Baryon Spectroscopy using the CLAS Spectrometer and the Frozen Spin Target (FROST) at Jefferson Laboratory

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Nuclear Physics Seminar
April 09, 2010

Outline

1. Introduction
   - Baryon Spectroscopy
   - Polarization Observable
2. FROST Experiment
   - The CLAS at JLab
   - The FRozen-Spin Target (FROST)
   - The FROST-g9a run Period
3. Event Selection
   - The particle identification
   - The dilution factor
   - The beam and target polarization
4. The Preliminary Results

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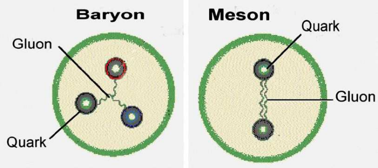
What are hadrons?

Hadrons are composed of quarks bound by the strong interaction.

- Baryon: qqq
- Meson: $q\bar{q}$



Quantum Chromodynamics (QCD)

- The theory of how quarks and gluons interact with themselves and each other

➜The study of the properties of baryon resonances

➜The N^* Program

One of the Goals of the N^* Program ...

Search for *missing* or yet unobserved resonances

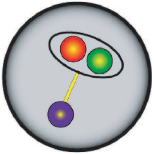
Quark models predict many more baryons than have been observed

	****	***	**	*
N Spectrum	11	3	6	2
Δ Spectrum	7	3	6	6

➜according to PDG
(Phys. Lett. B **667**, 1 (2008))
➜little known
(many open questions left)

Possible solutions:

1. Quark-diquark structure



one of the
internal degrees
of freedom
is frozen

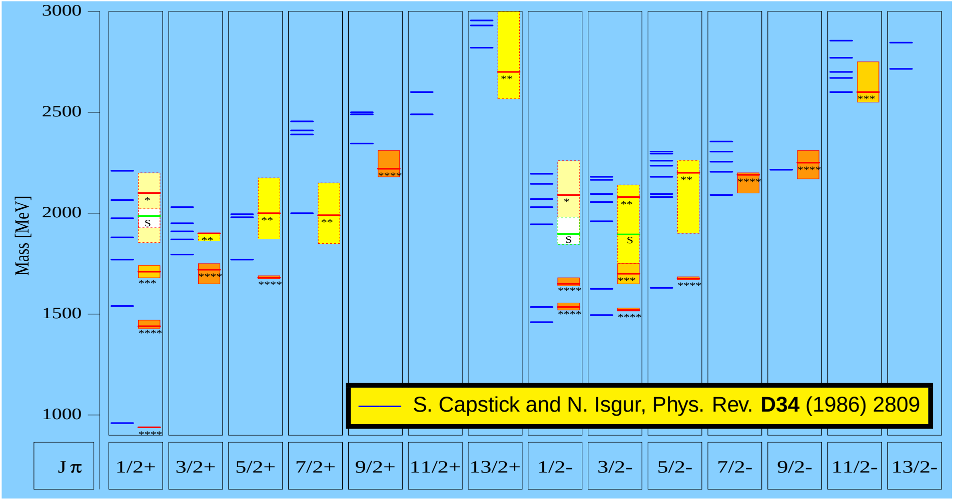
2. Have not been observed, yet

Nearly all existing data result from
πN scattering experiments

➜ If the missing resonances did not couple to Nπ, they would not have been discovered!!

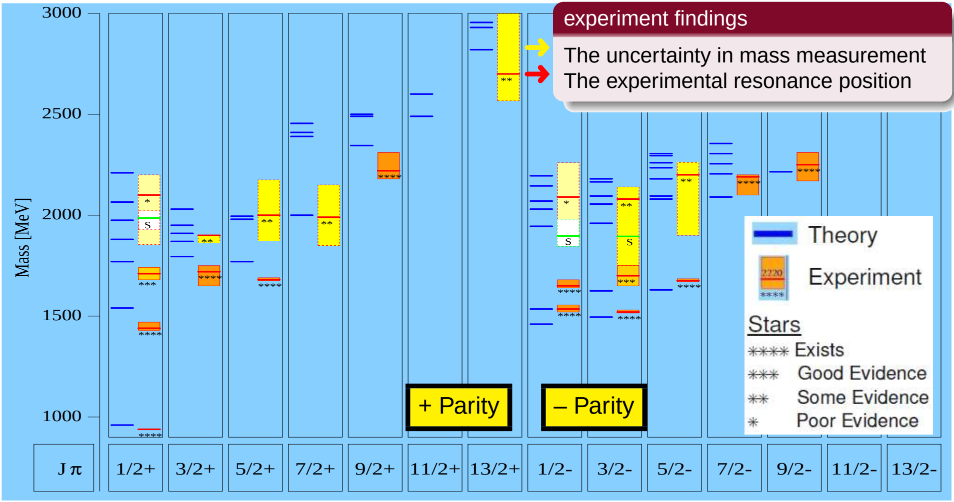
The excited states of the nucleon

Constituent quark models: Gluon-exchange model



The excited states of the nucleon

Constituent quark models: N^* resonances (Isospin $\frac{1}{2}$)



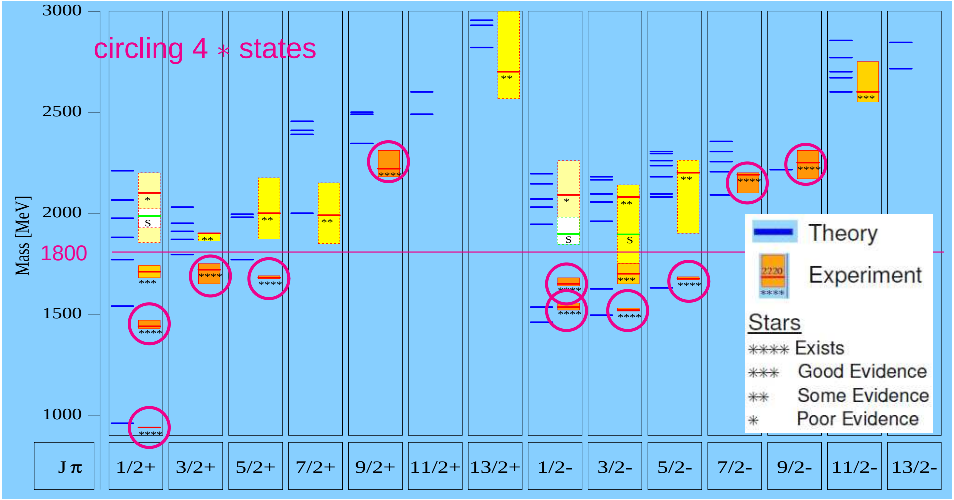
experiment findings

The uncertainty in mass measurement

The experimental resonance position

The excited states of the nucleon

Constituent quark models: N^* resonances (Isospin $\frac{1}{2}$)



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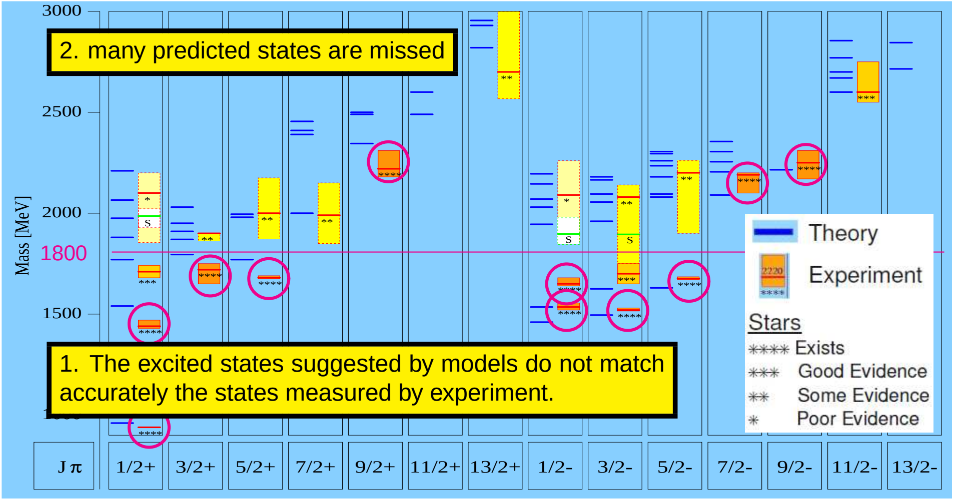


2. many predicted states are missed

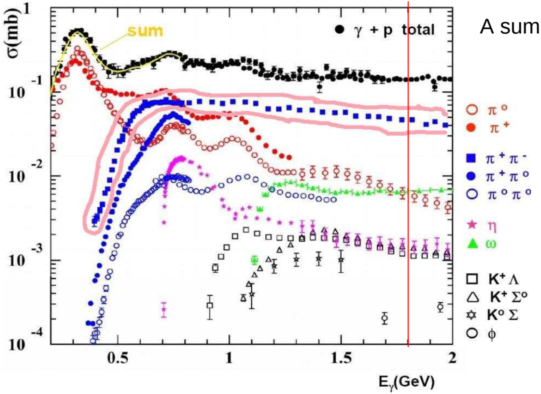
1. The excited states suggested by models do not match accurately the states measured by experiment.

