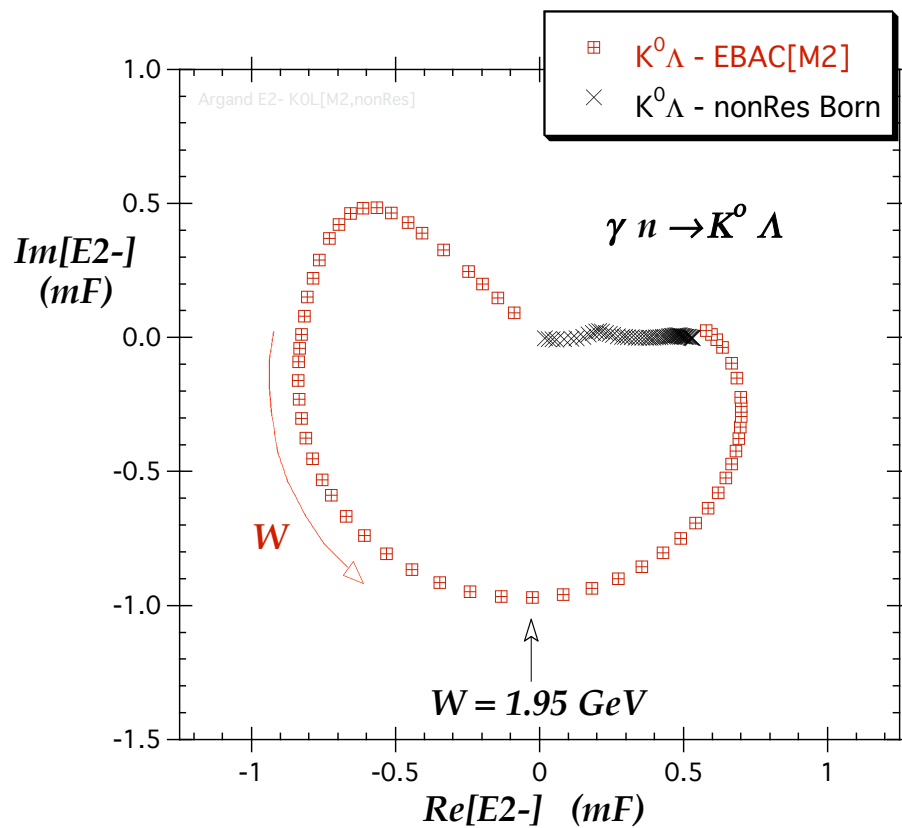
- using EBAC(M2) multipoles  $L=0-4$ , generate observables for energies of JLab experiments with expected angular coverage; Gaussian smear predictions with the expected statistical widths.
- start multipole search at non-resonant Born values;
- lock phase of  $M(1-)$ , or  $E(0+)$ , to EBAC(M2) values;
- $10^7$  Monte Carlo sample of parameter space for  $L=0-3$ ; Gradient search whenever  $\chi^2$  is within  $10^4$  of best value;

# E2- multipole of EBAC(m2) with $D_{13}$ (1950) resonance



## E2- multipole of EBAC(M2) with $D_{13}$ (1950) resonance

fit to expected JLab data, with phase locked to M1- of EBAC(m2)

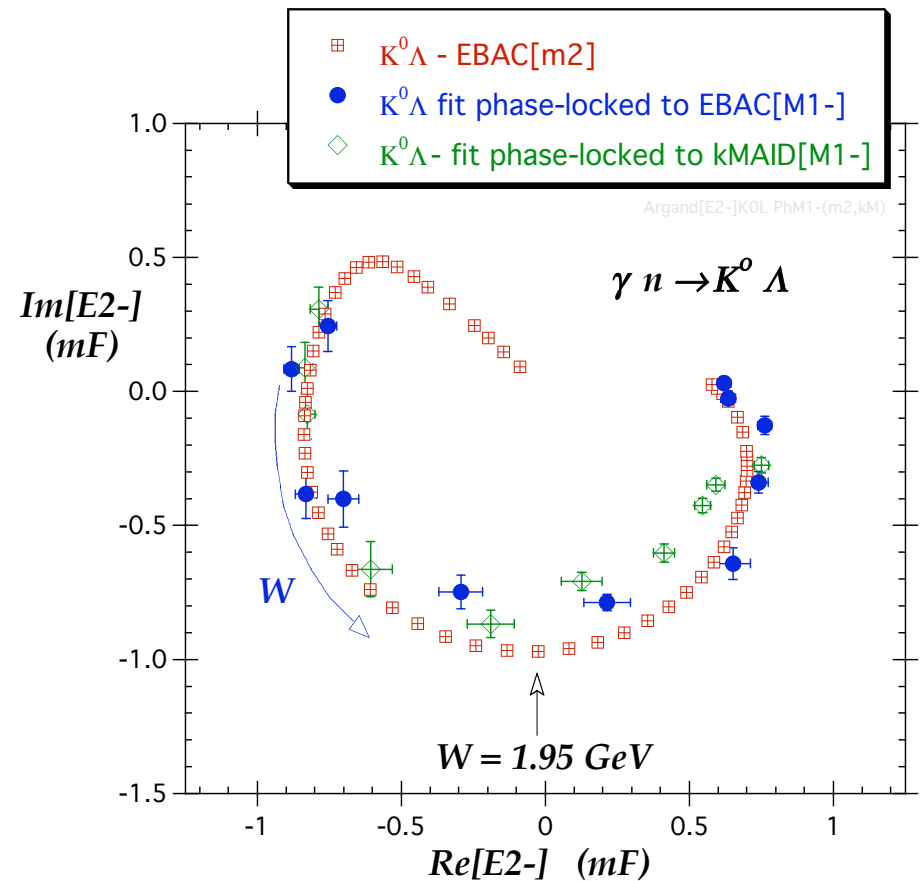
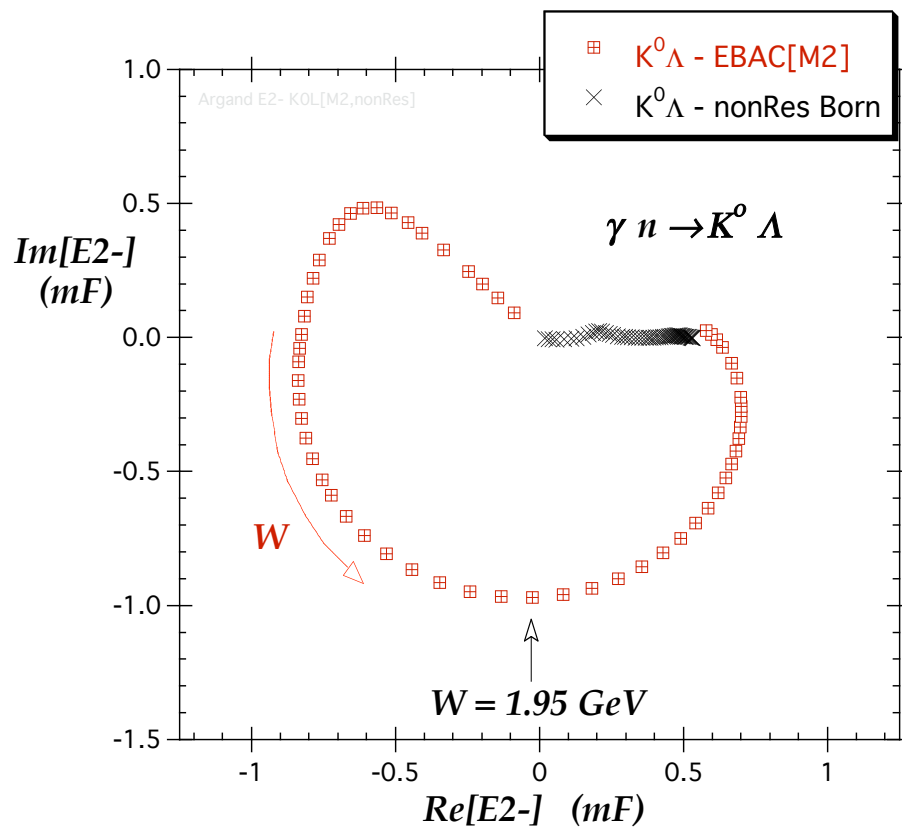