- Theory
- Experiment

Stars

- **** Exists
- *** Good Evidence
- ** Some Evidence
- * Poor Evidence

The Motivation for the $\pi^+\pi^-$ photoproduction

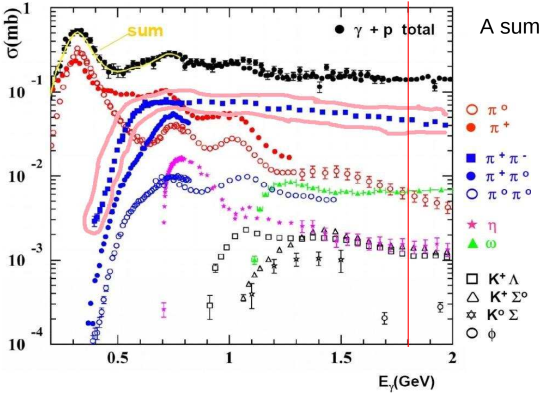


A summary of photoproduction cross section

$$\gamma p \rightarrow p\pi^+\pi^-$$

- The cross section of the $\pi^+\pi^-$ photoproduction dominates above W ≈ 1.8GeV

The Motivation for the $\pi^+\pi^-$ photoproduction



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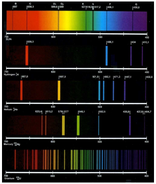

- The cross section of the $\pi^+\pi^-$ photoproduction dominates above W ≈ 1.8GeV
- The excited states are found as broadly overlapping resonances

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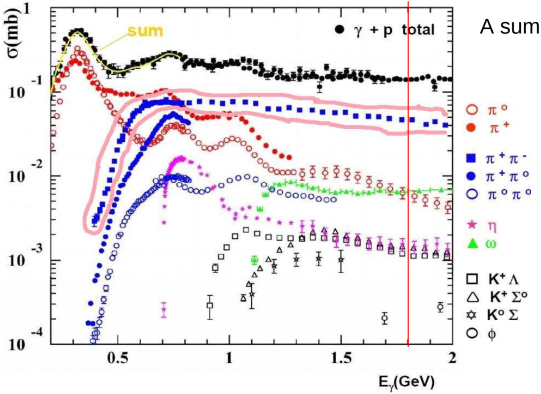

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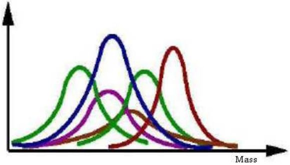

- The cross section of the $\pi^+\pi^-$ photoproduction dominates above W ≈ 1.8GeV
- The excited states are found as broadly overlapping resonances

→ The polarization observables can isolate single resonances from other interference terms

The differential cross section for $\gamma p \rightarrow p\pi^+\pi^-$

The differential cross section for $\gamma p \rightarrow p\pi^+\pi^-$

(without measuring the polarization of the recoiling nucleon)

$$\frac{\mathrm{d}\,\sigma}{\mathrm{d}\,\mathrm{x_i}} = \sigma_0 \left\{ \left(1 + \vec{\Lambda}_i \cdot \vec{\mathbf{P}}\right) + \delta_{\odot} \left(\mathbf{I}^{\odot} + \vec{\Lambda}_i \cdot \vec{\mathbf{P}}^{\odot}\right) + \delta_{l} \left[\sin 2\beta \left(\mathbf{I}^{\mathbf{s}} + \vec{\Lambda}_i \cdot \vec{\mathbf{P}}^{\mathbf{s}}\right) + \cos 2\beta \left(\mathbf{I}^{\mathbf{c}} + \vec{\Lambda}_i \cdot \vec{\mathbf{P}}^{\mathbf{c}}\right) \right] \right\}$$

- σ_0: The unpolarized cross section
- β: The angle between the direction of polarization and the x-axis
- $\delta_{\odot, l}$: The degree of polarizaton of the photon beam ⇒ $\delta_{\odot}$, and δ_l
- $\vec{\Lambda}_i$: The polarization of the initial nucleon ⇒ $(\Lambda_x, \Lambda_y, \Lambda_z)$
- $\mathbf{I}^{\odot,\mathbf{s},\mathbf{c}}$: The observable arising from use of polarized photons ⇒ $\mathbf{I}^{\odot}$, $\mathbf{I}^{\mathbf{s}}$, $\mathbf{I}^{\mathbf{c}}$
- $\vec{\mathbf{P}}$: The polarization observable ⇒ $(\mathbf{P_x}, \mathbf{P_y}, \mathbf{P_z})$ $(\mathbf{P}_x^{\odot}, \mathbf{P}_y^{\odot}, \mathbf{P}_z^{\odot})$ $(\mathbf{P}_x^{s}, \mathbf{P}_y^{s}, \mathbf{P}_z^{s})$ $(\mathbf{P_x^c}, \mathbf{P_y^c}, \mathbf{P_z^c})$

15 Observables

Polarization Observable

The differential cross section for $\gamma p \rightarrow p\pi^{+}\pi^{-}$

(without measuring the polarization of the recoiling nucleon)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\,\mathrm{x_i}} = \sigma_0 \{ (1 + \vec{\Lambda}_i \cdot \vec{\mathbf{P}}) + \delta_{\odot} (\mathbf{I}^{\odot} + \vec{\Lambda}_i \cdot \vec{\mathbf{P}}^{\odot}) \quad \text{15 Observables}$$

$$+ \delta_l [\sin 2\beta (\mathbf{I}^{\mathbf{s}} + \vec{\Lambda}_i \cdot \vec{\mathbf{P}}^{\mathbf{s}}) + \cos 2\beta (\mathbf{I}^{\mathbf{c}} + \vec{\Lambda}_i \cdot \vec{\mathbf{P}}^{\mathbf{c}})]\}$$

The circularly-polarized beam → $\delta_l = 0$

The longitudinally-polarized target → $\Lambda_x = \Lambda_y = 0$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\,\mathrm{x_i}} = \sigma_0 \{ (1 + \Lambda_z \cdot \mathbf{P_z}) + \delta_{\odot} (\mathbf{I}^{\odot} + \Lambda_z \cdot \enclose{circle}{\mathbf{P}_z^{\odot}})\} \quad \text{3 Observables}$$

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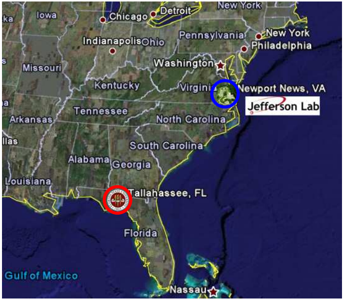
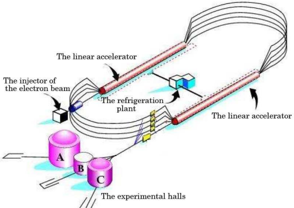
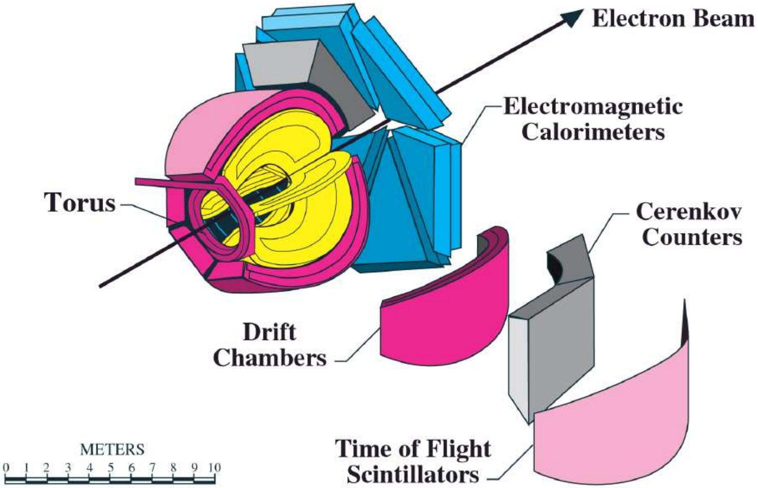
Jefferson Laboratory in Newport News, VA





The continuous electron beam accelerator facility (CEBAF) can deliver a continuous electron beam up to 6 GeV.

CEBAF Large Acceptance Spectrometer (CLAS)



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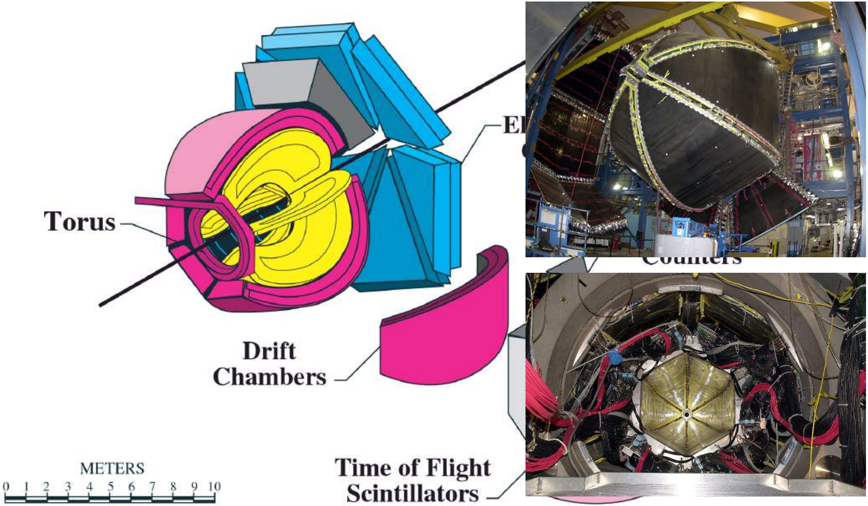
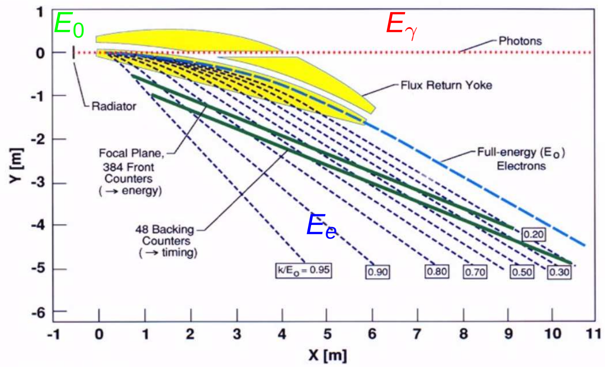
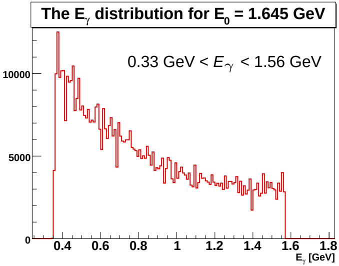