## E2- multipole of EBAC(M2) with $D_{13}$ (1950) resonance

fit to expected JLab data, with phase locked to M1- of EBAC(m2)

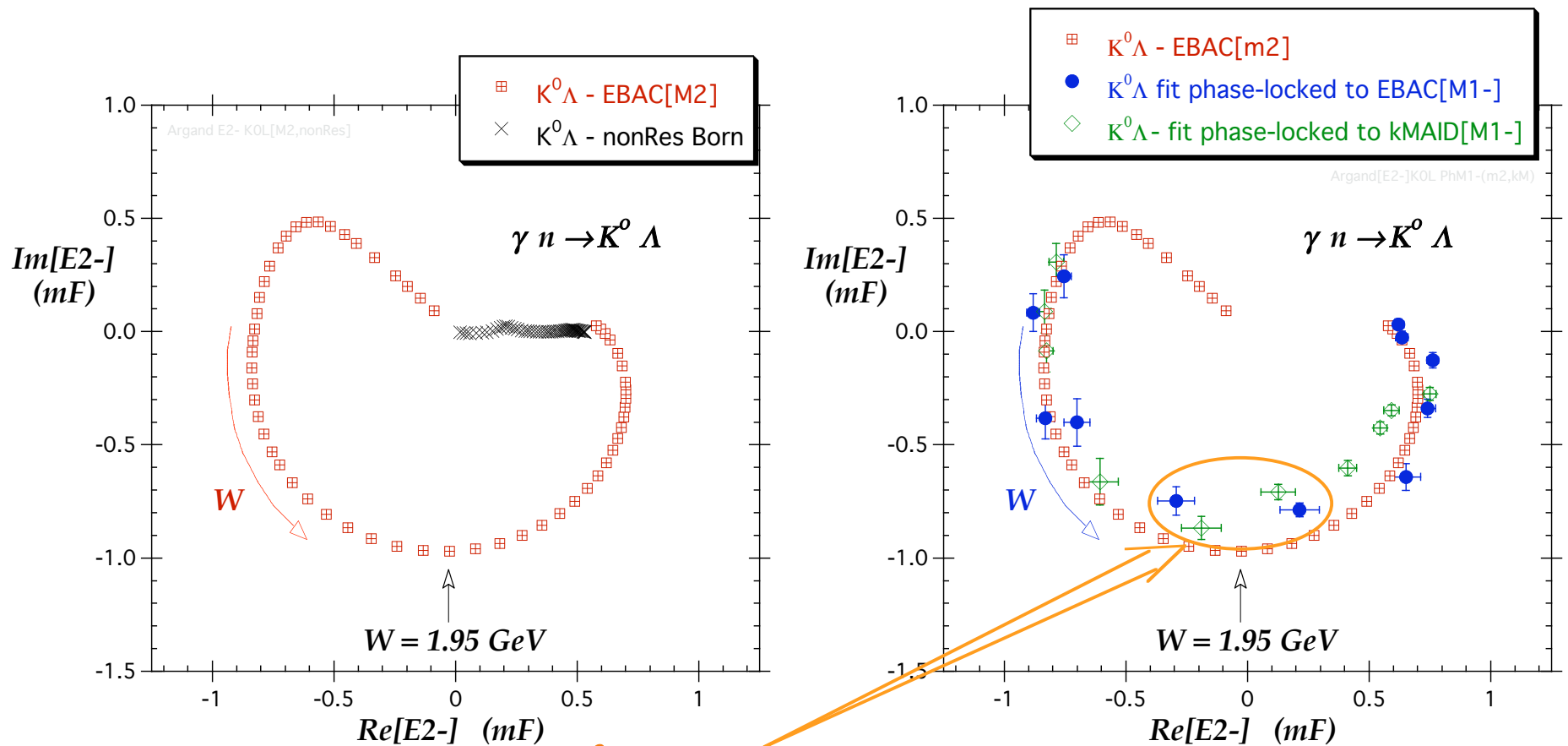
fit to expected JLab data, with phase locked to M1- of KaonMAID



## E2- multipole of EBAC(M2) with $D_{13}$ (1950) resonance

fit to expected JLab data, with phase locked to M1- of EBAC(m2)