The tagging system at CLAS

JLAB Hall B bremsstrahlung photon tagger

- E_γ = 20-95 % of E_0
- E_γ up to ∼5.5 GeV




The E_γ distribution for E_0 = 1.645 GeV

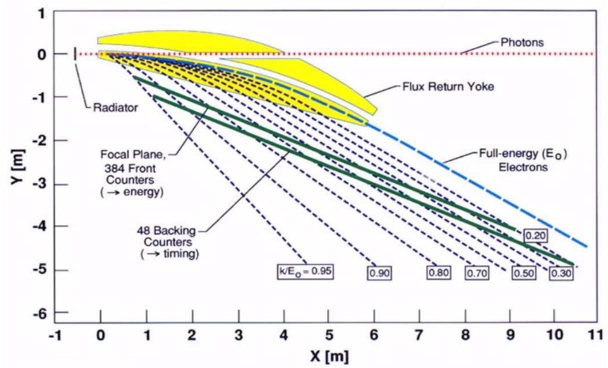
$$E_\gamma = E_0 - E_e$$

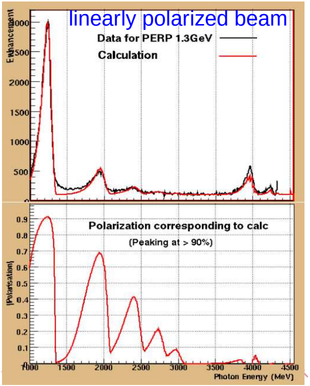
E_γ: The energy of the emitted photon
E_0: The energy of the incident electron
E_e: the energy of the outgoing electron

The tagging system at CLAS

JLAB Hall B bremsstrahlung photon tagger

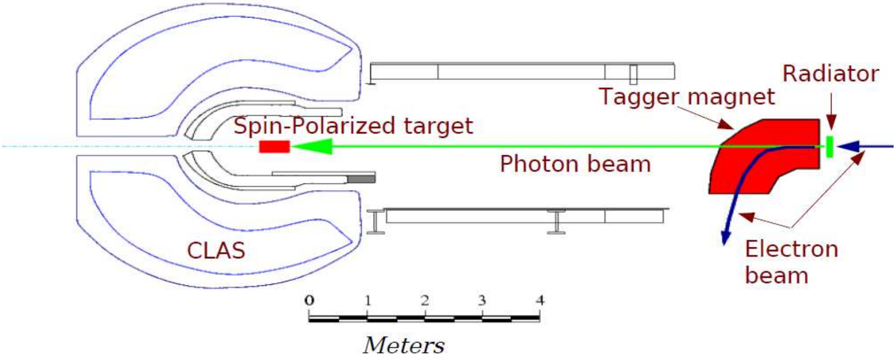
- Circular polarized photon beam: amorphous radiator; longitudinally polarized electron beam
- Linearly polarized photon beam: oriented diamod radiator; unpolarized electron beam



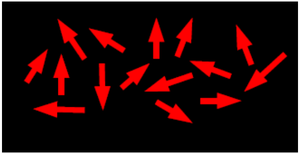


The things we need for the FROST experiment

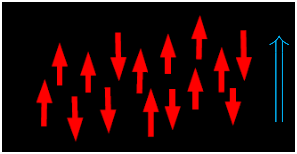
- The polarized photon beam - The tagging system at CLAS
- The polarized proton - The Frozen-Spin Target



The Basics of Polarization



In absence of a magnetic field,
a collection of spins is randomly oriented.



With the magnetic field,
the spins either parallel or anti-parallel to the field will be oriented

Polarization = excess of one orientation over the other

- Oscillating EM fields, produced by atomic vibration, tends to randomize (de-polarize) the spins.
- Strength of vibrations decreases at low temperature.

Polarization and Thermal Equilibrium

Any ensemble of atoms or nuclei with a magnetic moment can be polarized via the Zeeman interaction: $\vec{\mu} \cdot \vec{B}$

In general, the populations of the Zeeman levels (once equilibrium has been reached) will obey a Boltzmann distribution:

$$\frac{N(\uparrow)}{N(\downarrow)} = \mathrm{e}^{\frac{-2\,\vec{\mu}\cdot\vec{B}}{kT}} \qquad P_{\mathrm{te}} = \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)} = \tanh\left(\frac{\vec{\mu}\cdot\vec{B}}{kT}\right)$$

(T = Temperature, P_{te} = Thermal Equilibrium Polarization)

The polarization will approach thermal equilibrium with a characterisitc 1/e time constant t_1:

$$P(t) = P_{\mathrm{te}}\,(1 - \mathrm{e}^{-t/t_1})$$

"t_1: Spin-Lattice Relaxation Time"

A Simple Way to Polarize

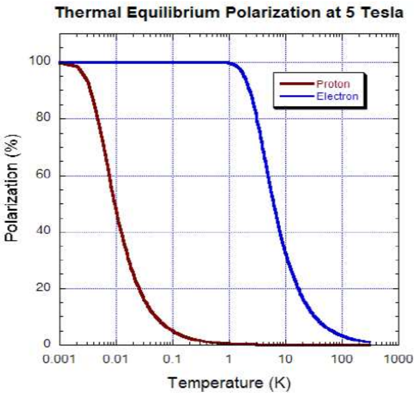
Brute Force Polarization

$$P_{\mathrm{te}} = \tanh\left(\frac{\vec{\mu} \cdot \vec{B}}{kT}\right)$$

To get high polarization

maximize B

minimize T



Disadvantages:

1. Requires very large magnet
2. Low temperatures require low luminosity
3. Polarization can take a very long time (protons slow, electrons fast)

A Better Way – **D**ynamic **N**uclear **P**olarization

(1) Use brute force to polarize free electrons in the target material.

(2) Use microwaves to “transfer” this polarization to nuclei.

Mutual electron-nucleus spin flips re-arranges the nuclear Zeeman populations to favor one spin state over the other.

For best results:

DNP is performed at B/T conditions where electrons t_1 is short (ms) and nuclear t_1 is long (minutes):

$$\text{JLab:}\ \begin{aligned} B &= 5\ \text{T} \\ T &= 1\ \text{K} \end{aligned}$$

Materials for DNP Targets

Choice of material dictated by:

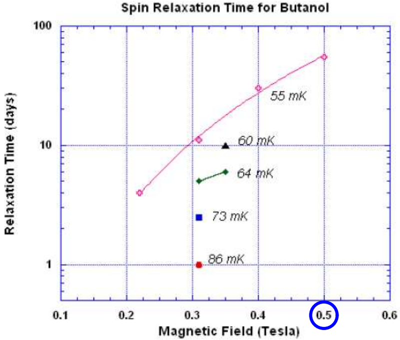
1. Maximum polarization
2. Resistance to ionizing radiation
3. Presence of unpolarized nuclei
4. Presence of unwanted, polarized nuclei

Compromise: Butanol (C_4H_9OH)

- Quality (dilution) factor:

$$f = \vec{N}/N_{\text{total}} = 10/74 \approx 0.13$$

The holding magnet for FROST : 0.5 T



Ch. Bradtke, phD Thesis, Univ. Bonn, 1999

The Frozen-Spin Target (FROST)

Operation is more complicated:

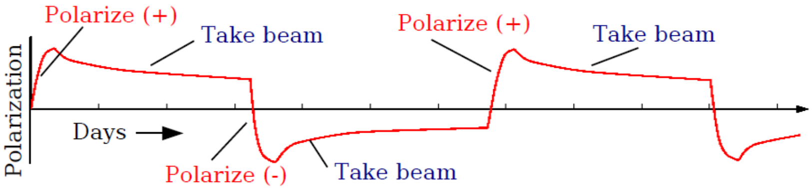
(1) Polarize target material via DNP at 5 T and 0.5 K (Polarizing Mode)

(2) After optimum polarization is obtained, turn off microwaves and 5 T magnet

(3) Use a 2^{nd} magnet ($\sim$0.5 T) and very low temperatures to “freeze” the polarization (Frozen Spin Mode)

(4) Polarization will decay very slowly with a time constant of several days

(5) After polarization decays to about 50 % of its initial value, go back to step 1



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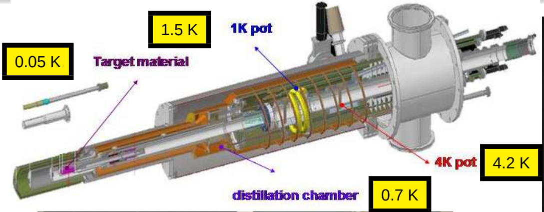

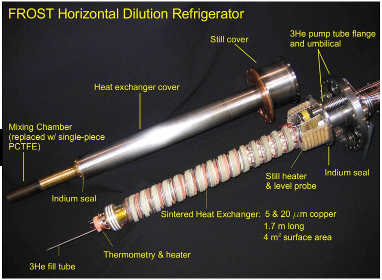

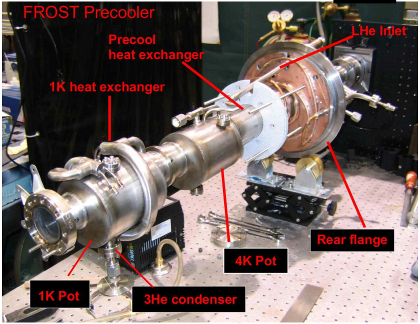

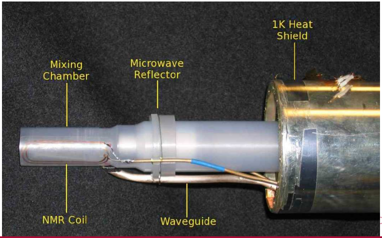

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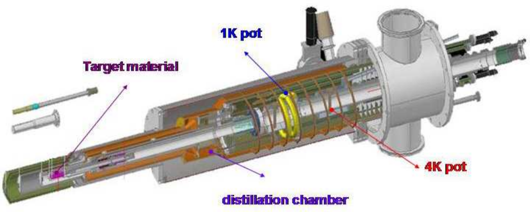
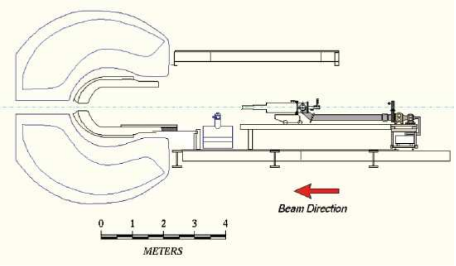


How to polarize the FROST?



The magnets in the FROST experiment

- (a) The longitudinal holding magnet. (About 0.5 T)
- (b) The transvere holding magnet. (g9b) (Charles Hanretty)
- (c) The polarizing magnet. (5 Tesla internal solenoid)

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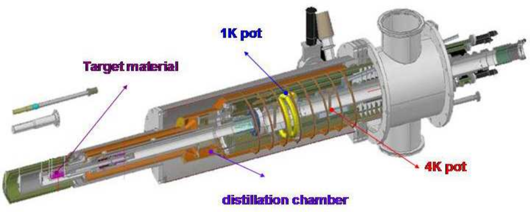

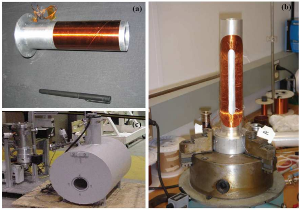

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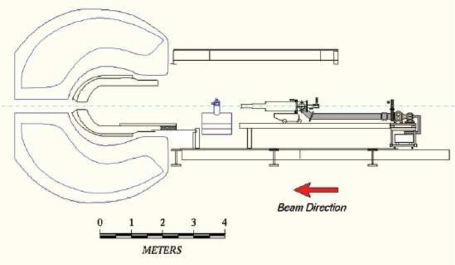

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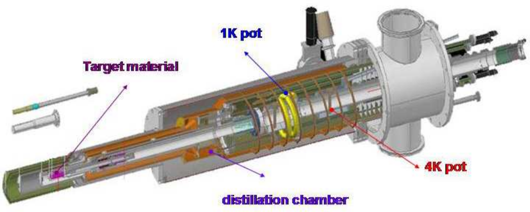

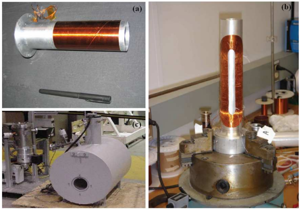

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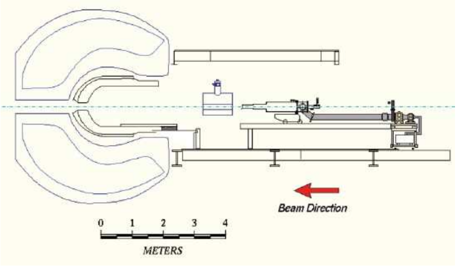

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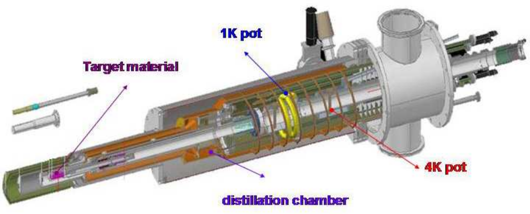

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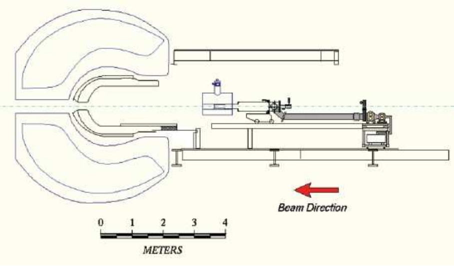

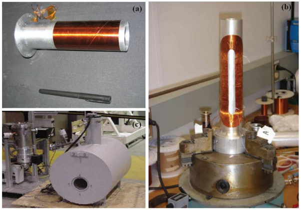

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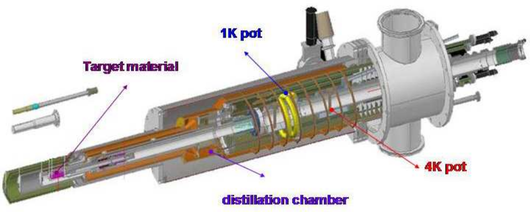
Polarizing Mode

- * Temperature 0.5 K
- * 5T magnet ON - (polarize the electrons)
- * Microwave ON - (transfer this polarization to nuclei)
- * Photon beam OFF

The Frozen-Spin Target (FROST)



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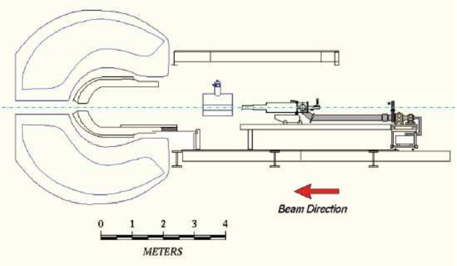

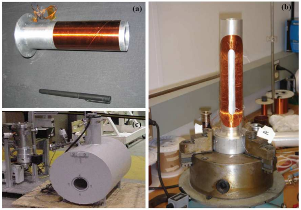

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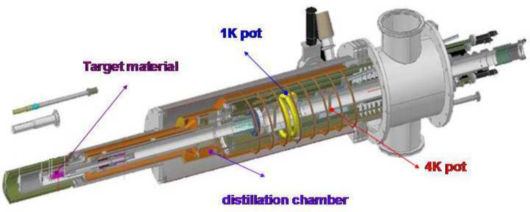
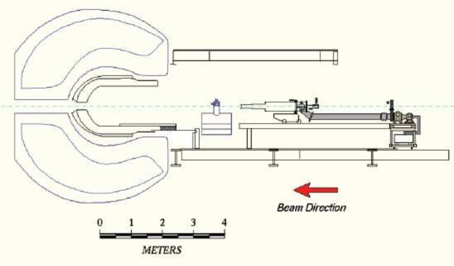
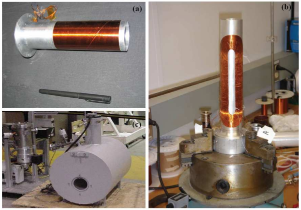


How to polarize the FROST?





The magnets in the FROST experiment

- (a) The longitudinal holding mqgnet. (About 0.5 T)
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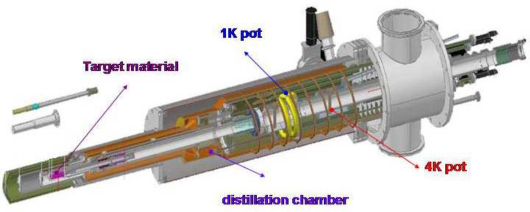

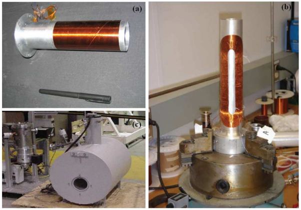

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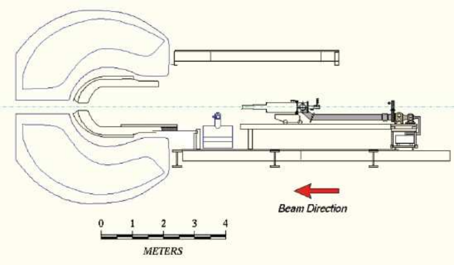

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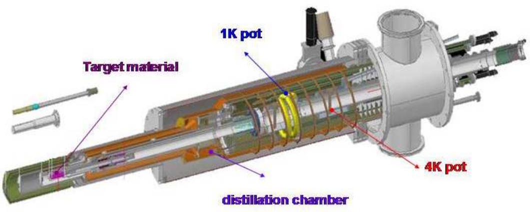

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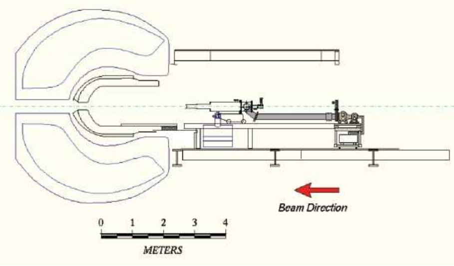