fit to expected JLab data, with phase locked to M1- of KaonMAID



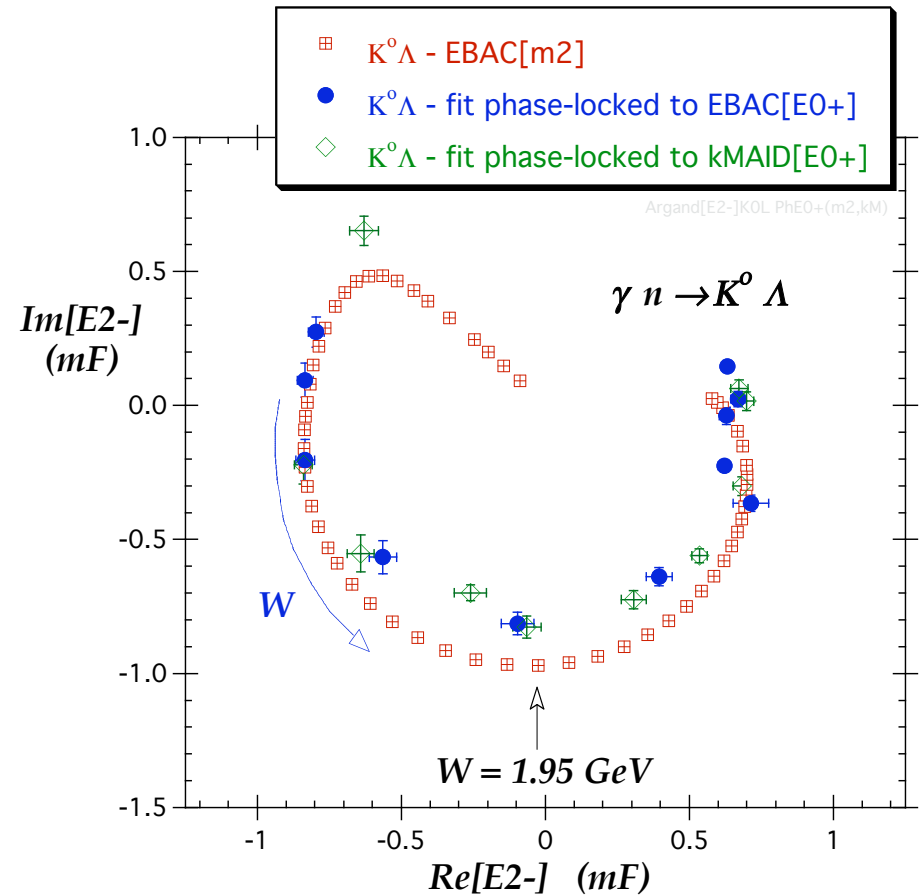
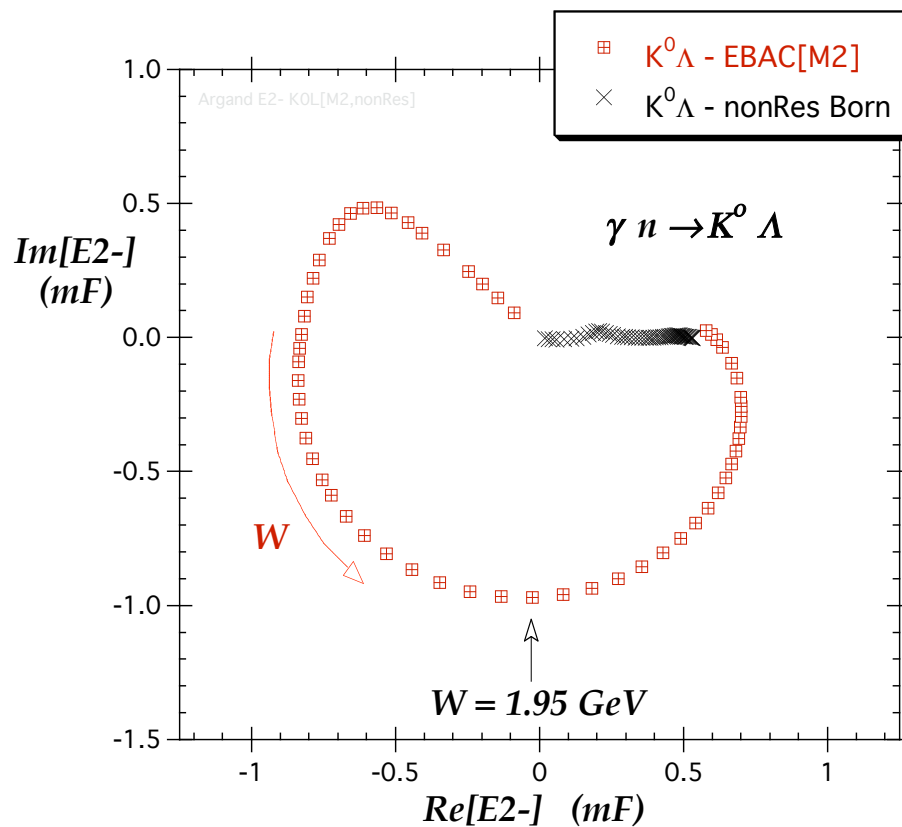
$\chi^2_{df}$  {fits to mock data}  $\sim 1.2$

$\sim 15\%$  less than  $\chi^2_{df}$  {mock data to generating Amp}

## E2- multipole of EBAC(M2) with $D_{13}$ (1950) resonance

fit to expected JLab data, with phase locked to E0+ of EBAC(m2)