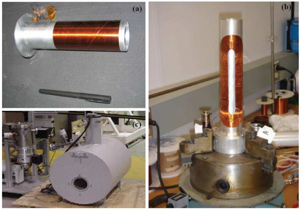

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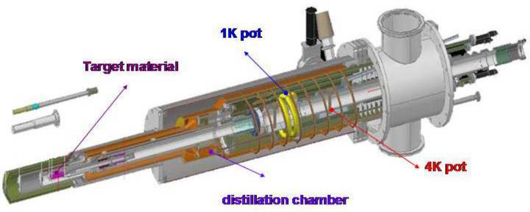

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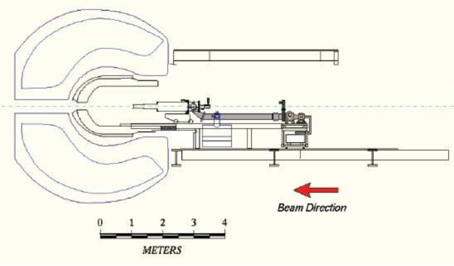

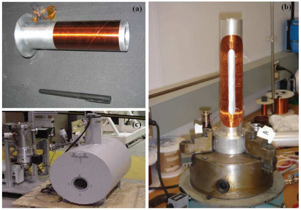

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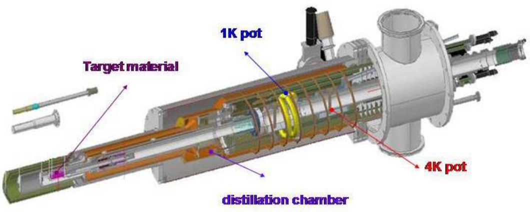

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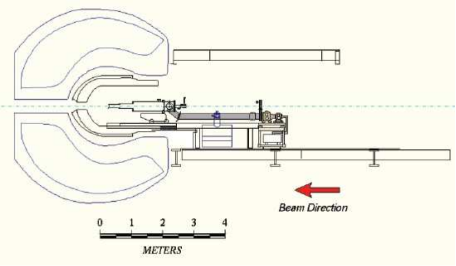

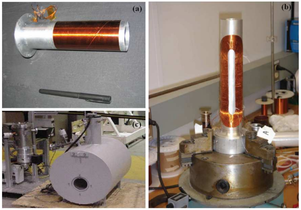

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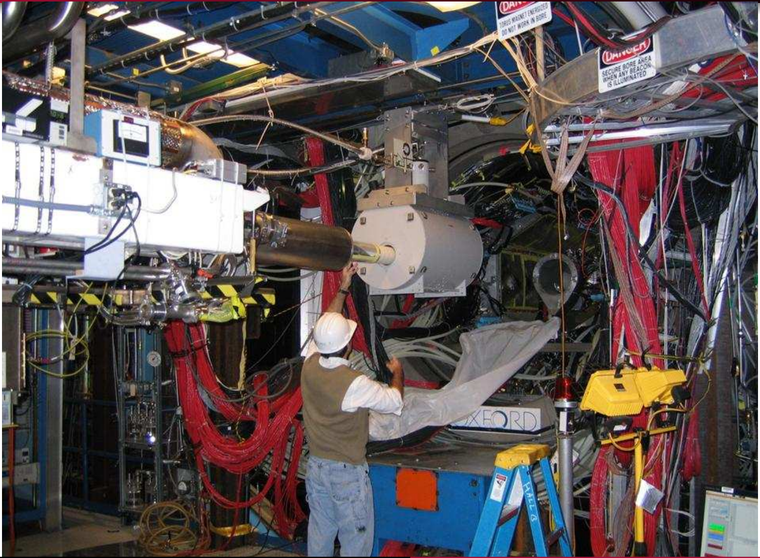
Frozen Spin Mode

- 5T magnet OFF
- Microwave OFF
- Temperature $\sim$ 0.05 K
- 0.5T magnet ON - (holding magnet)
- Photon beam ON

The Frozen-Spin Target (FROST) - polarizing mode



The Frozen-Spin Target - Summary of Results

	Expectation	Result
Base temperature:	50 mK	28 mK (w/o beam) 30 mK (w/ beam)
Cooling Power:	10 μW (Frozen) 20 mW (Polarizing)	800 μW @ 50mK 60mW @ 300 mK
Polarization:	80 %	+ 82 % - 85 %
1/e Relaxation Time:	500 hours	2700 hours (+ Pol.) 1600 hours (-Pol.)

The FROST-g9a run Data

The FROST run period: Nov. 3, 2007 - Feb. 12, 2008

Data set: 35 TBytes

Production Data

Target:

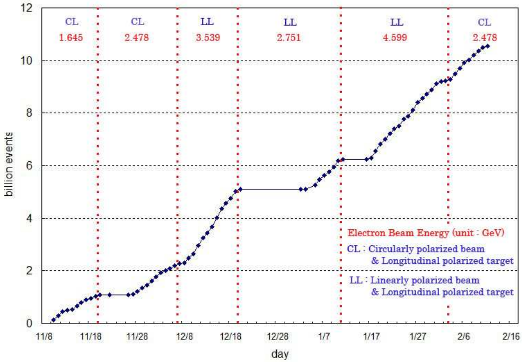
- Longitudinal polarized target
- Average target polarization
  ~ 82 % (+Pol) and 85 % (-Pol)

Photon beam:

- Circularly and linearly polarized photon beam
  0.5 - 4.5 GeV
- Electron beam polarization ~ 85 %

Trigger: - at least one charged particle in CLAS

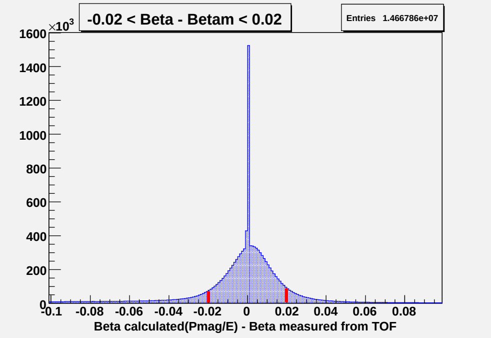
10.5 Billion events

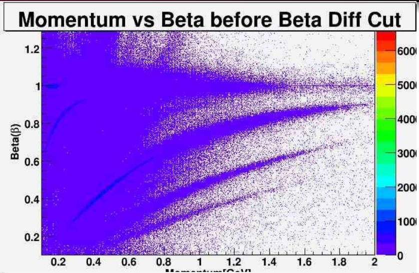


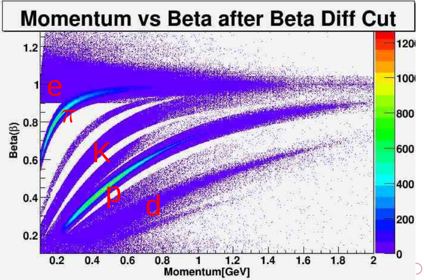
Outline

1. Introduction
   - Baryon Spectroscopy
   - Polarization Observable
2. FROST Experiment
   - The CLAS at JLab
   - The FRozen-Spin Target (FROST)
   - The FROST-g9a run Period
3. Event Selection
   - The particle identification
   - The dilution factor
   - The beam and target polarization
4. The Preliminary Results

The particle identification - The beta cut







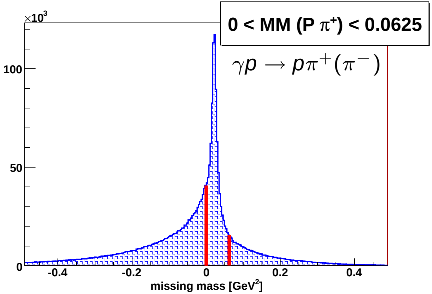
The beta cut

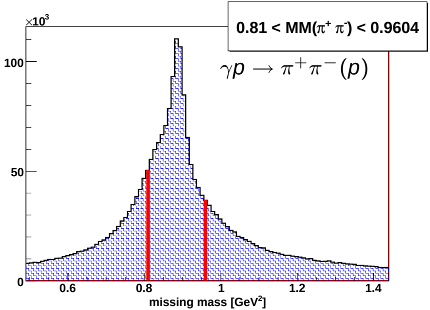
Calculated beta - Measured beta

$\left(\frac{momentum}{energy}\right)$ (TOF)

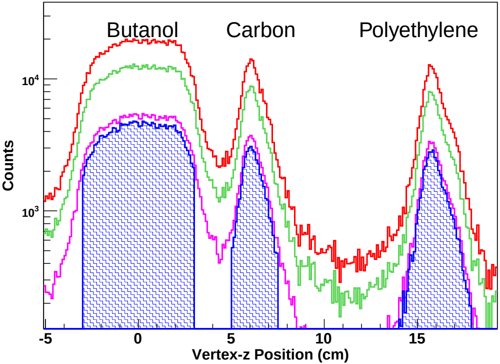
The 4 different topologies of $\gamma p \to p\pi^+\pi^-$

- ◇ The topology : $\gamma p \to p\pi^+(\pi^-)$
- ◇ The topology : $\gamma p \to p\pi^-(\pi^+)$
- ◇ The topology : $\gamma p \to \pi^+\pi^-(p)$
- ◇ The topology : $\gamma p \to p\pi^+\pi^-$





selecting the target



- The red line:

  the raw data
- The green line:

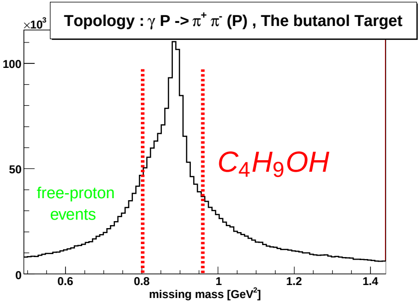
  the data after beta difference cut
- The pink line:

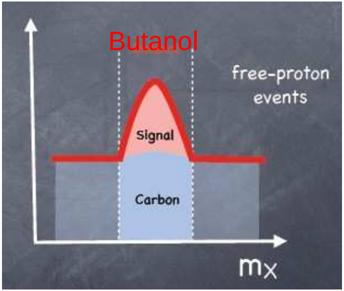
  the data included in $\pi^+ \pi^-$ photoproduction
- The blue part
  - the Z vertex difference cut
  - the three kinds of targets

The target	X and Y axis	Z axis
Butanol	radius 3	[-3.0,3.0]
Carbon		[5,7.5]
Polyethylene		[14,18]

- Polarized Butanol (C_4H_9OH) ($L = 5.0$ cm, $\phi = 1.5$ cm) ~ 5 g
- Carbon (^{12}C) ($L = 0.15$ cm) (6 cm from CLAS center)
- Polyethylene (CH_2) ($L = 0.35$ cm) (16 cm from CLAS center)

What is the dilution factor?

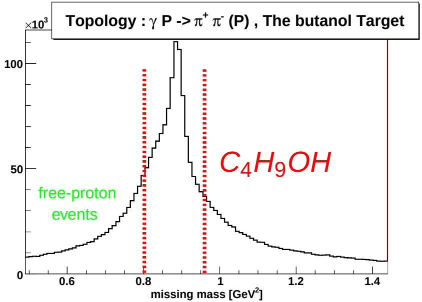


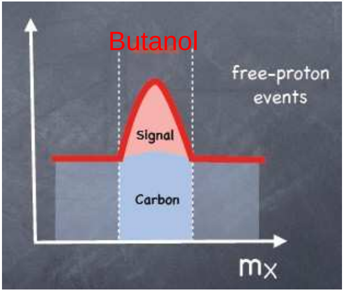


- The hydrongen atoms are polarized longitudinally in FROST experiment
- The butanol (C_4H_9OH) target has the unpolarized atoms like the carbon (C) or the oxyzen (O).

The dilution factor

What is the dilution factor?

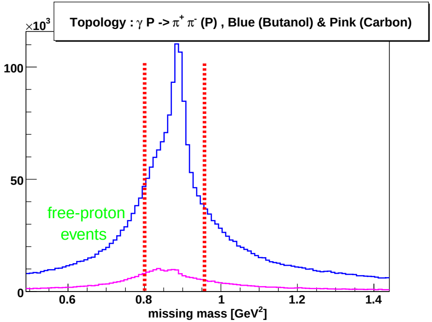


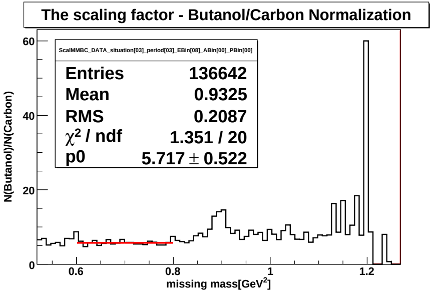


- The dilution factor is defined as the ratio between the polarized hydrogen and the full butanol contribution to the cross section

$$\text{The dilution factor} = \frac{\sigma_H}{\sigma_{C_4H9OH}} = \frac{N_{butanol} - S \cdot N_{carbon}}{N_{butanol}}$$ (where S : The scaling factor)

The scaling factor

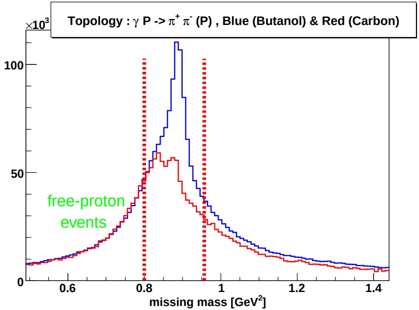


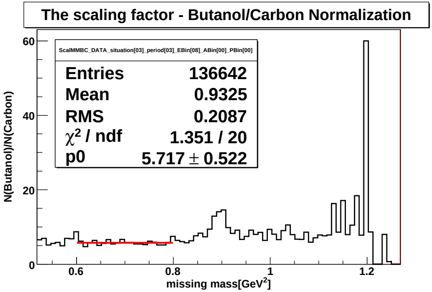


- ◇ The scaling factor normalizes the distribution of the two targets.
- ◇ comparing [0.6,0.8] of the two targets; the butanol and carbon.

$$\text{The scaling factor} = \frac{\textit{The blue part}}{\textit{The pink part}}$$

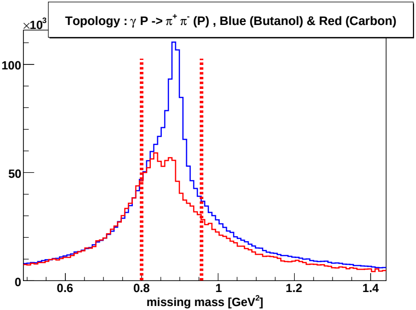
The scaling factor





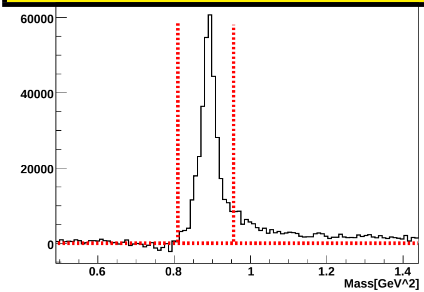
- ◇ The scaling factor normalizes the distribution of the two targets.
- ◇ comparing [0.6,0.8] of the two targets; the butanol and carbon.
- ◇ (The red plot) = (The pink plot) X (The scaling factor)

The dilution factor



- ◇ The blue - ($N_{butanol}$)
- ◇ The red - scaled (N_{carbon})
- ◇ The difference = The blue - The red ($N_{butanol}$) - scaled (N_{carbon}) = ($N_{hydrongen}$)

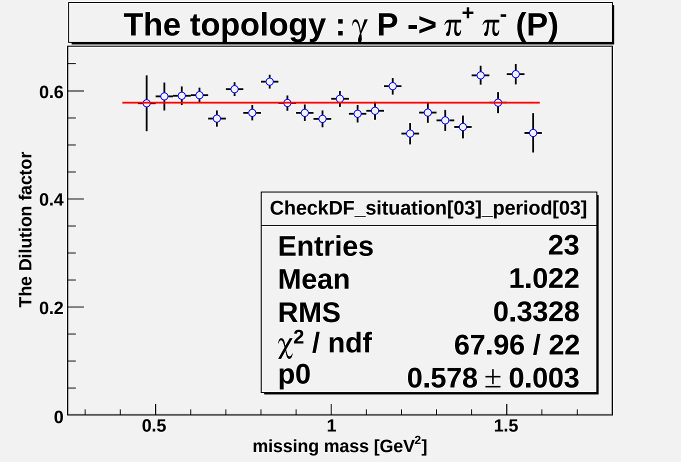
Topology : $\gamma\, p \;\longrightarrow\; \pi^+\, \pi^-\, (p)$ in the polarized hydrogen target



The dilution factor

$$\boxed{\frac{N_{hydrogen}}{N_{butanol}}}$$

The dilution factor

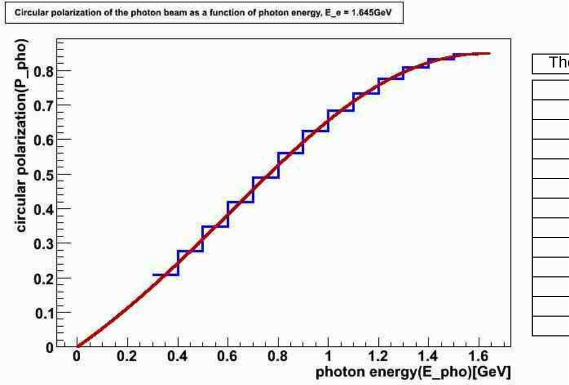


- The average dilution factor is 0.578

the beam and target polarization

- ◇ Target polarization, $\Lambda_z \sim 0.8$
- ◇ Electron beam polarization, $P_e \sim 0.85$

$$P_\gamma = P_e \cdot \frac{\left(\frac{4}{E_e}\right) E_\gamma - \left(\frac{4}{E_e}\right)^2 E_\gamma^2}{4 - \left(\frac{4}{E_e}\right) E_\gamma + 3\left(\frac{4}{E_e}\right)^2 E_\gamma^2}$$



The photon energy [GeV]	The photon polarization
[0.3,0.4]	0.209
[0.4,0.5]	0.277
[0.5,0.6]	0.348
[0.6,0.7]	0.419
[0.7,0.8]	0.490
[0.8,0.9]	0.559
[0.9,1.0]	0.624
[1.0,1.1]	0.683
[1.1,1.2]	0.734
[1.2,1.3]	0.777
[1.3,1.4]	0.810
[1.4,1.5]	0.833
[1.5,1.6]	0.846

Outline

Polarization Observable $\mathbf{P_z^{\odot}}$

$$\mathbf{P_z^{\odot}} = \frac{1}{f \cdot \delta_{\odot} \cdot \Lambda_z} \left\{ \frac{\Big(N(\rightarrow\Rightarrow)+N(\leftarrow\Rightarrow)\Big) - \Big(N(\rightarrow\Leftarrow)+N(\leftarrow\Leftarrow)\Big)}{\Big(N(\rightarrow\Rightarrow)+N(\leftarrow\Rightarrow)\Big) + \Big(N(\rightarrow\Leftarrow)+N(\leftarrow\Leftarrow)\Big)} \right\}$$