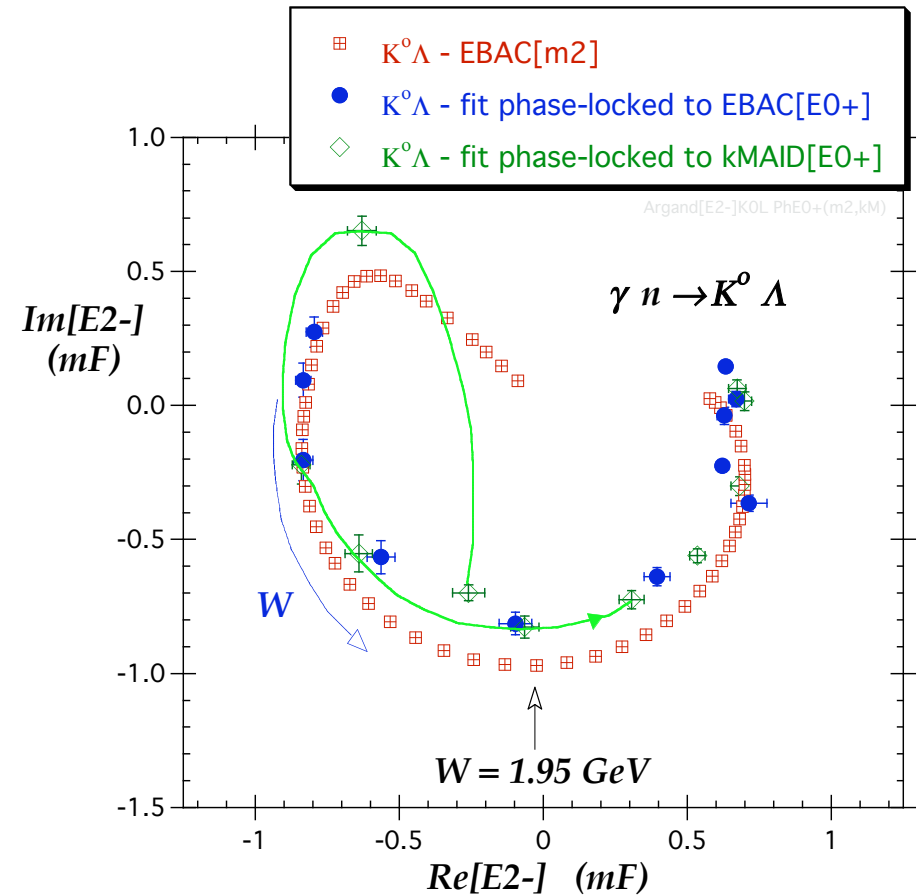
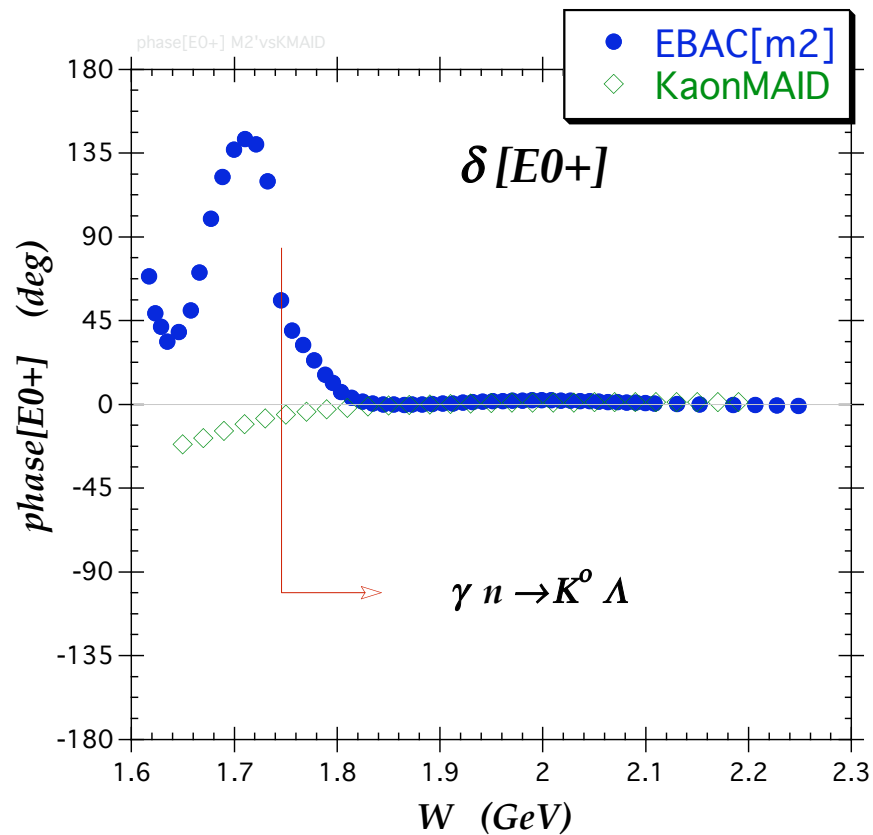
fit to expected JLab data, with phase locked to E0+ of KaonMAID



## E2- multipole of EBAC(M2) with $D_{13}$ (1950) resonance

fit to expected JLab data, with phase locked to E0+ of EBAC(m2)

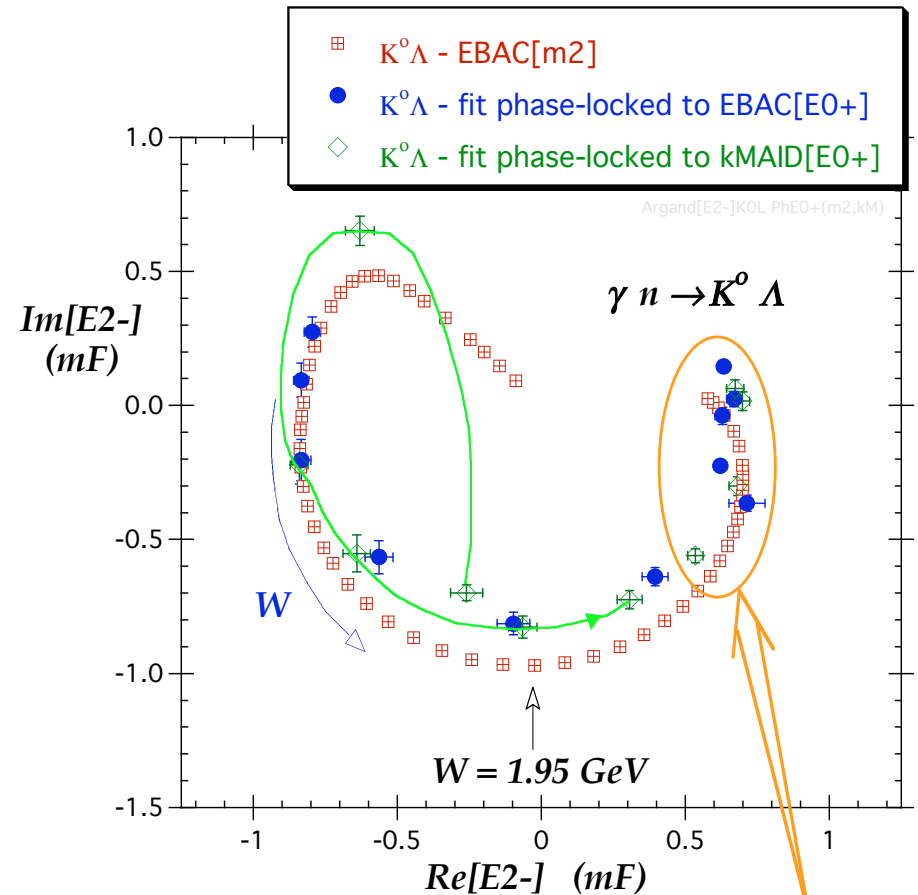
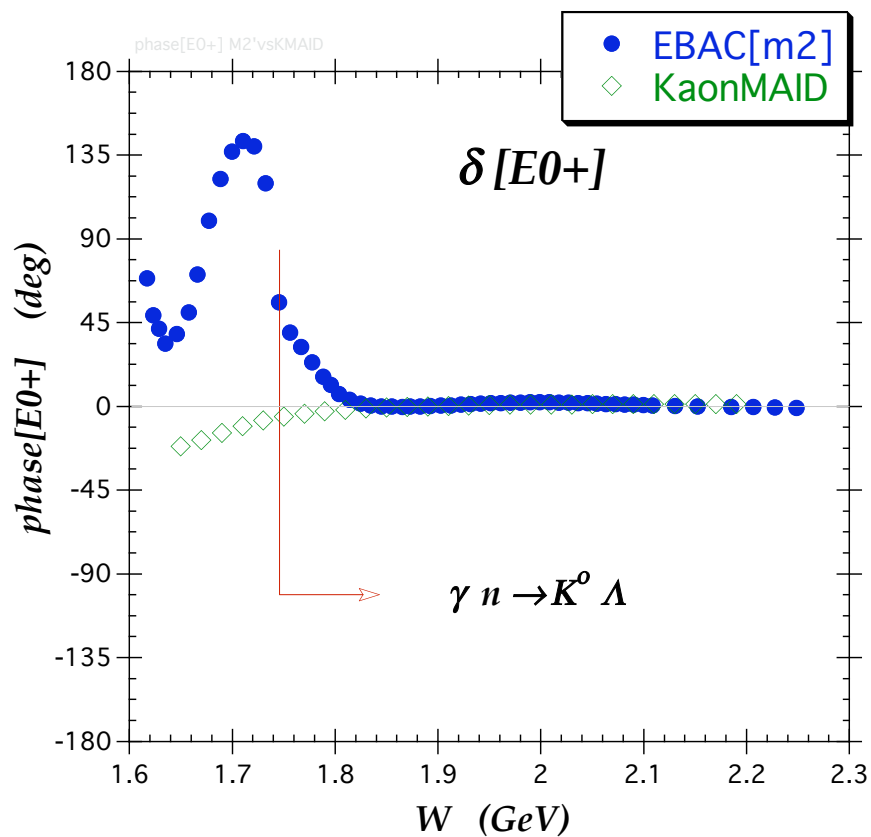
fit to expected JLab data, with phase locked to E0+ of KaonMAID



## E2- multipole of EBAC(M2) with $D_{13}$ (1950) resonance

fit to expected JLab data, with phase locked to E0+ of EBAC(m2)

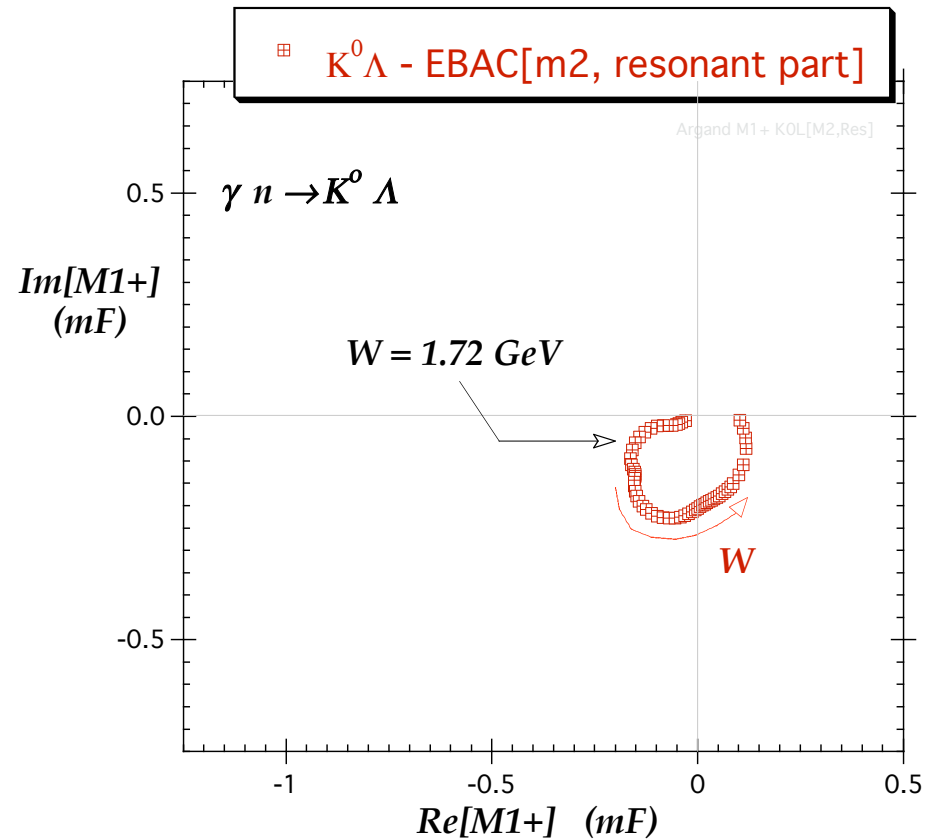
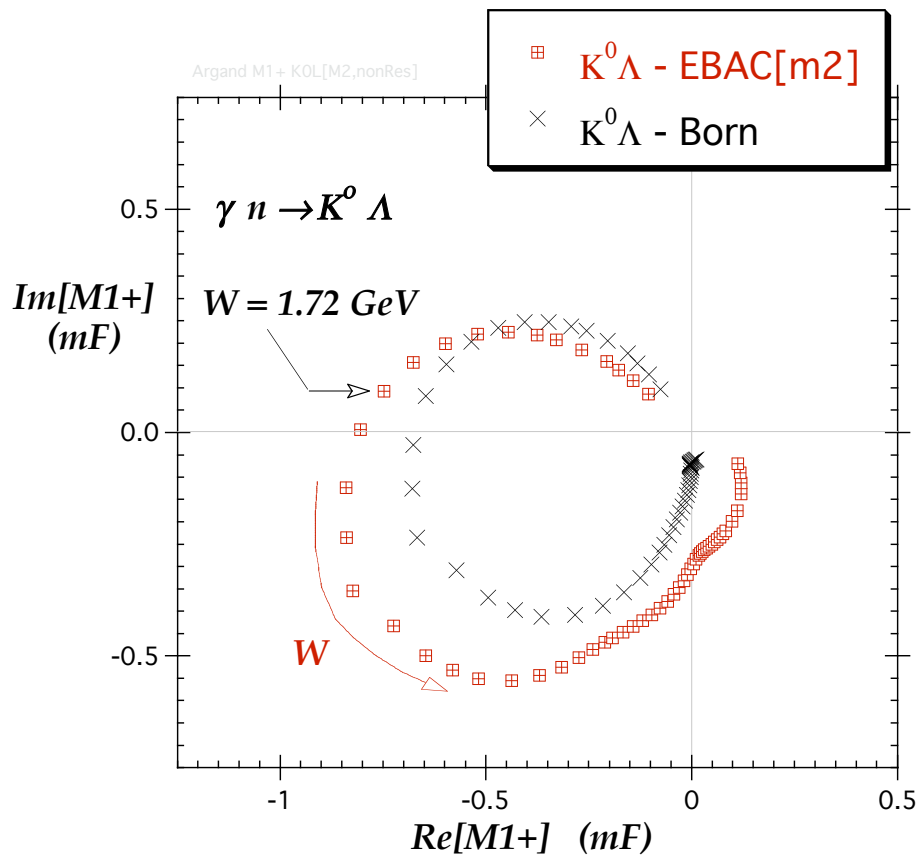
fit to expected JLab data, with phase locked to E0+ of KaonMAID



$\chi^2_{df}$  {fits to mock data} ~ 2-4  
 $\rightarrow \chi^2$  valleys very narrow !