- f - dilution factor
- $\delta_{\odot}$ - beam polarization
- Λ_z - target polarization
- $N(\rightarrow\Rightarrow)$ - the number of events with the circular beam polarization and longitudinal target polarization

$\rightarrow$ and $\leftarrow$: circular polarization of the beam in its two possible settings

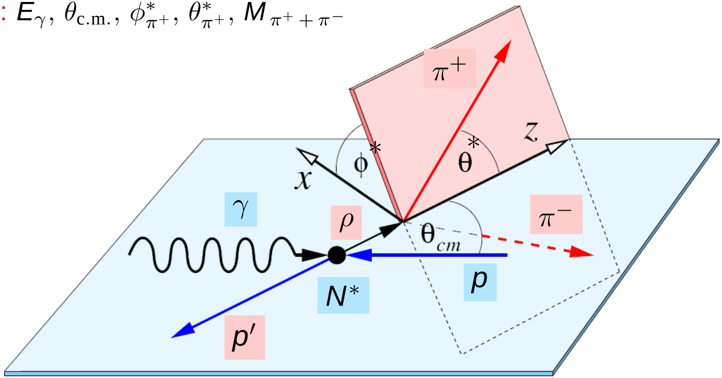
$\Rightarrow$ and $\Leftarrow$: longitudinal target polarization parallel or anti-parallel to the beam

Photoproduction of $\pi^+\pi^-$ off the Proton: Kinematics

The π^+ π^- in the final state require 5 independent variables!

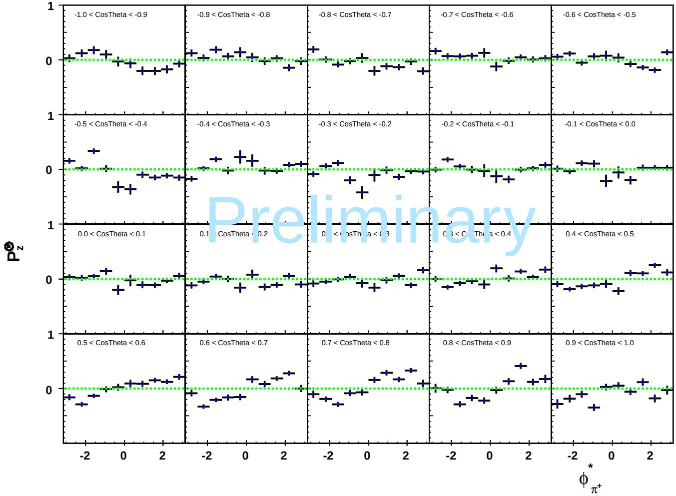
$\gamma p \rightarrow N^* \rightarrow p'\rho \rightarrow p'\pi^+\pi^-$

ex: $E_\gamma,\ \theta_{\mathrm{c.m.}},\ \phi^*_{\pi^+},\ \theta^*_{\pi^+},\ M_{\pi^+ + \pi^-}$



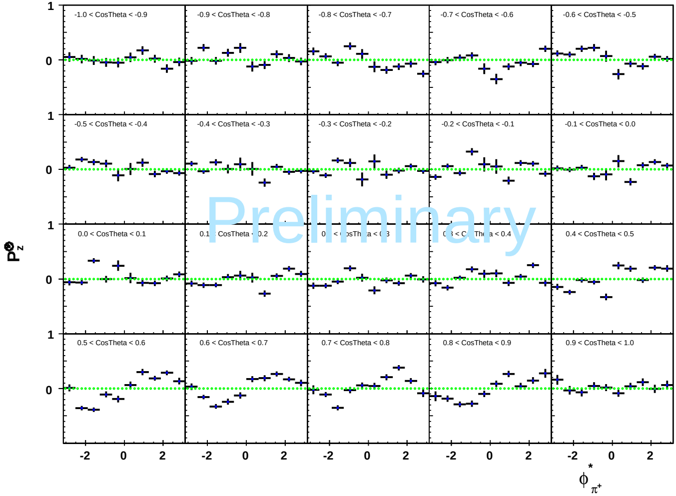
The asymmetry plot for $\mathbf{P_z^{\odot}}$

The topology $\gamma p \rightarrow \pi^+\pi^-(p)$ (Energy Bin 1100 MeV - 1200 MeV)



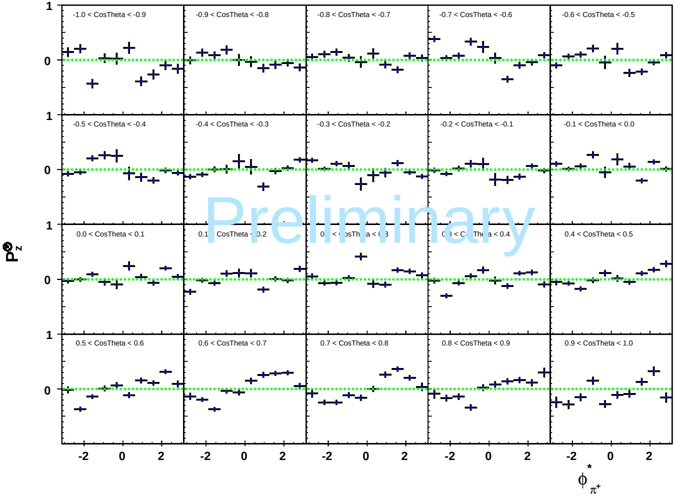
The asymmetry plot for $\mathbf{P_z}$ ⊙

The topology $\gamma p \rightarrow \pi^+\pi^-(p)$ (Energy Bin 1200 MeV - 1300 MeV)



The asymmetry plot for $\mathbf{P_z^{\odot}}$

The topology $\gamma p \rightarrow \pi^+\pi^-(p)$ (Energy Bin 1300 MeV - 1400 MeV)



Summary

- The first part of FROST with a longitudinally polarized target has been completed
- Preliminary results for $\mathbf{P_z}^{\odot}$ in $\pi^+\pi^-$ photoproduction
- The second part of FROST with a transversely polarized target already start from March 2010 to July 2010

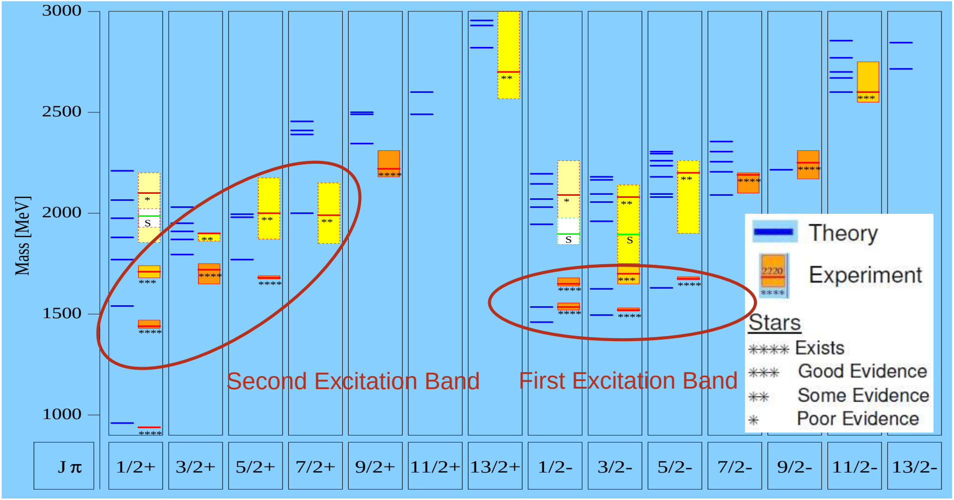
To do:

- Energy Loss Correction
- Momentum Correction
- Kinematic Fitting
- Normalization

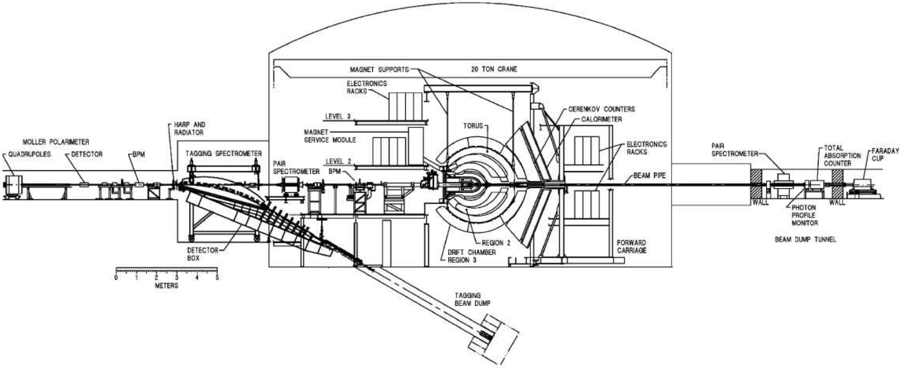
Thank you

The excited states of the nucleon

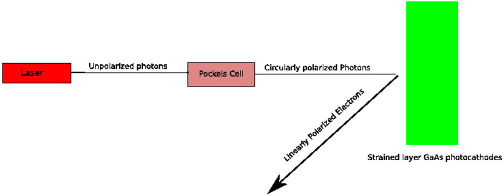
Constituent quark models: N^* resonances (Isospin $\frac{1}{2}$)