# M1+ multipole of EBAC(M2) with $P_{13}$ (1720) resonance

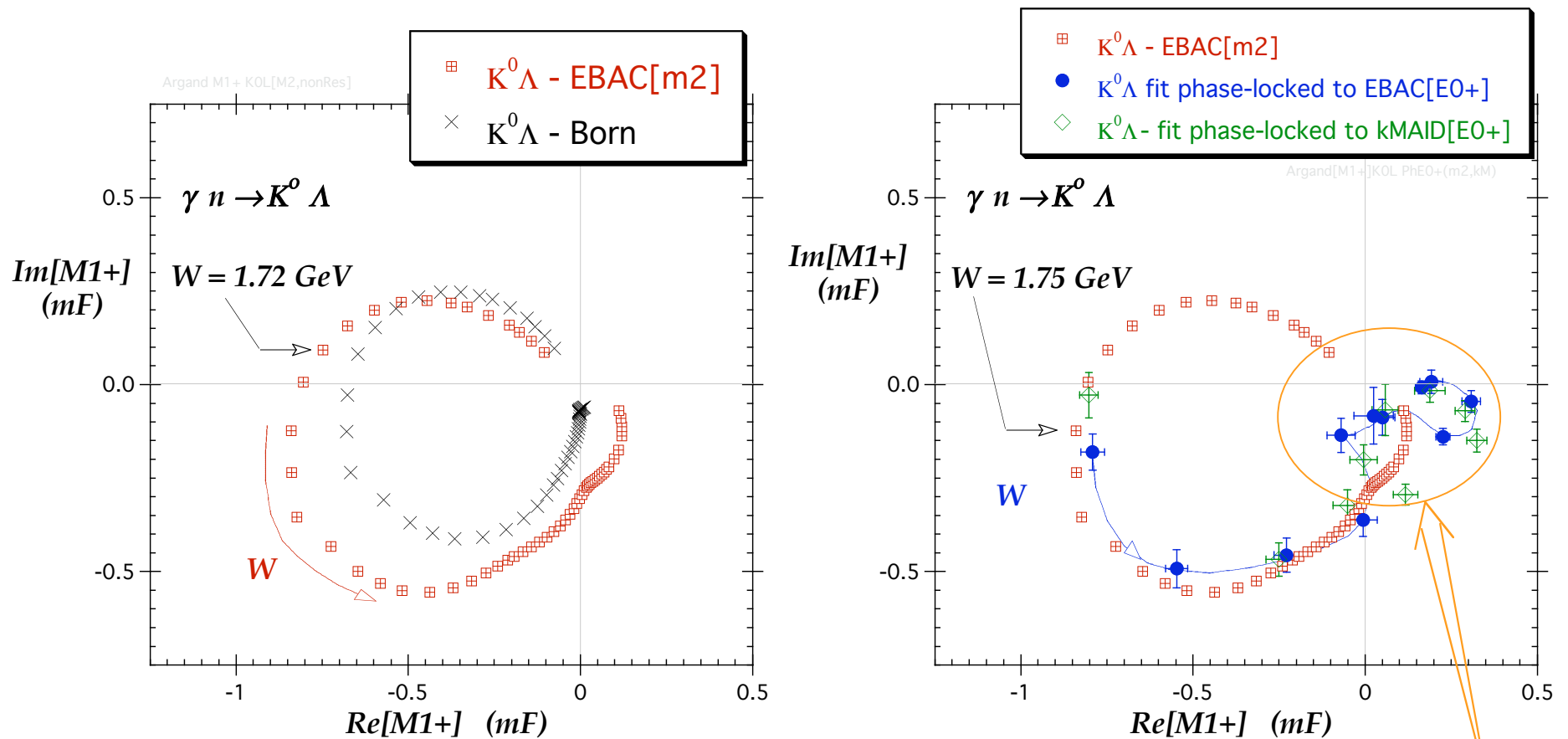
- \*\*\*\*  $P_{13}$  with small resonant part and large background;
- EBAC PRC73 (06); not seen in latest EBAC analysis (09)



# M1+ multipole of EBAC(M2) with $P_{13}$ (1720) resonance

fit to expected JLab data, with phase locked to E0+ of EBAC(m2)