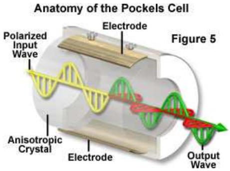


Side view of the CLAS spectrometer



Creating a Circular Polarized Photon Beam





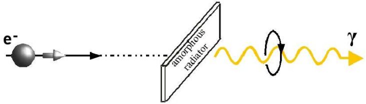
1. Laser → Unpolarized Photons
2. Pockels Cell → Circularly Polarized Photons
3. Electron Gun → Low Energy Linearly Polarized Electrons
4. Accelerator → High Energy Linearly Polarized Electrons
5. Møller Detector → Measures Degree of Polarization of Beamline at Radiator
6. Radiator → Circularly Polarized Photons

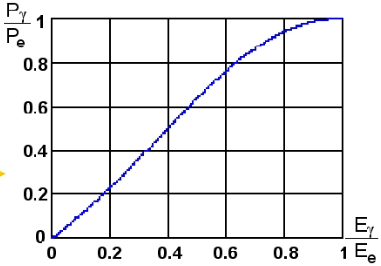
Creating a Circular Polarized Photon Beam

Bremsstrahlung Radiation is described by QED exactly

Helicity Transfer

$$P_\odot = P_{el}\,\frac{4z-z^2}{4-4z+3z^2} \quad \text{Where } z = \frac{E_\gamma}{E_{el}}$$

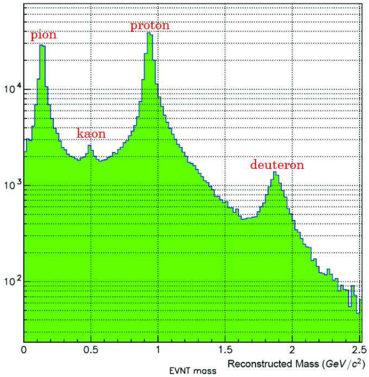




Crede, Volker. $\pi^0\eta$ *Helicity Difference CB-ELSA Proposal to PAC*, 2005

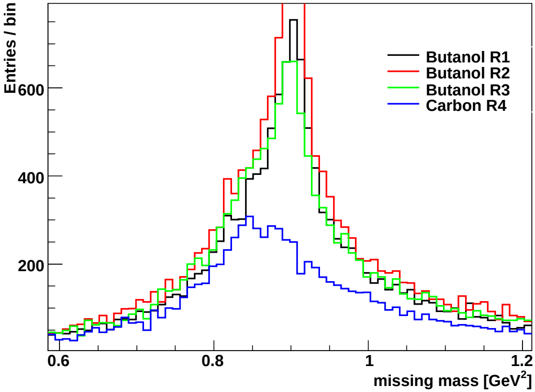
Kammer, Susanne. *Strahlpolarimetrie am CBELSA/TAPS Experiment*, DPG Meeting 2008

Particle identification



- Particle identification in CLAS relies on the combination of measured charged-particle momenta (from DC) and the flight time from the target to the respective TOF counters.

The contamination



- Butanol R1 [-3,-1]
- Butanol R2 [-1,+1]
- Butanol R3 [+1,+3]
- Carbon R4 [+5,7.5]

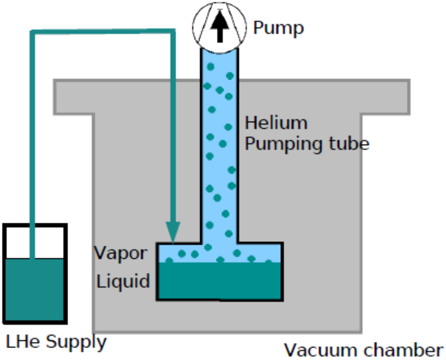
How do we make the low temperature?

Refrigeration below 4.2 K - Evaporative cooling

Liquid 4He boils at 4.2K under atmospheric conditions (3He at 3.1K).

Liquid Temperature can be lowered by reducing the vapor Pressure

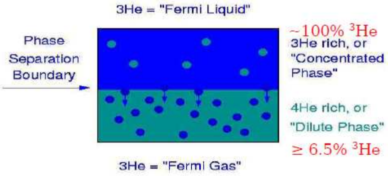
Practical Limit:
-- about 0.9K using 4He
-- about 0.3K using 3He



How do we make the low temperature?

3He/4He Dilution Refrigeration: The Basics

- below 0.8 K, a 3He/4He mixture will separate into two phases



Practical Limit:
-- about 0.005 K!

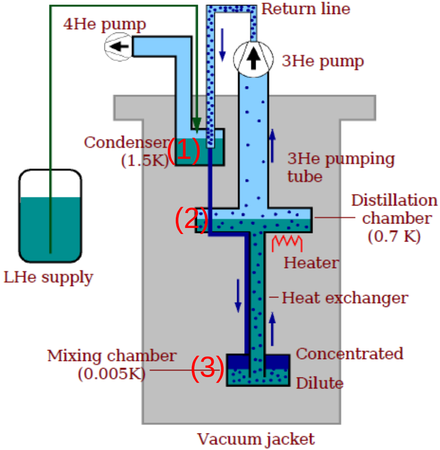
-The specific heat of a 3He atom is higher in the lower, dilute phase than in the upper, concentrated phase.

$$C_d > C_c$$

-Therefore, 3He will absorb energy when it dissolves into the dilution phase.

How do we make the low temperature?

Dilution Refrigeration



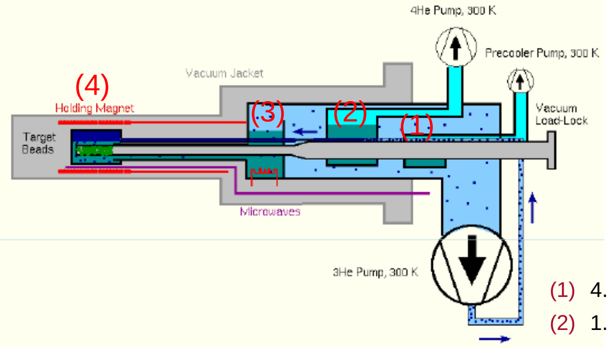
The cooling precess of 3He

(1) 1.5 K : in Condenser
(2) 0.7 K : in Distillation chamber
(3) 0.005 K : in mixing chamber

- Things near mixing chamber are cooled to around 0.005 K

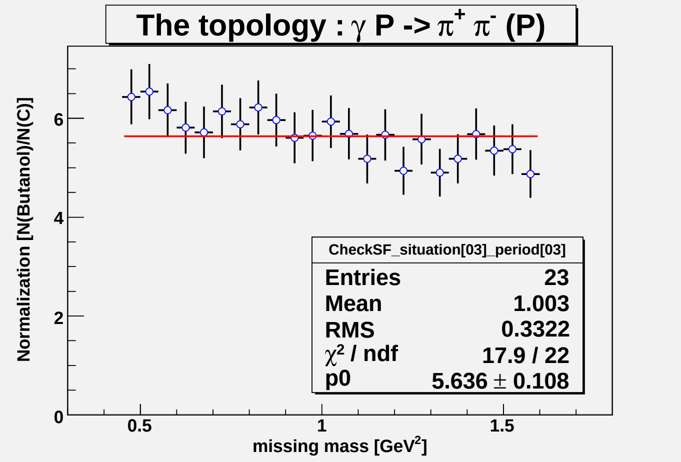
How do we make the low temperature?

Horizontal Dilution Refrigerator: T.O. Niinikoski, CERN 1971



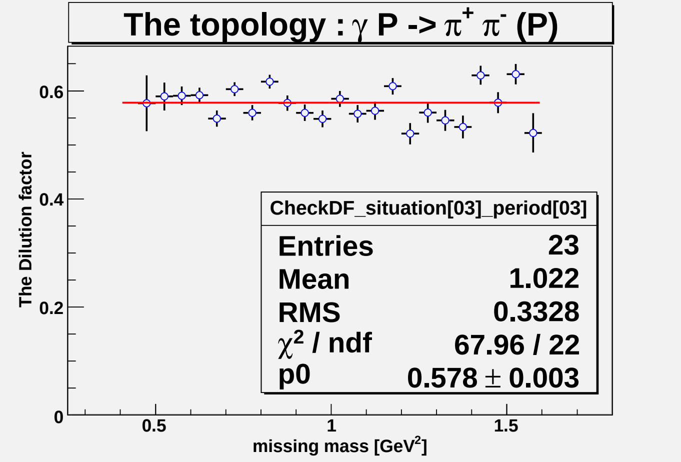
(1) 4.2 K : in 4K pot
(2) 1.5 K : in 1K pot
(3) 0.7 K : in distillation chamber
(4) 0.005 K : in target part

The Scaling factor

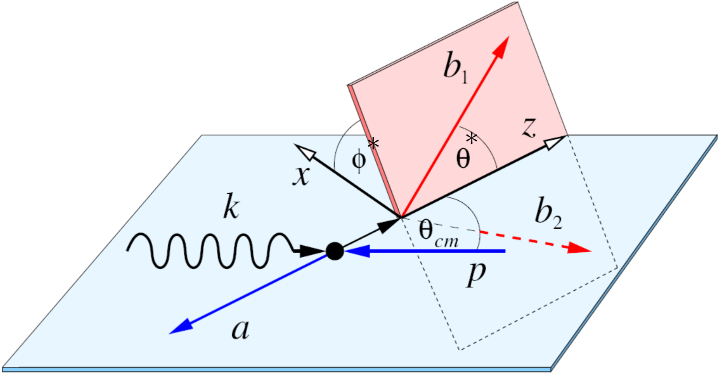


- The average scaling factor is 5.636

The dilution factor



- The average dilution factor is 0.578


b_1
z
θ^*
ϕ^*
x
k
b_2
θ_{cm}
p
a