fit to expected JLab data, with phase locked to E0+ of KaonMAID



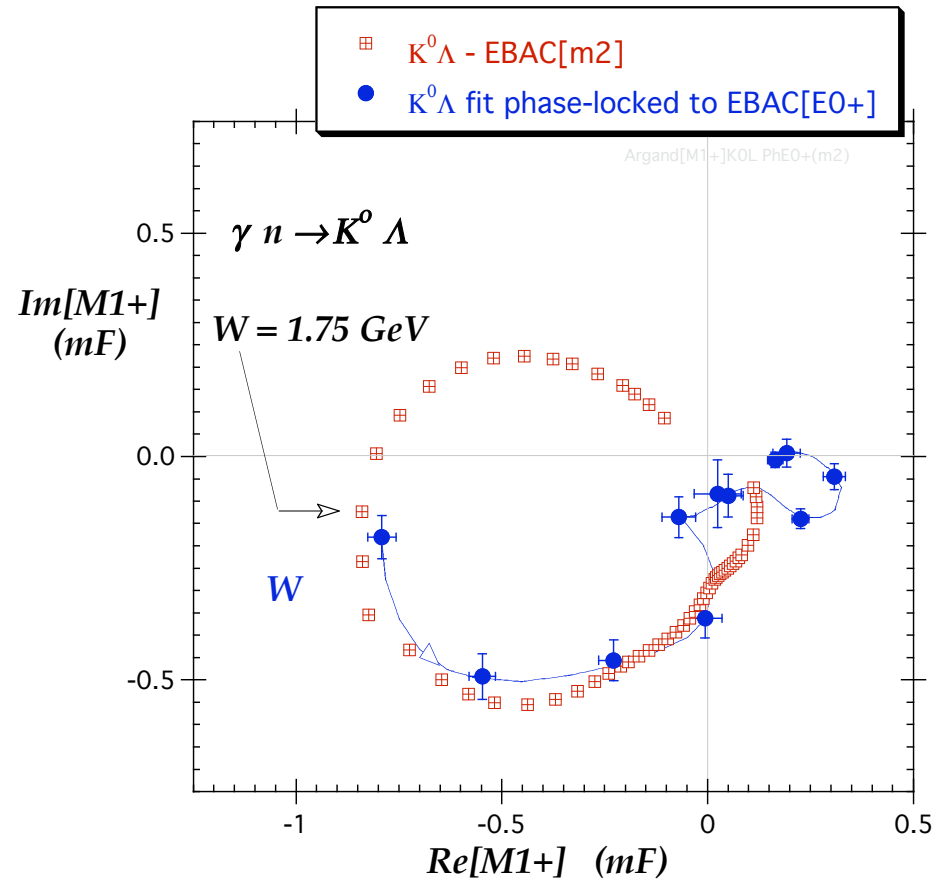
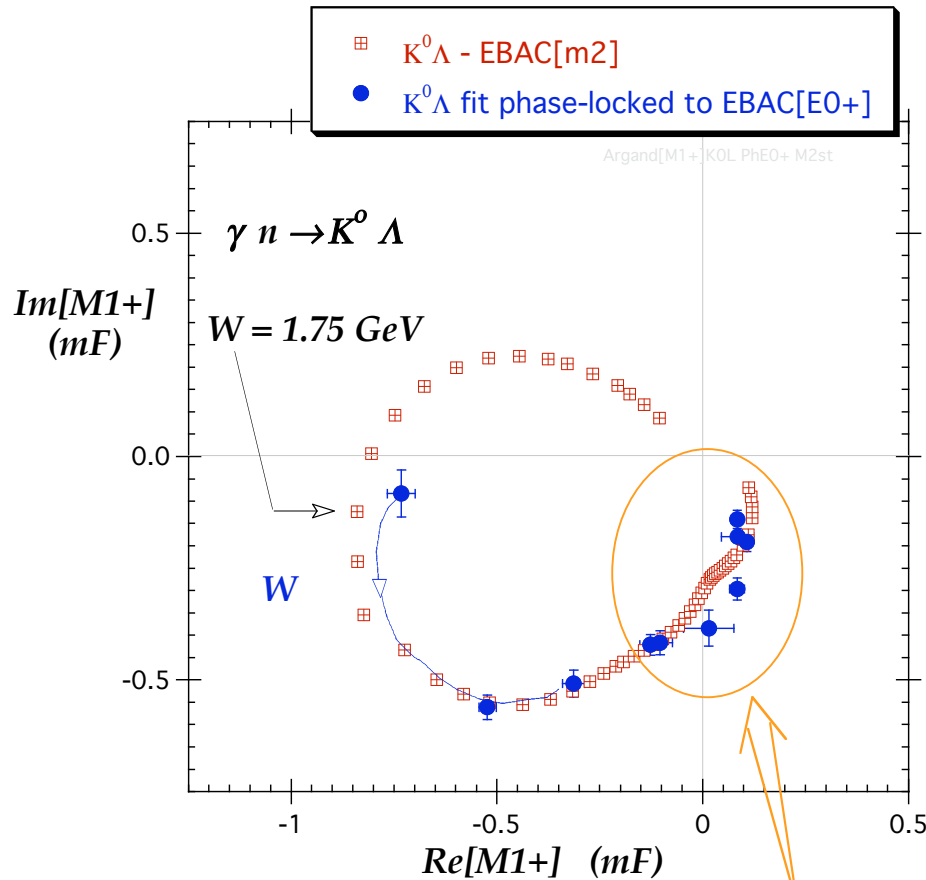
$\chi^2_{df} \sim 2-4$  for  $W > 2050$

# M1+ multipole of EBAC(M2) with $P_{13}$ (1720) resonance

fit to expected JLab data, with phase locked to E0+ of EBAC(m2)

- start at EBAC(m2)
- Gradient search

- start at Born
- $10^7$  Monte Carlo + Gradient



$$\chi^2_{df} \sim 1$$

small errors  $\rightarrow$  very narrow minima

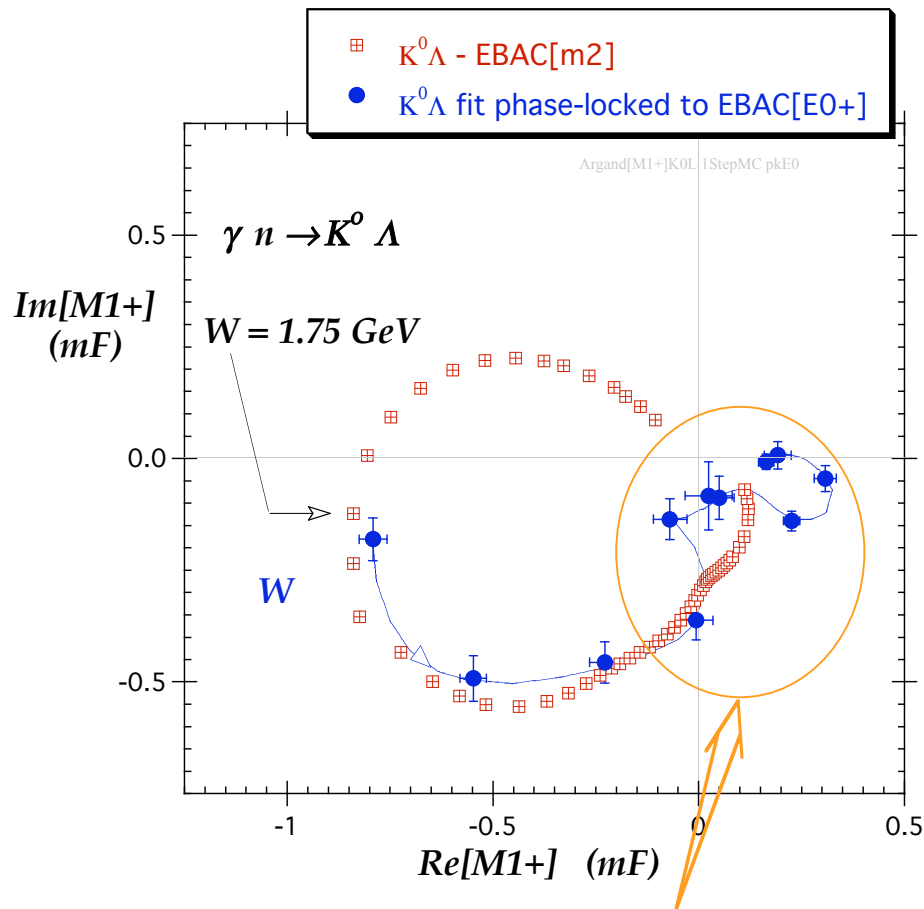


# M1+ multipole of EBAC(M2) with $P_{13}$ (1720) resonance

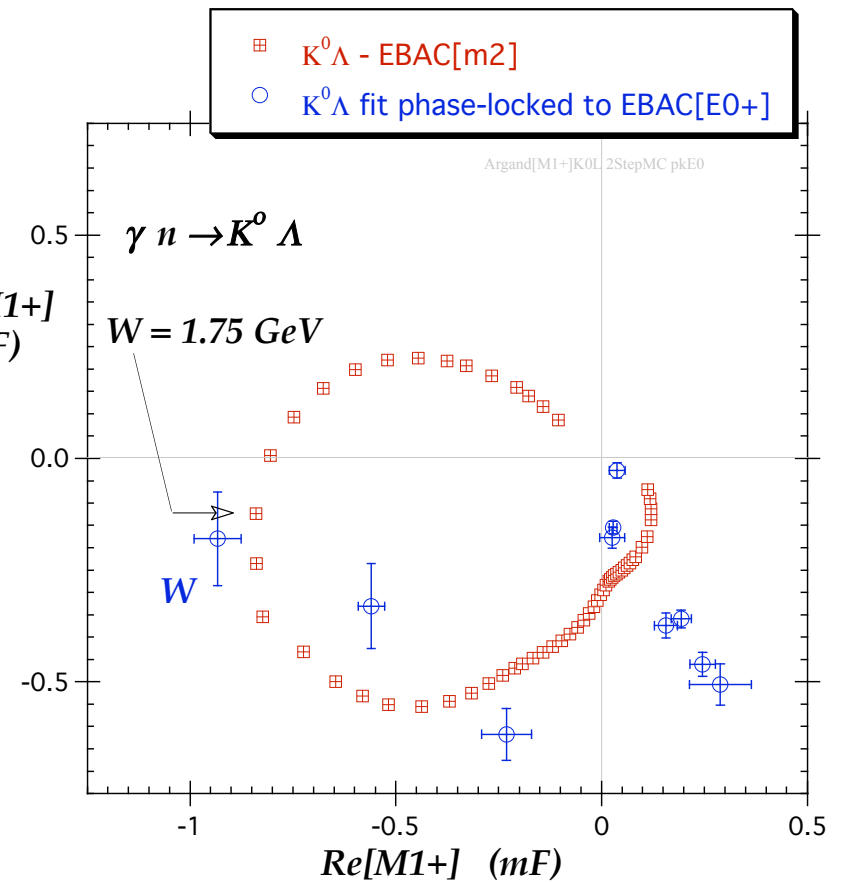
fit to expected JLab data, with phase locked to E0+ of EBAC(m2)

(1) start at Born(L=0-4); vary L=0-3;  
-  $10^7$  Monte Carlo + wide Gradient

(2) start at soln(1); vary L=0-4 ;  
-  $10^7$  Monte Carlo + narrow Gradient



$\chi^2_{df} \sim 2-4 \text{ for } W > 2050$



$\chi^2_{df} \sim 1 \text{ everywhere}$

## Coupled-channel constraints on the phase:

- $S_{\beta\alpha} = 1 + 2i T_{\beta\alpha}$  , for reactions  $\alpha \rightarrow \beta$
- $O_{exp} = \left| \sum c T_{\beta\alpha} \right|^2 \Leftrightarrow \left| \sum c e^{i\phi_{\beta\alpha}} T_{\beta\alpha} \right|^2$
- $S^+ S = 1 \Rightarrow i \left[ T_{\beta\alpha} - T_{\beta\alpha}^+ \right] = \sum_n T_{\beta n}^+ T_{n\alpha}$
- phase rotation  $\Rightarrow i \left[ T'_{\beta\alpha} - T'^+_{\beta\alpha} \right] = \sum_n T'^+_{\beta n} T'_{n\alpha}$   
 but  $e^{i\phi_{\beta\alpha}} i \left[ T_{\beta\alpha} - T_{\beta\alpha}^+ \right] \neq \sum_n e^{-i(\phi_{\beta n} - \phi_{n\alpha})} T_{\beta n}^+ T_{n\alpha}$

With coupled channels, arbitrary phase may leave  $O_{exp}$  invariant, but this does not generally satisfy Unitarity. (Mark Paris, Alfred Svarc)

$\Rightarrow$  *complete* experiments in two channels could constrain phases  
 - obvious choice is  $\gamma N \rightarrow \pi N$ ; challenge is recoil polarization !

## Partial Summary - of a work in progress

Complete sets of spin-observables will soon be available for  $\gamma+N \rightarrow K\Lambda$   
(and possibly  $\gamma n \rightarrow \pi^- p$ ) from g9 and g14

Prospects for directly fitting out the full (Res+Bgk+cc+...) multipoles:

- 16 observables X N angles fitted to 24 parameters (L=0-3) at each W;  $\chi^2$  space has a large number of valleys (even when the Chiang-Tabakin requirements are met)
  - in principle, phase can be constrained by one of the following:
    - the high L Born limit  
(which requires sufficient statistics in ALL non-Born partial waves),
    - by CC-Unitarity (which requires *complete* data in at least 2 channels),
    - or by locking phase to that of one partial wave  
(if models can agree on one with minimal uncertainty - eg. E0+ ?)
- from a combination of Monte Carlo sampling and Gradient searches, with the phase of one multipole determined, it will be possible to deduce the full amplitude, which will reveal the resonant structure in the multipoles

## **Caviats:**

- fits to the current round of experiments will probably be limited to L=0-3; fits to L=4 improve  $\chi^2$  but produce meaningless results  
⇒ such analyses should provide reliable L=0-2 multipoles.
- below W=2000,  $\chi^2$ {to "smeared data"} <  $\chi^2$ {"data" to unsmeared pts};  
⇒ "better" solutions than the generating amplitude,  
but still reasonably close;  
  
⇒ statistical errors, coupled with many local minima, lead to systematic uncertainties in multipoles; statistics could help and some observables may be more useful than others - still to come
- for W > 2050, present  $\chi^2_{df} > 2$  and valleys are very narrow;  
which will certainly complicate searches for potentially *new* states

## Next steps:

(a) expand MC search:

MC iterations	net samples/parameter
$10^7$	2
$10^9$	2.4
$10^{11}$	3
$10^{24}$	10

(b) test effects of systematic errors on data

⇔ will lead to systematic uncertainties in deduced multipoles

- asymmetry ratios can cancel many exp. systematics,  
BUT, observables no longer bi-linear combinations of multipoles  
⇒ greatly increased number of local minima
- potentially larger in  $\gamma p \rightarrow K^+ \Lambda$   
since BR cannot be measured simultaneously with BT and TR
- more to come

Extras